CHAPTER 7
POST-TENSIONED CONCRETE GIRDERS

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CHAPTER 7
POST-TENSIONED CONCRETE GIRDERS

7.1 INTRODUCTION

7.1.1 General

Post-tensioned concrete box girders are widely used in the highway bridges in California. Figure 7.1-1 shows the San Luis Rey River Bridge – a typical cast-in-place post-tensioned (CIP/PT) concrete box girder bridge.

Figure 7.1-1 San Luis Rey River Bridge: A Concrete Box CIP/PT Bridge

Basic concepts, definitions and assumptions are first discussed in this Chapter. An example problem with "longhand" solution is then worked through to illustrate typical design procedure.

7.1.2 Basic Concepts

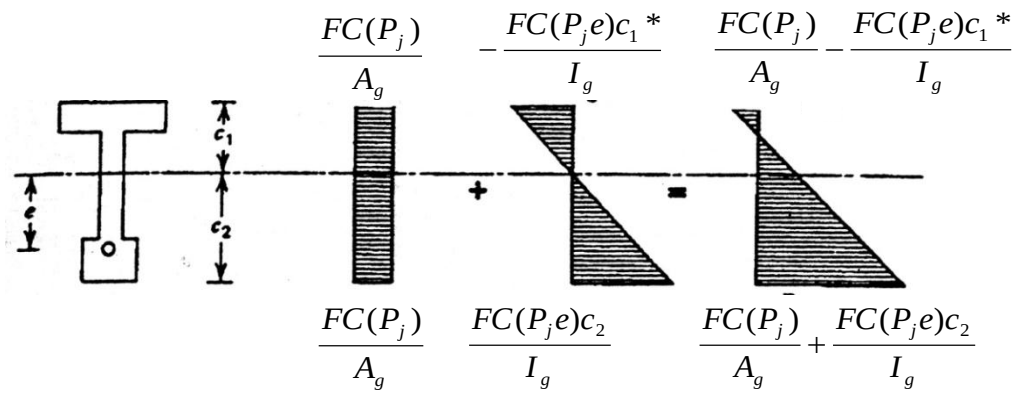
Post tensioning is one of methods of prestressing concrete. The concrete members are cast first. Then after the concrete has gained sufficient strength, tendons (strands of high strength steel wire) are inserted into preformed has ducts and tensioned to induce compressive stresses in the expected tensile stress regions of the member. Concrete must be free to shorten under the precompression. The strands are then anchored and a corrosion protection such as grout or grease, is installed (Gerwick, 1997).

Before further discussing prestressing, we should compare it with conventionally reinforced concrete. Prior to gravity loading, the stress level in conventional reinforced concrete is zero. The reinforcing steel is only activated by the placement of the gravity load. The concrete and reinforcing steel act as a composite section. However, once the tensile capacity of the concrete surrounding the longitudinal reinforcement has been surpassed, the concrete cracks. Prestressed concrete activates the steel prior to gravity loading through prestressing the reinforcement. This prevents cracking at service loads in prestressed concrete.

Prestressed concrete utilizes high strength materials effectively. Concrete is strong in compression, but weak in tension. High tensile strength of prestressing steel and high compressive strength of concrete can be utilized more efficiently by pre-tensioning high strength steel so that the concrete remains in compression under service loads activated while the surrounding concrete is compressed. The prestressing operation results in a self-equilibrating internal stress system which accomplishes tensile stress in the steel and compressive stress in the concrete that significantly improves the system response to induced service loads (Collins and Mitchell, 1997).

The primary objectives of using prestressing is to produce zero tension in the concrete under dead loads and to have service load stress less than the cracking strength of the concrete along the cross section. Thus the steel is in constant tension. Because of this the concrete remains in compression under service loads throughout the life of the structure. Both materials are being activated and used to their maximum efficiency.

Figure 7.1-2 shows elastic stress distribution for a prestressed beam after initial prestressing.



Note: *Component of Equation is negative because *c* is on opposite side of center of gravity from the tendon. Tension is denoted as negative (-), compression is denoted as positive (+)

Figure 7.1-2 Elastic Stresses in an Uncracked Prestress Beam. Effects of Initial Prestress by Component (Nilson, 1987)

The stress at any point of the cross-section can be expressed as:

$$f_{pe} = \frac{FC(P_j)}{A_g} \pm \left(\frac{FC(P_j e)y}{I_g} + \frac{MC_s(P_j)y}{I_g} \right) \tag{7.1-1}$$

Where:

A_g	=	gross area of section (in.^{2})
e	=	eccentricity of resultant of prestressing with respect to the centroid of the cross section. Always taken as a positive (ft)
FC	=	force coefficient for loss
f_{pe}	=	effective stress in the prestressing steel after losses (ksi)
I_g	=	moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})
P_j	=	force in prestress strands before losses (ksi)
MC_s	=	secondary moment force coefficient for loss (ft)
y	=	distance from the neutral axis to a point on member cross-section (in.)

The prestressing force effect is accomplished by two components of the general equation shown above as Equation 7.1-1. The first component is uniform compression stress due to the axial prestressing force. The second component is the bending stress caused by eccentricity of the prestressing steel with respect to the center of gravity of the cross section. This creates a linear change in stress throughout the beam cross

section (Figure 7.1-2). It is noted that the distance from the neutral axis to the fiber in question y, (y is the general term, c_1 and c_2 which are more specific terms shown in Figure 7.1-2), may result in a negative value for the bending part of the equation. It is possible the prestressing force will create tension across the center of gravity from the tendon, and therefore part of the beam section may be in tension prior to applying load.

The use of prestressed concrete has its advantages and limitations. Some limitations are its low superstructure ductility, the need for higher concrete compressive strengths, and larger member sizes to accommodate ducts inside the girders.

Post-tension box girder superstructures are commonly used due to their low costs, their performance throughout the life of the structure, and contractor's experience with the structure type. Post-tensioning also allows for thinner superstructures. A continuous superstructure increases the stiffness of the bridge frame in the longitudinal direction and gives the designer the option to fix the columns to the superstructure, reducing foundation costs.

7.2 MATERIAL PROPERTIES

At first glance, prestressed concrete and reinforced concrete make use of the same two core materials: concrete and steel. However, the behavior of the materials vary due to usage. Conventional concrete structures use deformed bars for reinforcement. Most prestressed applications use tightly wrapped, low-relaxation (lo-lax) seven-wire strand. As shown in Figure 7.2-1, the stress strain curves for those steel are quite different.

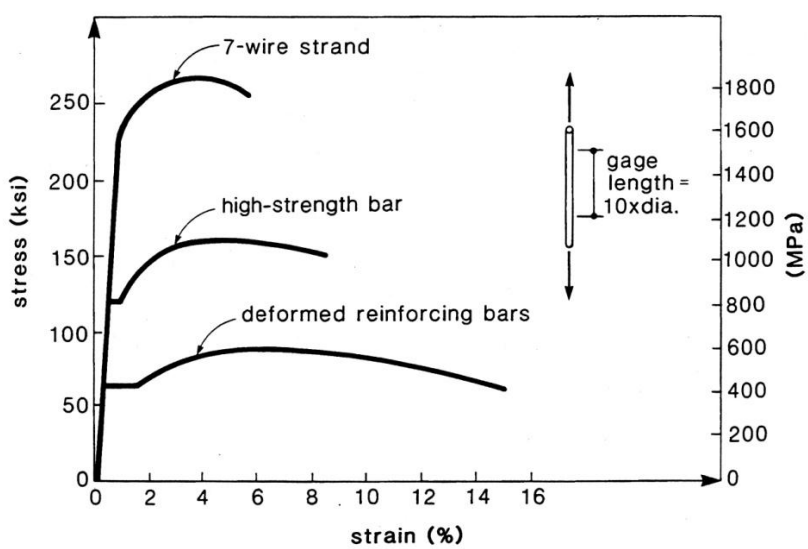


Figure 7.2-1 Stress-Strain Curves of Mild Steel (Deformed Reinforcing Bars) and Prestress Steel (7-Wire Strand) (Collins and Mitchell, 1997)

Table 7.2-1 shows the steel material properties for ASTM A706 Grade 60 and ASTM A416 PS Strand Grade 270. The mild reinforcement steel (ASTM A706 Grade 60) used for reinforced concrete has a much lower yield strength and tensile strength than the prestressing strands (ASTM A416 PS Strand Grade 270). Prestressing steel shall be high strength and possesses superior material properties. This enables a smaller quantity of steel to be used to support the bridge. Higher strength steel is also used because the ratio of effective prestess (prestress force after losses in force) to initial prestress (prestress force before losses in force) of high strength steel is much higher than that of mild steel (Figure 7.2-2). This is because losses, which will be discussed below, consume a large percentage of the strain in the elastic range of the mild steel, but a small portion of the prestressed steel.

Table 7.2-1 Steel Material Properties for Reinforced and Prestressed Concrete

	Grade 60 ASTM A706	Grade 270 ASTM A416 PS Strand
f_y	60 ksi	250 ksi
f_u	80 ksi	270 ksi

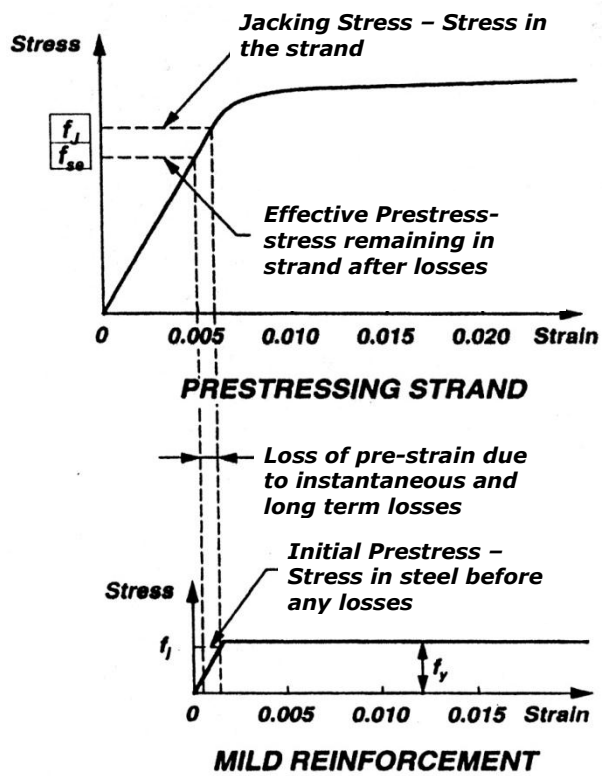


Figure 7.2-2 Loss Effects Comparison Between Prestressing Strand and Mild Reinforcement (Based inpart on Portland Cement Association 2001 Bridge Professors' Seminar Chicago, IL)

7.3 GIRDER LAYOUT AND STRUCTURAL SECTION

Section design is a very important tenant of structure design. An efficient section maximizes the ability of a structure to carry applied loads while minimizing self-weight. Basic mechanics of materials theory shows that the further away the majority of a material lies from the centroid of the shape, the better that shape is at resisting moment. A shape such as a basic "I" is perfect for maximizing flexural strength and minimizing weight. The placement of I-girders side by side results in a box; which is easier to construct and has seismic advantages over individual I-girders.

Determination of the typical section of a bridge has been made a simple process. Creation of a typical section begins with the calculation of structure depth for a given span length. Table 7.3-1 lists (AASHTO, 2012) minimum structural depth for various structural spans.

Table 7.3-1 Traditional Minimum Depth for Constant Depth Superstructures (AASHTO Table 2.5.2.6.3-1, 2012)

Superstructure		Minimum Depth (Including Deck) When variable depth members are uses, values may be adjusted to account for changes in relative stiffness of positive and negative moment sections	
Material	Type	Simple Spans	Continuous Spans
Reinforced Concrete	Slabs with main reinforcement parallel to traffic	$\frac{1.2\,(S+10)}{30}$	$\frac{S+10}{30} \geq 0.54$ ft
	T-Beams	$0.070L$	$0.065L$
	Box Beams	$0.060L$	$0.055L$
	Pedestrian Structure Beams	$0.035L$	$0.033L$
Prestressed Concrete	Slabs	$0.030L \geq 6.5$ in.	$0.027L \geq 6.5$ in.
	CIP Box Beams	$0.045L$	$0.040L$
	Precast I-Beams	$0.045L$	$0.040L$
	Pedestrian Structure Beams	$0.033L$	$0.030L$
	Adjacent Box Beams	$0.030L$	$0.025L$
Steel	Overall Depth of Composition I-Beam	$0.040L$	$0.032L$
	Depth of I-Beam Portion of Composite I-Beam	$0.033L$	$0.027L$
	Trusses	$0.100L$	$0.100L$

In order to use MTD 10-20 (Caltrans, 2008), a simple rule of thumb is girder spacing for a prestressed box girder should not exceed twice the superstructure depth. A larger girder spacing may require a customized deck and soffit slab design and may result in a larger web thickness. MTD 10-20 (Caltrans, 2008) provides predetermined soffit and deck thickness based on girder to girder spacing as well as overhang length.

7.4 PRESTRESSING CABLE LAYOUT

To induce compressive stress along all locations of the bridge girder, the prestressing cable path must be raised and lowered along the length of the girder. A typical continuous girder is subjected to negative moments near fixed supports, and positive moments near mid-span. As Equation 7.1-1 shows, eccentricity determines the stress level at a given location on the cross-section. In order to meet the tension face criteria the location of the prestressing cable path will be high (above the neutral axis) at fixed supports, low (below the neutral axis) at midspans, and at the centroid of the section near simply supported connections (Figure 7.4-1). The shape of the cable path is roughly the same as the opposite sign of the dead load moment diagram shown in Figure 7.4-2.

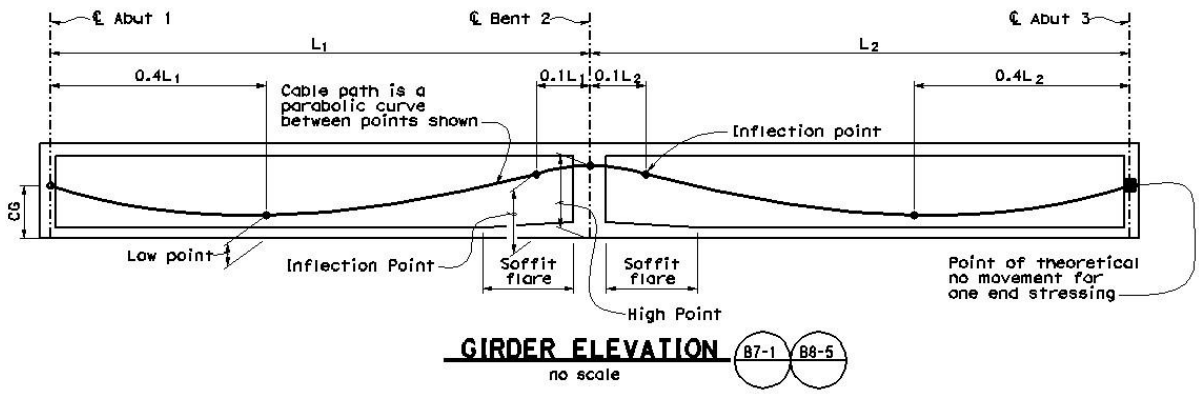


Figure 7.4-1 Typical Prestressing Path for a Two-Span Bridge

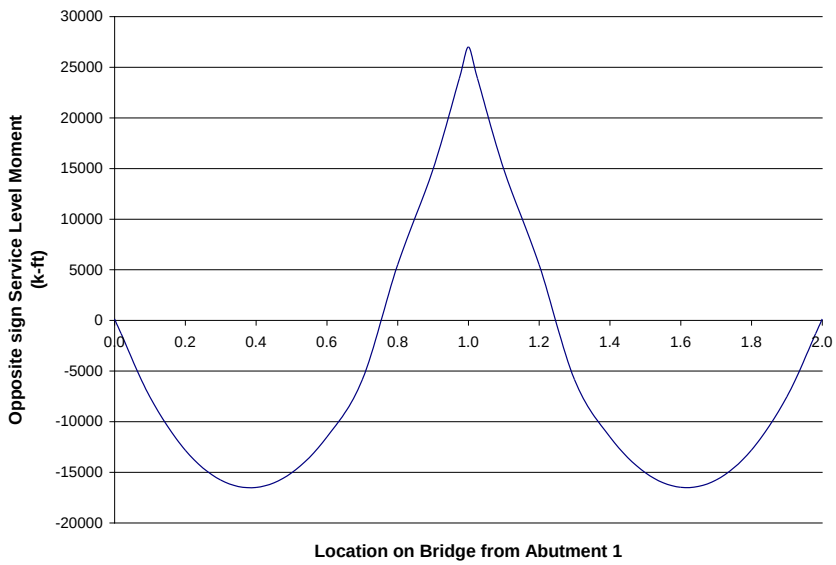


Figure 7.4-2 Opposite Sign of Dead Load Moment Diagram

Determining the most efficient possible final pre-stress cable layout is an iterative process. This requires using Bridge Design Aids (BDA) Sections 11-13 through 11-18 (Caltrans, 2005) to determine a best "guess" initial prestressing force (P_{jack}) as a function of deck area, span length, and span configuration. Using tables and charts provided in MTD 11-28 (Caltrans, 2010), the designer can then locate critical points along the cable path. The highest point should occur at the locations of highest negative moments, on our example bridge that would be at the bents. At its highest, the duct will fit in just below the bottom transverse mat of steel in the deck. The lowest point will occur near mid-span, and is limited by the location of the top mat of transverse soffit steel.

7.5 PRESTRESS LOSSES FOR POST-TENSIONING

Throughout the life of a prestressed concrete girder, the initial force applied to the pre-stress tendons significantly decreases. This decrease in the force is called loss. Loss of stress in a girder can revert a location previously in compression to tension, or increase a tension stress. This may be very dangerous when stress in concrete is near its stress limit. Because of the significant impact to the structure of these losses, losses must be quantified and accounted for in design.

In Caltrans practice, coefficients are usually used to estimate the reduction factor in initial force to find a final prestressing force. These coefficients are called force (Equation 7.5-1) and moment coefficients (Equation 7.5-2). Both coefficients are used to determine a jacking force, as most losses are both functions of and dependant of jacking force. These force coefficients are given as the sum of lost force of each component of loss divided by the allowable stress in the tendon (modification of AASHTO, 2012, Equations 5.9.5.1-1, 5.9.5.1-2).

$$FC_{pT} = (1 - \frac{\sum \Delta f_i}{f_{ps}}) \tag{7.5-1}$$

where:

FC_{pT} = force coefficient for loss

Δf_i = change in force in prestressing tendon due to an individual loss (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi) (5.7.3.1.1-1)

$$MC_P = (FC_{pT})(e_x) \tag{7.5-2}$$

where:

MC_p = primary moment force coefficient for loss (ft)

FC_{pT} = total force coefficient for loss

e_x = eccentricity as a function of x along parabolic segment (ft)

The force coefficient is defined as one at the jacking location and begins decreasing towards zero to the point of no movement. The point of no movement is a finite point of the strand that does not move when jacked and is defined as the location where internal strand forces are in equilibrium. For single-end post tensioning, the point of no movement is at the opposite anchorage from stressing. For two-end tensioning the location is where the movement in one direction is countered by movement from the other direction, and is generally near the middle of the frame.

Force coefficients are determined at each critical point along the girder. The product of the force coefficients and strand eccentricities (e) are called moment coefficients. The coefficients determined from the locked in moments at fixed supports

are used to convert initial strand moment resistant capacities into capacities after losses, or final capacities.

7.5.1 Instantaneous Losses

There are two types of losses: instantaneous and long term. The instantaneous losses are due to anchor set, friction, and elastic shortening. Instantaneous losses are bridge specific, yet still broad enough to be estimated in user friendly equations. Therefore a lump sum value is not used and a bridge specific value is calculated. Given below are three different types of instantaneous losses.

7.5.1.1 Anchor Set Loss

Anchor Set is caused by the movement of the tendon prior to seating of the anchorage gripping device. This loss occurs prior to force transfer between wedge (or jaws) and anchor block. Anchor set loss is the reduction in strand force through the loss in stretched length of the strand. Once a force is applied to the strands, the wedges move against the anchor block, until the wedges are "caught". Because of the elasticity of the strands, this movement will cause a loss in strain, stress, and force. This movement and the resulting loss of force prior to being "caught" is the anchor set loss. The force necessary to pull the movement out, will not be captured as the effective force. Even though the size of the slip is small, it still manifests as a reduction in prestressing force. AASHTO 5.9.5.2.1 suggests a common value for anchor set as 3/8 inch. This anchor set loss represents the amount of slip in Caltrans approved anchorage systems. Equation 7.5.1.1-1 puts the Anchor Set into a more familiar change in force and force coefficient form.

$$\Delta FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2(\Delta f_L)(x_{pA})}{L(f_{ps})} \qquad (7.5.1.1\text{-}1)$$

$$x_{pA} = \sqrt{\frac{E_p(\Delta_{Aset})L}{12\Delta f_L}} \qquad (7.5.1.1\text{-}2)$$

where:

FC_{pA}	=	force coefficient for loss from anchor set
x_{pA}	=	influence length of anchor set (ft)
E_p	=	modulus of elasticity of prestressing
Δ_{Aset}	=	anchor set length (in.)
L	=	distance to a point of known stress loss (ft)
Δf_L	=	friction loss at the point of known stress loss (ksi)
Δf_{pA}	=	jacking stress lost in the *P*/S steel due to anchor set (ksi)

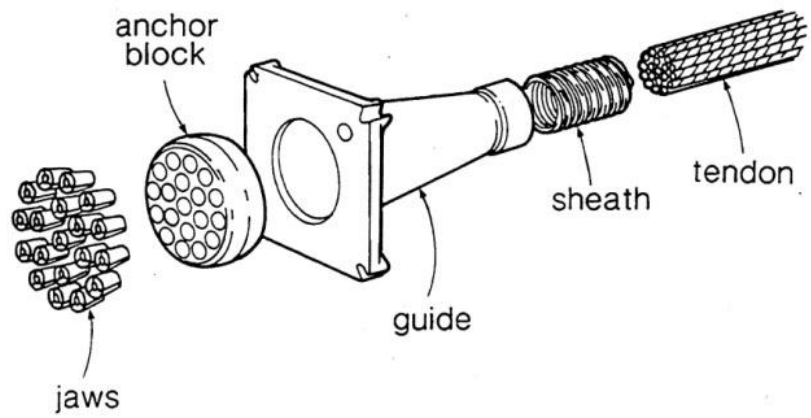


Figure 7.5.1-1 Anchorage System for Multi-Strand Tendon (Collins and Mitchell, 1997)

7.5.1.2 Friction Loss

Friction loss is another type of instantaneous loss, which occurs when the prestressing tendons get physically caught on the ducts. This is a significant loss of force on non-linear prestressing paths because of the angle change of the ducts. Friction loss has two components: curvature and wobble frictional losses. Modified Equation AASHTO 5.9.5.2.2b-1 results in Equation 7.5.1.2-1, the equation used to obtain friction losses.

Curvature loss occurs when some fraction of the jacking force is used to maneuver a tendon around an angle change in a duct. An example would be: as a tendon is bending around a duct inflection point near a pier or bent, the bottom of the tendon is touching (and scraping) the bottom of the duct. This scraping of the duct is loss of force via friction.

$$FC_{pF} = \frac{\Delta f_{pF}}{f_{pj}} = e^{-(Kx+\mu\alpha)} \qquad (7.5.1.2\text{-}1)$$

where:

e	=	e is the base of Napierian logarithms
FC_{pF}	=	force coefficient for loss from friction
f_{pj}	=	stress in the prestressing steel at jacking (ksi)
K	=	wobble friction coefficient (per ft of tendon)
x	=	general distance along tendon (ft)
α	=	total angular change of prestressing steel path from jacking end to a point under investigation (rad)
Δf_{pF}	=	change in stress due to friction loss
μ	=	coefficient of friction

Table 7.5-1 Provides Wobble Friction Coefficient and Coefficient of Friction as Specified in the California Amendments (Caltrans, 2014).

Table 7.5-1 Friction Coefficient *K* and Coefficient of Friction μ

Type of Steel	Type of Duct	*K* (1/ft)	μ
Wire or strand	Rigid and semi-rigid galvanized metal sheathing	~~0.0002~~	~~0.15-0.25~~
	Tendon Length:		
	< 600 ft	0.0002	0.15
	600 ft < 900 ft	0.0002	0.20
	900 ft < 1200 ft	0.0002	0.25
	> 1200 ft	0.0002	>0.25
	Polyethylene	0.0002	0.23
	Rigid steel pipe deviators for external tendons	0.0002	0.25
High-strength bars	Galvanized metal sheathing	0.0002	0.30

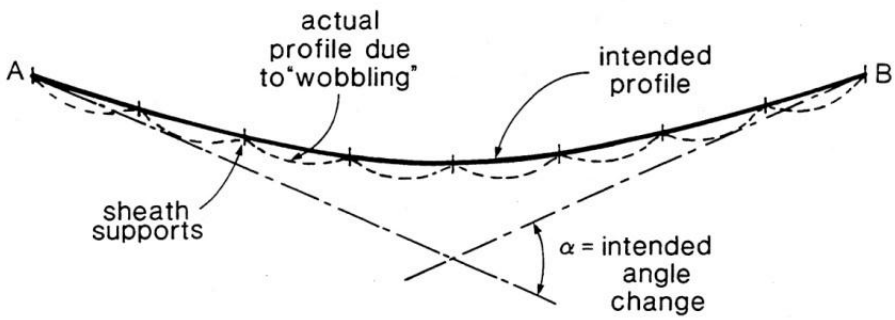


Figure 7.5.1-2 Wobble Friction Losses (Collins & Mitchell, 1997)

Wobble losses result from unintended angle changes of the tendon along the length of the cable path (Figure 7.5.1-2). These losses depend on the properties of the duct such as rigidity, diameter, support spacing, and type of duct. Wobble losses are the accumulation of the wobble coefficient over the length of the cable path.

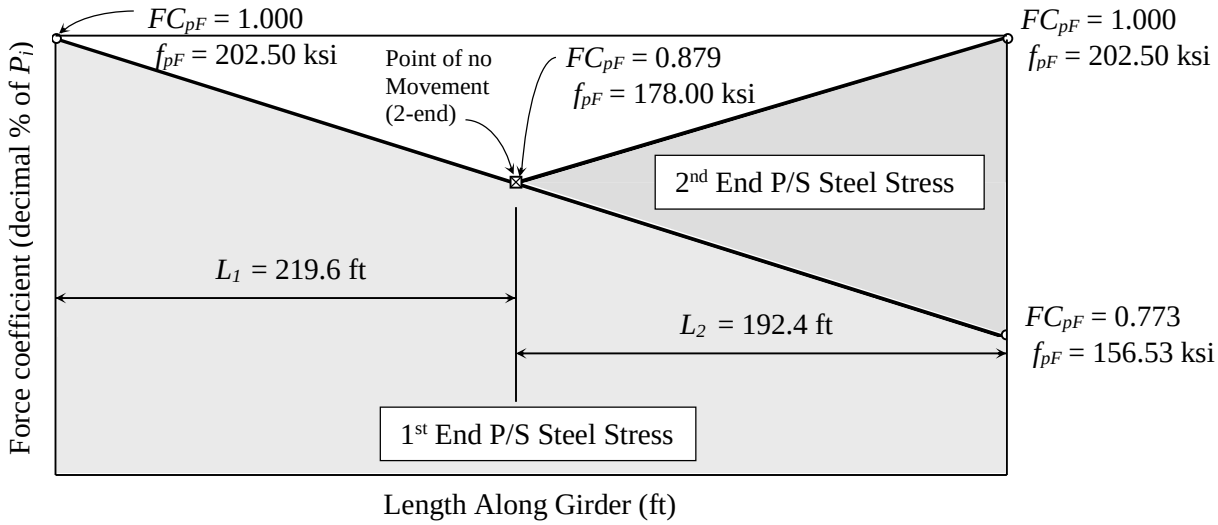


Figure 7.5.1-3 Two-End Stressing Versus One-End Stressing

Friction losses over a long girder begin to add up to a high percentage of the prestressing force. High friction losses can be counteracted by using two-end stressing. As stated above, two-end stressing moves the point of no movement from the anchored end to a point close to the middle of the frame. By jacking the end that was previously anchored, friction stress that was building up in the tendons between the point of no movement and the second end, is effectively pulled out thus reducing the friction losses between the second end and the point of no movement. Figure 7.5.1-3 shows the difference in stress when using two-end stressing instead of one for the design example in Section 7.12.

7.5.1.3 Elastic Shortening

When the pre-stressing force is applied to a concrete section, an elastic shortening of the concrete takes place simultaneously with the application of the pre-stressing force to the pre-stressing steel. It is caused by the compressive force from the tendons pulling both anchors of the concrete towards the center of the frame. Therefore, the distance between restraints has been decreased. Because of the elastic nature of the strand decreasing the distance between restraints after the pre-stressing force has been applied, thus reducing the strain, stress, and force levels in the tendons.

The equations for elastic shortening in pre-tensioned (such as precast elements) members are shown in AASHTO Equation 5.9.5.2.3a-1.

$$\Delta f_{pES} = \frac{E_p}{E_{ct}} f_{cgp} \qquad \text{(AASHTO 5.9.5.2.3a-1)}$$

The equations for elastic shortening in post-tensioned members other than slabs are shown in Equation AASHTO 5.9.5.2.3b-1.

$$\Delta f_{pES} = \frac{N-1}{2N}\frac{E_p}{E_{ct}} f_{cgp} \qquad \text{(AASHTO 5.9.5.2.3b-1)}$$

where:

E_{ct} = modulus of elasticity of concrete at transfer or time of load application (ksi)

E_p = modulus of elasticity of prestressing tendons (ksi)

N = number of identical prestressing tendons

f_{cgp} = concrete stress at the center of gravity of prestressing tendons, that results from the prestressing force at either transfer or jacking and the self-weight of the member at sections maximum moment (ksi)

Δf_{pES} = change in stress due to elastic shortening loss

The California Amendments to the AASHTO LRFD specify that as the number of tendons increase, the first fractional term converges to 1/2, and the formula is simplified as follows:

$$\Delta f_{pES} = 0.5\frac{E_p}{E_{ct}} f_{cgp} \qquad \text{(CA Amendments 5.9.5.2.3b-1)}$$

$$f_{cgp} = f_g + f_{ps} = \frac{M_{DL}e}{I_g} - (\frac{P_j}{A_g} + \frac{P_j e^2}{I_g}) \qquad \text{(7.5.1.3-1)}$$

where:

A_g = gross area of section (in.^{2})

e = eccentricity of the anchorage device or group of devices with respect to the centroid of the cross section. Always taken as a positive (in.)

f_g = stress in the member from dead load (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

I_g = moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

M_{DL} = dead load moment of structure (kip-in.)

P_j = force in prestress strands before losses (ksi)

Turning this into a force coefficient results in:

$$FC_{pES} = \frac{\Delta f_{pES}}{f_{pi}} = 0.5\frac{E_p}{E_{ct}}\frac{f_{cgp}}{f_{pi}}$$

where:

FC_{pES} = force coefficient for loss from elastic shortening

7.5.2 Long Term Loss

Long term, time-dependent losses are the losses of presstress force in the tendon during the life of the structure. When using long term losses on post-tensioned members it is acceptable to use a lump sum value (CA Amendments to AASHTO 5.9.5.3) in lieu of a detailed analysis. In the CA Amendments to AASHTO 2012, this value is 20 ksi. When completing a detailed analysis, long term losses are the combination of the following three losses.

7.5.2.1 Shrinkage of Concrete

The evaporation of free water in concrete causing the concrete to lose volume is a process known as shrinkage. The amount of shrinkage, and therefore the amount of loss caused by shrinkage, is dependent on the composition of the concrete and the curing process (Libby, 1990). Empirical equations for calculating shrinkage have been developed which rely on concrete strength, and relative humidity of the region where the bridge will be placed. Combining and modifying AASHTO 2012 Equations 5.9.5.3-1, 5.9.5.3-2, and 5.9.5.3-3 results in Equation 7.5.2.1-1.

$$\Delta f_{pSR} = 12.0(1.7 - .01H)\frac{5}{(1 + f'_{ci})} \qquad (7.5.2.1\text{-}1)$$

where:

Δf_{pSR} = change in stress due to shortening loss

H = average annual ambient mean relative humidity (percent)

f'_{ci} = specified compressive strength of concrete at time of initial loading or prestressing (ksi); nominal concrete strength at time of application of tendon force (ksi)

7.5.2.2 Creep

Creep is a phenomenon of gradual increase of deformation of concrete under sustained load. There are two types of creep, drying creep and basic creep. Drying creep is affected by moisture loss of the curing concrete and is similar to shrinkage, as it can be controlled by humidity during the curing process. Basic creep is the constant stress of the post-tensioning steel straining the concrete. Creep is determined by relative humidity at the bridge site, concrete strengths, gross area of concrete, area of

prestressing steel, and initial prestressing steel. Combining and modifying AASHTO 2012 Equations 5.9.5.3-1, 5.9.5.3-2, and 5.9.5.3-3 results in Equation 7.5.2.2-1.

$$\Delta f_{pCR} = 10.0 \frac{f_{pi} A_{ps}}{A_g} (1.7 - .01H) \frac{5}{(1 + f'_{ci})} \qquad (7.5.2.2\text{-}1)$$

where:

Δf_{pCR}	=	change in stress due to creep loss
f_{pi}	=	prestressing steel/stress immediately prior to transfer (ksi)
A_{ps}	=	area of prestressing steel (in.^{2})
A_g	=	gross area of section (in.^{2})
H	=	average annual ambient mean relative humidity (percent)
f'_{ci}	=	specified compressive strength of concrete at time of initial loading or prestressing (ksi); nominal concrete strength at time of application of tendon force (ksi)

7.5.2.3 Relaxation of Steel

The relaxation of steel is a phenomenon of gradual decrease of stress when the strain is held constant over time. As time goes by, force is decreasing in the elongated steel. Relaxation losses are dependent on how the steel was manufactured (Figure 7.5.2.3-1). The manufacturing processes used to create prestressing strands result in significant residual stresses in the strand.

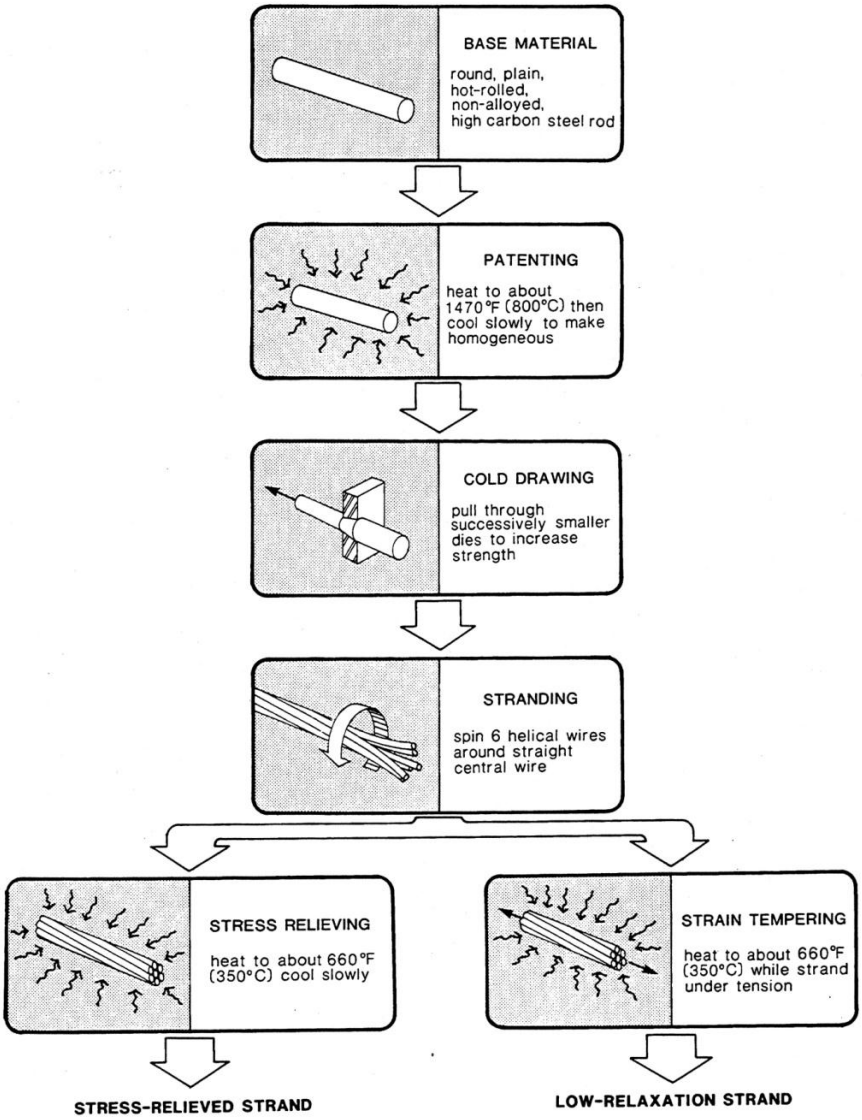


Figure 7.5.2.3-1 7-Wire Strand Production Method (Collins & Mitchell, 1997)

The steel can be manufactured to reduce relaxation as much as possible; this steel is called low relaxation (lo-lax). Lo-lax is generally the type of prestressing steel used in Caltrans post-tensioned girder bridges. A lo-lax strand goes through the production of high strength steel (patenting, cold drawing, stranding) and is then heated and cooled under tension. This process removes residual stresses and reduces the time dependant losses due to the relaxation of the strand. AASHTO 2012 allows for the use of lump-sum values. (AASHTO, 2012. 5.9.5.3-1). These are given as 2.4 ksi for lo-lax and 10.0 ksi for stress relieved steel.

7.6 SECONDARY MOMENTS AND RESULTING PRESTRESS LOSS

Another type of loss exists based on the frame configuration and support boundary conditions. A continuous prestressed flexural member which is free to deform (i.e. unrestrained by its supports), will deform axially and deflect from its original shape (Libby, 1990). If the prestress reactions are restrained by the supports, moments and shear forces are created as a result of the restraint. Distortions due to primary prestress moments generate fixed end moments at rigid supports. These fixed end moments are always positive, due to the geometry of the cable path, and always enhance the effects of prestressing. These locked in secondary moments decrease the effect of prestressing by lowering the effective prestressing force.

For statically indeterminate concrete flexural members, loss of prestress can be tabulated by using the moment distribution method; accounting for eccentricities, and curvature of tendons. Secondary moments vary linearly between supports.

7.7 STRESS LIMITATIONS

7.7.1 Prestressing Tendons

Tensile stress is limited to a portion of the ultimate strength to provide a margin of safety against tendon fracture or end-anchorage failures. Stress limits are also used to avoid inelastic tendon deformation, and to limit relaxation losses. Table 7.7.1-1 provides the stress limitations for prestresseing tendons as specified in the California Amendments (Caltrans, 2014). Those limitations can be increased if necessary in long bridges where losses are high.

Table 7.7.1-1 Stress Limitations for Prestressing Tendons CA Amendments Table CA5.9.3-1, (Caltrans, 2014)

Condition	Tendon Type		
	Stress-Relieved Strand and Plain High-Strength Bars	Low Relaxation Strand	Deformed High-Strength Bars
Pretensioning			
Prior to Seating : short-term f_{pbt} may be allowed	$0.90f_{py}$	$0.90\ f_{py}$	$0.90f_{py}$
Immediately prior to transfer (f_{pbt})	$0.70f_{pu}$	$0.70f_{pu}$	–
At service limit state after all losses (f_{pe})	$0.80f_{py}$	$0.80f_{py}$	$0.80f_{py}$
Post-tensioning			
~~Prior to Seating short term f_{pbt} may be allowed~~	~~$0.90f_{py}$~~	~~$0.90f_{py}$~~	~~$0.90f_{py}$~~
Maximum Jacking Stress: short-term f_{pbt} may be allowed	$0.75f_{pu}$	$0.75f_{pu}$	$0.75f_{pu}$
At anchorages and couplers immediately after anchor set	$0.70f_{pu}$	$0.70f_{pu}$	$0.70f_{pu}$
Elsewhere along length of member away from anchorages and complers immediately after anchor set	$0.70f_{pu}$	$0.70f_{pu}$	$0.70f_{pu}$
At service limit state after losses (f_{pe})	$0.80f_{py}$	$0.80f_{py}$	$0.80f_{py}$

7.7.2 Concrete

Stress in concrete varies at discrete stages within the life of an element. These discreet stages vary based on how the element is loaded, and how much pre-stress loss the element has experienced. The stages to be examined are the Initial Stage: Temporary Stresses Before Losses, and the Final Stage: Service Limit State after Losses, as defined by AASHTO 2012. The prestress force is designed as the minimum force required to meet the stress limitations in the concrete as specified in AASHTO Article 5.9.4 (AASHTO, 2012).

During the time period right after stressing, the concrete in tension is especially susceptible to cracking. This is before losses occur when prestress force is the highest and the concrete is still young. At this point the concrete has not completely gained strength. Caltrans project plans should show an initial strength of concrete that must be met before the stressing operation can begin. This is done to indicate a strength required to resist the post tensioning during the concrete's vulnerable state. During this initial temporary state, the Table 7.7.2-1 (AASHTO, 2012) allows for a higher tensile stress limit and the concrete is allowed to crack. The concrete is allowed to crack because as the losses reduce the high tension stress and the young concrete strengthens, the crack widths will reduce. Then the high axial force from prestressing will pull the cracks closed.

Table 7.7.2-1 Temporary Tensile Stress Limits in Prestressed Concrete Before Losses in Non-Segmental Bridges (AASHTO Table 5.9.4.1.2-1, 2012)

Location	Stress Limit
• In precompressed tensile zone without bonded reinforcement	N/A
• In area other than the precompressed tensile zone and without bonded reinforcement	$0.0948\sqrt{f'_{ci}} \leq 0.2$(ksi)
• In areas with bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of 0.5 f_y, not to exceed 30 ksi.	$0.24\sqrt{f'_{ci}}$ (ksi)
• For handling stresses in prestressed piles	$0.158\sqrt{f'_{ci}}$ (ksi)

The final stage of a bridge's lifespan is known as the "in place" condition. Stresses resisted by concrete and prestressing steel in this condition are from gravity loads. At the service limit, the bridge superstructure concrete should not crack. The code provides for this by setting the stress limit (Table 7.7.2-2) to be less than the tensile strength of the concrete. Under permanent loads tension is not allowed in any concrete fiber (Caltrans, 2014).

Table 7.7.2-2 Tensile Stress Limits on Prestressed Concrete at Service Limit State, After Losses, Fully Prestressed Components (CA Amendements Table 5.9.4.2.2-1, 2014)

Location	Stress Limit
Precompresssed Tensile Zone Bridges, Assuming Uncracked Sections—components with bonded prestressing tendons or reinforcement, subjected to permanent loads only.	No tension
Tension in the Precompressed Tensile Zone Bridges, Assuming Uncracked Sections	
• For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions, and/or are located in Caltrans' Environmental Areas I or II.	$0.19\sqrt{f'_c}$ (ksi)
• For components with bonded prestressing tendons or reinforcement that are subjected to severe corrosive conditions, and/or are located in Caltrans' Environmental Area III.	$0.0948\sqrt{f'_c}$ (ksi)
• For components with unbonded prestressing tendons.	No tension

Note that Caltrans' Environmental Areas I and II correspond to Non-Freeze Thaw Area, and Environmental Area III corresponds to Freeze Thaw Area, respectively.

Because concrete is strong in compression, the code-defined compression limits are much higher than corresponding tension limits. Compression limits as shown in Table 7.7.2-3 are in place to prevent the concrete from crushing. The code establishes a portion of the concrete strength to resist both gravity loads and compression from the prestressing tendons. Because compression limits are dependent on stage, like tension limits, careful consideration must not only be taken for force level in the steel but for the loading conditions as well.

Table 7.7.2-3 Compressive Stress Limits in Prestressed Concrete at Service Limit State, Fully Prestressed Components (AASHTO Table 5.9.4.2.1-1, 2012)

Location	Stress Limit
• In other than segmentally constructed bridges due to the sum of effective prestress and permanent loads	0.45 f_c' (ksi)
• In segmentally constructed bridges due to the sum of effective prestress and permanent loads	0.45 f_c' (ksi)
• Due to the sum of effective prestress, permanent loads, and transient loads and during shipping and hanling	0.60 ϕ_w f_c' (ksi)

7.8 STRENGTH DESIGN

Prestressing force and concrete strength are usually determined for the Service Limit States, while mild steel are determined for Strength Limit States. Flexural and shear design are discussed in detail in Chapter 6.

7.9 DEFLECTION AND CAMBER

Deflection is a term that is used to describe the degree to which a structural element is displaced under a load. The California Amendments to the AASHTO 2012 code defines camber as the deflection built into a member, other than prestressing, in order to achieve a desired grade. Camber is the physical manifestation of removing deflection from a bridge by building that deformation into the initial shape. This is done in the long term to give the superstructure a straight appearance, which is more pleasing to the public and also for drainage purposes.

There are two types of deflections. Instantaneous deflections consider the appropriate combinations of dead load, live load, prestressing forces, erection loads, as well as instantaneous prestress losses. All deflections are based on the stiffness of the structure versus the stiffness of the supports. This stiffness is a function of the Moment of Inertia (I_g or I_e) and Modulus of Elasticity (E). AASHTO 2012 (Equation 5.7.3.6.2-1) has developed an equation to define I_e based on I_g. With I_e obtained an instantaneous

deflection can be determined by using a method such as virtual work, or a design software such as CT-BRIDGE.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \qquad \text{(AASHTO 5.7.3.6.2-1)}$$

where:

I_e = effective moment of inertia (in.^{4})

M_{cr} = cracking moment (kip-in.)

M_a = maximum moment in a member at the stage which the deformation is computed (kip-in.)

I_g = moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

I_{cr} = moment of inertia of the cracked section, transformed to concrete (in.^{4})

$$M_{cr} = f_y \frac{I_g}{y_t} \qquad \text{(AASHTO 5.7.3.6.2-2)}$$

where:

f_y = yield strength of mild steel (ksi)

y_t = distance from the neutral axis to the extreme tension fiber (in.)

The primary function of calculating long-term deflections is to provide a camber value. Permanent loads will deflect the superstructure down, and give the bridge a sagging appearance. Prestressing force and eccentricity cause upward camber in superstructures. When calculating long-term deflections of a bridge, creep, shrinkage, and relaxation of the steel must be considered. This is done by multiplying the instantaneous deflection (deflection caused by DC and PS case) by code defined factor (CA 5.7.3.6.2). This product of instantaneous deflection and long term coefficient is the long term deflection of the bridge. The opposite sign of these deflections is what is placed on the project plans of new structures as camber. Generally, negative cambers (upward deflection) is ignored.

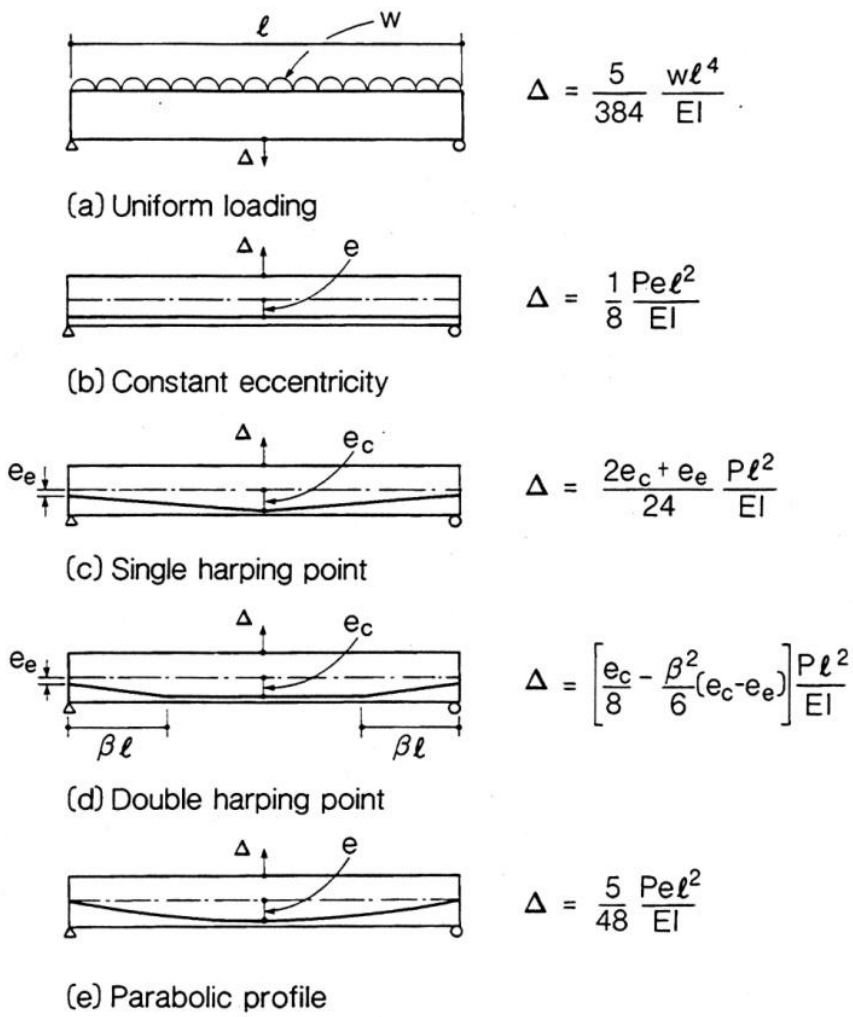
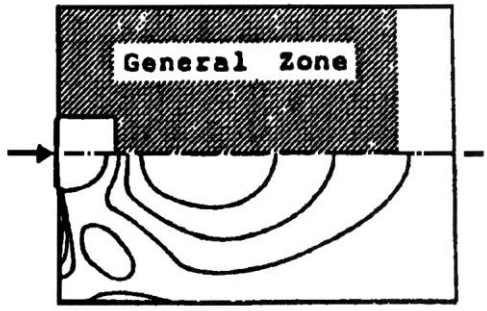


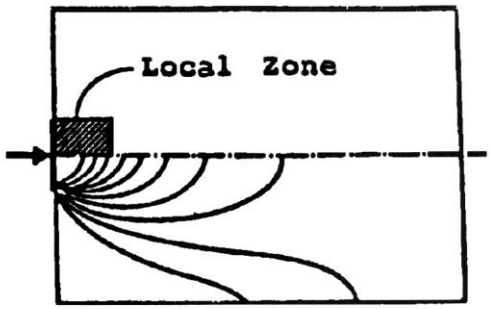
Figure 7.9-1 Expressions for Deflections Due to Uniform Load and Camber for Hand Checks (Collins and Mitchell, 1997)

7.10 POST-TENSIONING ANCHOR DESIGN

The abrupt termination of high force strands within the girder generates a large stress ahead of the anchorage. Immediately ahead of the girder are bursting stresses, and surrounding the anchorages are spalling stresses. The code specifies that for ease of design, the anchorage zone shall consist of two zones (Figure 7.10-1). One a "Local Zone" consisting of high compression stresses which lead to spalling. Also there is a General Zone consisting of high tensile stresses which lead to bursting (AASHTO, Equation 5.10.9.2.1).



a) Principal Tensile Stresses and the General Zone



b) Principal Compressive Stresses and the Local Zone

Figure 7.10-1 General and Local Zone (AASHTO, 2012)

The local zone of the anchorage system is dependent on the nearby crushing demand. Compression reinforcement is used within the local zone to keep concrete from spalling and eventually crushing. The local zone is more influenced by the characteristics of the anchorage device and its anchorage characteristics than by loading and geometry. Anchorage reinforcement is usually designed by the prestressing contractor and reviewed/approved by the design engineer during the shop drawing process.

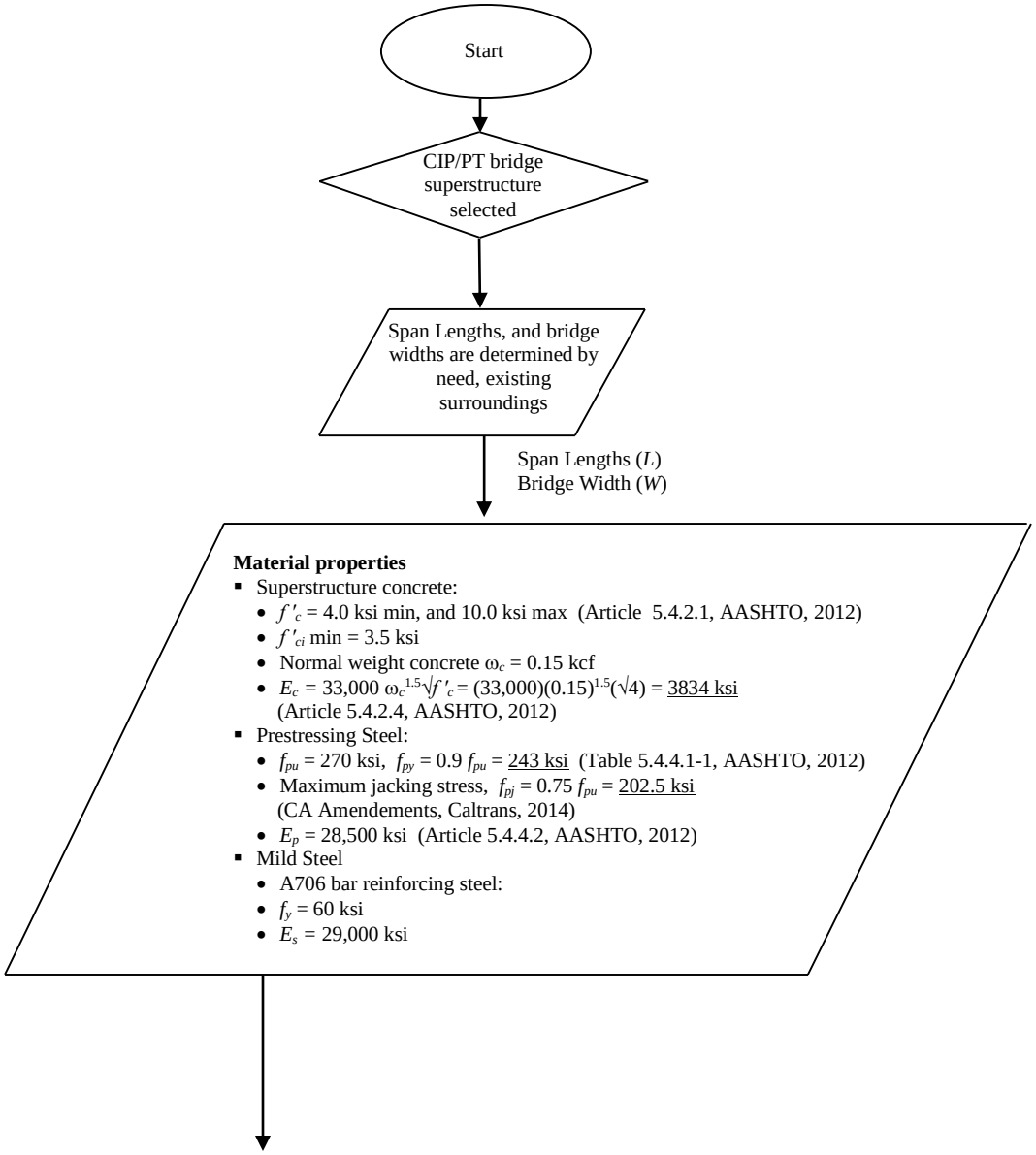
The general zone is defined by tensile stresses due to spreading of the tendon force into the structure. These areas of large tension stresses occur just ahead of the anchorage and slowly dissipate from there. Tension reinforcement is used in the

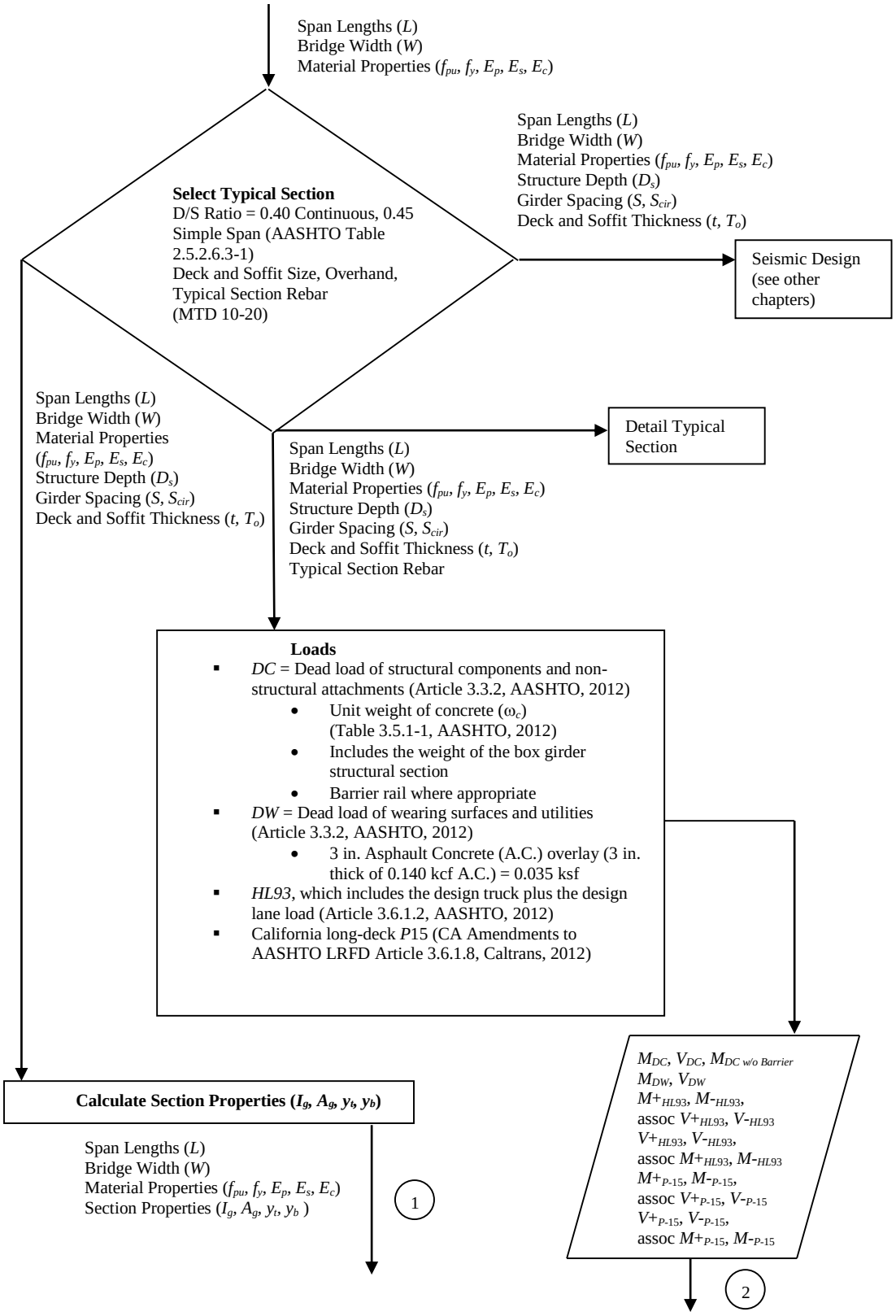
general zone as a means to manage cracking and bursting. The specifications permit the general anchorage zone to be designed using:

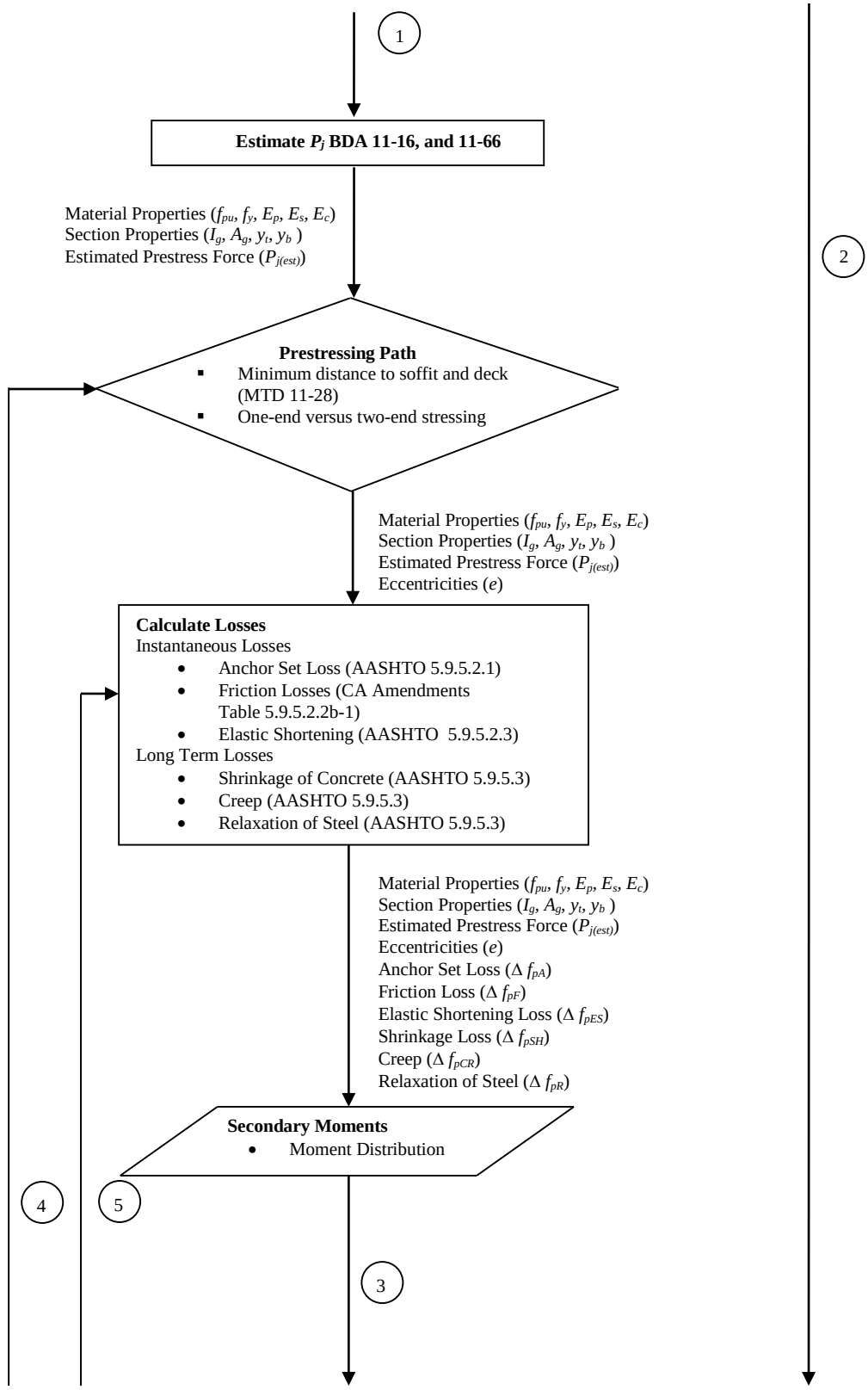
- the Finite Element Method
- the Approximate Method contained in the specifications
- the Strut and Tie method, which is the preferred method

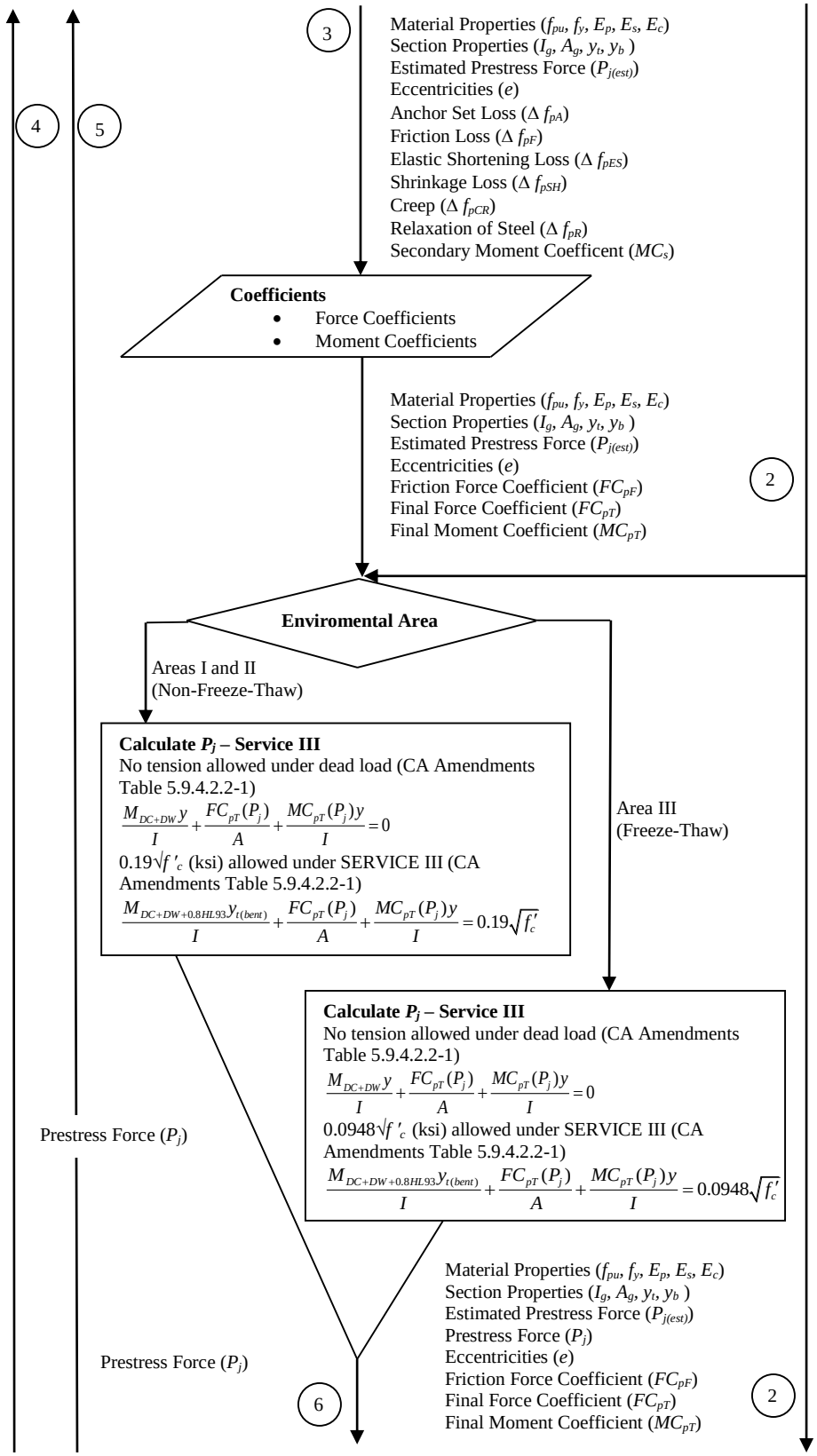
The result of this design yields additional stirrups and transverse bars into and near the end diaphragm.

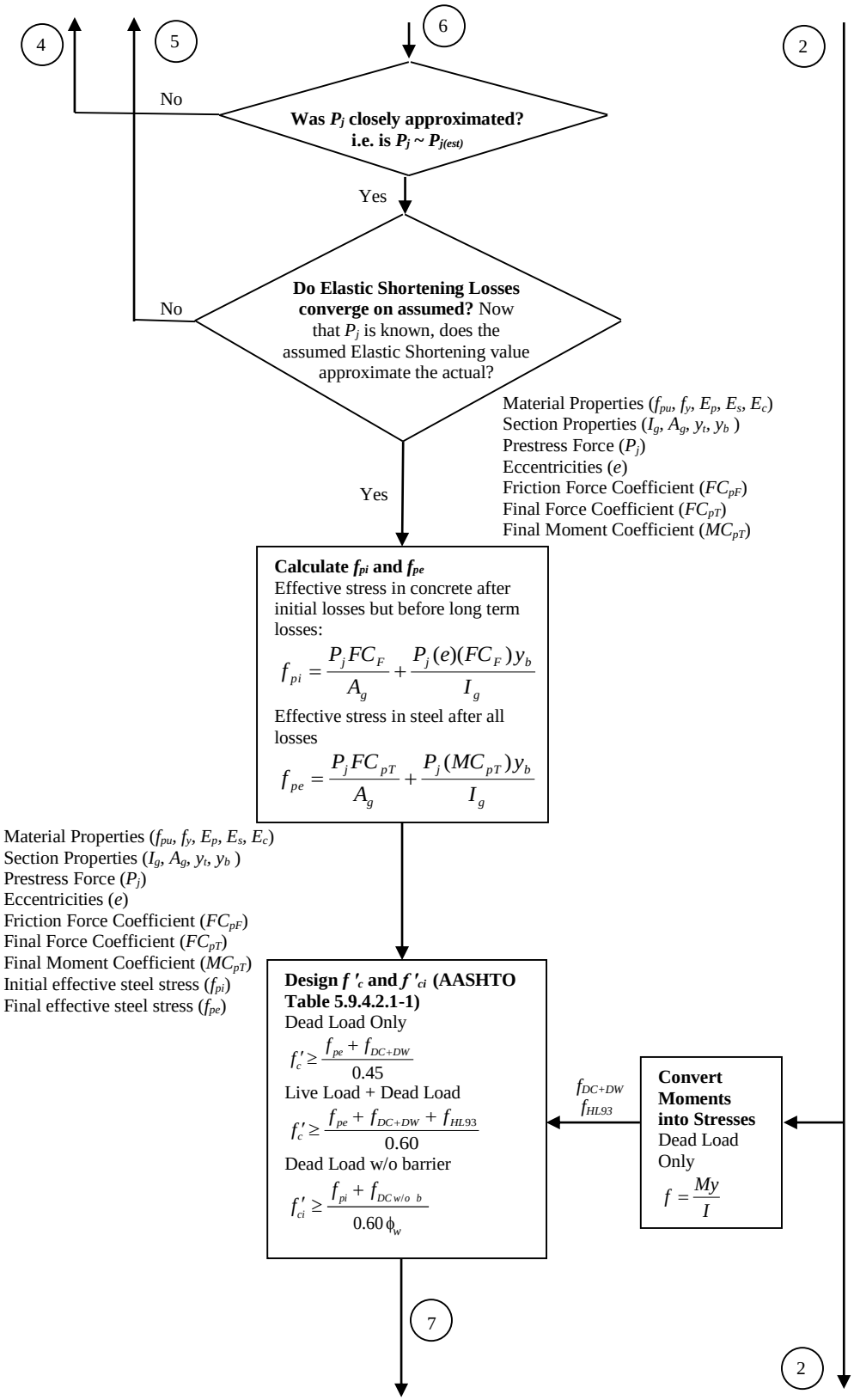
7.11 DESIGN PROCEDURE




Span Lengths (L)
Bridge Width (W)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Select Typical Section
D/S Ratio = 0.40 Continuous, 0.45 Simple Span (AASHTO Table 2.5.2.6.3-1)
Deck and Soffit Size, Overhand, Typical Section Rebar (MTD 10-20)
Span Lengths (L)
Bridge Width (W)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Structure Depth (D_s)
Girder Spacing (S, S_{cir})
Deck and Soffit Thickness (t, T_o)
Seismic Design (see other chapters)
Span Lengths (L)
Bridge Width (W)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Structure Depth (D_s)
Girder Spacing (S, S_{cir})
Deck and Soffit Thickness (t, T_o)
Detail Typical Section
Span Lengths (L)
Bridge Width (W)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Structure Depth (D_s)
Girder Spacing (S, S_{cir})
Deck and Soffit Thickness (t, T_o)
Typical Section Rebar
Loads
DC = Dead load of structural components and non-structural attachments (Article 3.3.2, AASHTO, 2012)
Unit weight of concrete (ω_c) (Table 3.5.1-1, AASHTO, 2012)
Includes the weight of the box girder structural section
Barrier rail where appropriate
DW = Dead load of wearing surfaces and utilities (Article 3.3.2, AASHTO, 2012)
3 in. Asphault Concrete (A.C.) overlay (3 in. thick of 0.140 kcf A.C.) = 0.035 ksf
HL93, which includes the design truck plus the design lane load (Article 3.6.1.2, AASHTO, 2012)
California long-deck P15 (CA Amendments to AASHTO LRFD Article 3.6.1.8, Caltrans, 2012)
M_{DC}, V_{DC}, $M_{DC\ w/o\ Barrier}$
M_{DW}, V_{DW}
$M+_{HL93}$, $M-_{HL93}$,
assoc $V+_{HL93}$, $V-_{HL93}$
$V+_{HL93}$, $V-_{HL93}$,
assoc $M+_{HL93}$, $M-_{HL93}$
$M+_{P-15}$, $M-_{P-15}$,
assoc $V+_{P-15}$, $V-_{P-15}$
$V+_{P-15}$, $V-_{P-15}$,
assoc $M+_{P-15}$, $M-_{P-15}$
Calculate Section Properties (I_g, A_g, y_t, y_b)
Span Lengths (L)
Bridge Width (W)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
1
2


1
Estimate P_j BDA 11-16, and 11-66
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Estimated Prestress Force ($P_{j(est)}$)
2
Prestressing Path
Minimum distance to soffit and deck (MTD 11-28)
One-end versus two-end stressing
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Calculate Losses
Instantaneous Losses
Anchor Set Loss (AASHTO 5.9.5.2.1)
Friction Losses (CA Amendments Table 5.9.5.2.2b-1)
Elastic Shortening (AASHTO 5.9.5.2.3)
Long Term Losses
Shrinkage of Concrete (AASHTO 5.9.5.3)
Creep (AASHTO 5.9.5.3)
Relaxation of Steel (AASHTO 5.9.5.3)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Anchor Set Loss (Δf_{pA})
Friction Loss (Δf_{pF})
Elastic Shortening Loss (Δf_{pES})
Shrinkage Loss (Δf_{pSH})
Creep (Δf_{pCR})
Relaxation of Steel (Δf_{pR})
Secondary Moments
Moment Distribution
4
5
3


3
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Anchor Set Loss (Δf_{pA})
Friction Loss (Δf_{pF})
Elastic Shortening Loss (Δf_{pES})
Shrinkage Loss (Δf_{pSH})
Creep (Δf_{pCR})
Relaxation of Steel (Δf_{pR})
Secondary Moment Coefficent (MC_s)
4
5
Coefficients
Force Coefficients
Moment Coefficients
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
2
Enviromental Area
Areas I and II
(Non-Freeze-Thaw)
Calculate P_j – Service III
No tension allowed under dead load (CA Amendments Table 5.9.4.2.2-1)
$\frac{M_{DC+DW}y}{I}+\frac{FC_{pT}(P_j)}{A}+\frac{MC_{pT}(P_j)y}{I}=0$
$0.19\sqrt{f'_c}$ (ksi) allowed under SERVICE III (CA Amendments Table 5.9.4.2.2-1)
$\frac{M_{DC+DW+0.8HL93}y_{t(bent)}}{I}+\frac{FC_{pT}(P_j)}{A}+\frac{MC_{pT}(P_j)y}{I}=0.19\sqrt{f'_c}$
Area III
(Freeze-Thaw)
Calculate P_j – Service III
No tension allowed under dead load (CA Amendments Table 5.9.4.2.2-1)
$\frac{M_{DC+DW}y}{I}+\frac{FC_{pT}(P_j)}{A}+\frac{MC_{pT}(P_j)y}{I}=0$
$0.0948\sqrt{f'_c}$ (ksi) allowed under SERVICE III (CA Amendments Table 5.9.4.2.2-1)
$\frac{M_{DC+DW+0.8HL93}y_{t(bent)}}{I}+\frac{FC_{pT}(P_j)}{A}+\frac{MC_{pT}(P_j)y}{I}=0.0948\sqrt{f'_c}$
Prestress Force (P_j)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Estimated Prestress Force ($P_{j(est)}$)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Prestress Force (P_j)
6
2


4
5
6
2
No
Was P_j closely approximated?
i.e. is $P_j \sim P_{j(est)}$
Yes
No
Do Elastic Shortening Losses converge on assumed? Now that P_j is known, does the assumed Elastic Shortening value approximate the actual?
Yes
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Calculate f_{pi} and f_{pe}
Effective stress in concrete after initial losses but before long term losses:
$f_{pi} = \frac{P_j FC_F}{A_g} + \frac{P_j(e)(FC_F)y_b}{I_g}$
Effective stress in steel after all losses
$f_{pe} = \frac{P_j FC_{pT}}{A_g} + \frac{P_j(MC_{pT})y_b}{I_g}$
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Initial effective steel stress (f_{pi})
Final effective steel stress (f_{pe})
Design f'_c and f'_{ci} (AASHTO Table 5.9.4.2.1-1)
Dead Load Only
$f'_c \geq \frac{f_{pe} + f_{DC+DW}}{0.45}$
Live Load + Dead Load
$f'_c \geq \frac{f_{pe} + f_{DC+DW} + f_{HL93}}{0.60}$
Dead Load w/o barrier
$f'_{ci} \geq \frac{f_{pi} + f_{DC\,w/o\ b}}{0.60\,\phi_w}$
f_{DC+DW}
f_{HL93}
Convert Moments into Stresses
Dead Load Only
$f = \frac{My}{I}$
7
2

7

2

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_t, y_b)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Compressive Strength of Concrete (f'_c)
Initial Compressive Strength of Concrete (f'_{ci})

Flexural Reinforcement Design – Strength I and II

AASHTO 5.7.3.1.1-1:

$$f_{ps} = f_{pu}\left(1 - \frac{kc}{d_p}\right)$$

AASHTO 5.7.3.1.1-4 (for rectangular section, see AASHTO for T-sections):

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b + k A_{ps} \dfrac{f_{ps}}{d_p}}$$

AASHTO 5.7.3.2.2-1

$$M_n = \underbrace{A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)}_{\text{Prestressing Steel only}} + \underbrace{A_s f_s\left(d_s \frac{a}{2}\right)}_{\text{Additional Flexural Steel}} - \underbrace{A'_s f'_s\left(d'_s - \frac{a}{2}\right)}_{\text{Compression Steel}} + \underbrace{0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)}_{\substack{\text{Flanged Section Component} \\ a = \beta_1 c \le h_f}}$$

Minimum Requirement $\dfrac{M_n}{\phi}$ shall be larger than or equal to the lesser of

- 1.33 M_u as defined in Article 5.7.3.3.2 (AASHTO, 2012)

Or AASHTO 5.7.3.3.2-1

$$M_{cr} = \gamma_3 \left[(\gamma_1 f_r + \gamma_2 f_{cpe}) S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$$

Modified AASHTO 5.7.3.1.1-4 (for rectangular section, see AASHTO for T-sections)

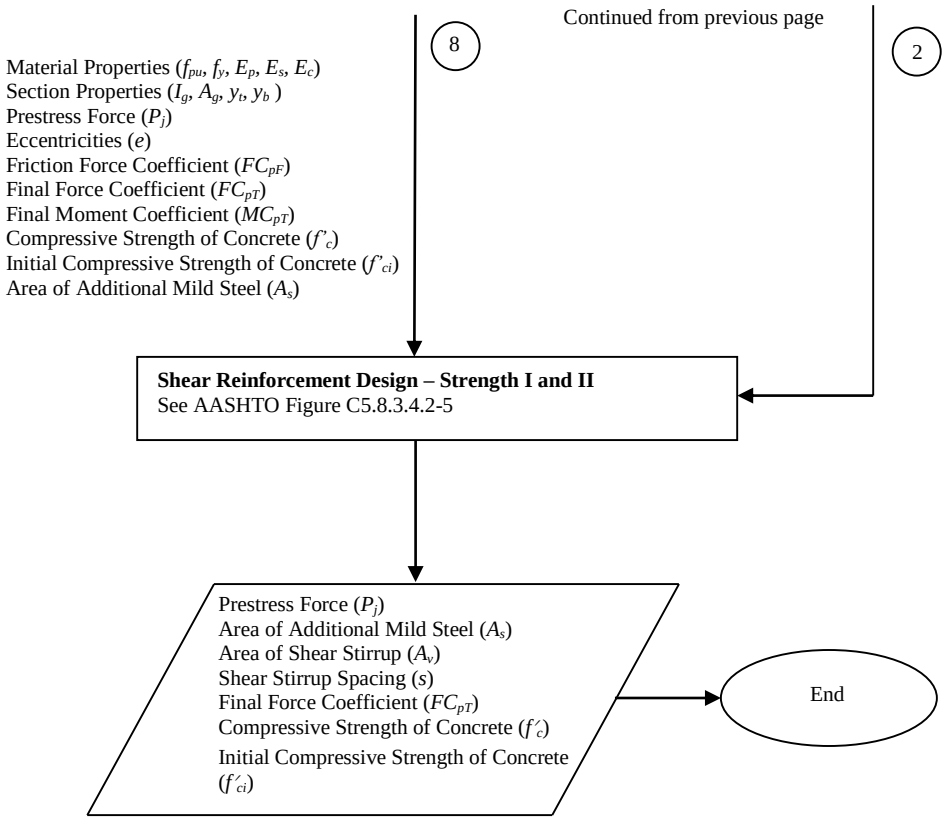
$$a = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{ps} + A_s f_s - A'_s f'_s}{0.85 f'_c b + \dfrac{k}{\beta_1} \dfrac{P_j}{0.75 f_{pu}} \dfrac{f_{ps}}{d_p}}$$

Use AASHTO 5.7.3.2.2-1 to solve for A_s once a is known.

Check AASHTO C5.7.3.3.1 to make sure that Mild Steel yields

8

2



7.12 DESIGN EXAMPLE

7.12.1 Prestressed Concrete Girder Bridge Data

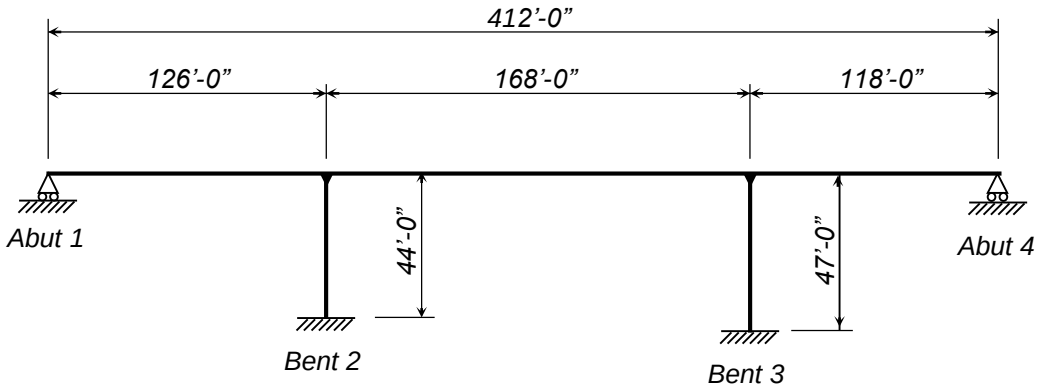


Figure 7.12.1-1 Elevation View of the Example Bridge

It is important, in a two-span configuration, to try and achieve equal, or nearly equal spans. In frames consisting of three or more spans, the designer should strive for 75% end spans, with nearly equal interior spans whenever possible.

The overall width of the bridge in this example is based on the following traffic requirements:

3 – 12-foot lanes with traffic flow in same direction
2 – 10-foot shoulders
2 – Type 732 barriers supporting Type 7 chain link railing

Overall bridge width, W = 58 ft – 10 in. (See Figure 7.12.3-1)

Materials

Superstructure Concrete:

f'_c = 4.0 ksi min, and 10.0 ksi max (AASHTO Article 5.4.2.1)

f'_{ci} min = 3.5 ksi

Normal weight concrete ω_c = 0.15 kcf

$E_c = 33{,}000\ \omega_c^{1.5}\sqrt{f'_c} = (33{,}000)(0.15)^{1.5}(\sqrt{4}) = 3834$ ksi (AASHTO Article 5.4.2.4)

Prestressing Steel:

f_{pu} = 270 ksi, $f_{py} = 0.9\ f_{pu}$ = 243 ksi (AASHTO Table 5.4.4.1-1)

Maximum jacking stress, $f_{pj} = 0.75\ f_{pu}$ = 202.5 ksi (Caltrans, 2014)

E_p = 28,500 ksi (AASHTO Article 5.4.4.2)

Mild Steel:

A706 bar reinforcing steel

f_y = 60 ksi, E_s = 29,000 ksi

7.12.2 Design Requirements

Perform the following design portions for the box girder in accordance with the AASHTO LRFD Bridge Design Specifications, 6th Edition (2012) with California Amendments 2014 (Caltrans, 2014).

7.12.3 Select Girder Layout and Section

Table 2.5.2.6.3-1 (AASHTO, 2012) states that the traditional minimum depth for a continuous CIP box girder shall be calculated using 0.040 L, where L is the length of the longest span within the frame.

Structure depth, $d \sim (0.040)(168) = 6.72$ ft use: d = 6.75 ft = 81 in.

Assuming an overhang width that is about 40 – 50% of the clear spacing between girders, and that the maximum girder spacing, S_{max}, should not exceed (2)(d), where d = structure depth, find the number and spacing of the girders. Overhang width should be limited to 6′-0″ max. When span lengths are of similar length on the same structure, it's generally a good idea to use the same depth for the entire frame.

Maximum girder spacing, S_{max} = (2)(6.75) = 13.50 ft

- Try 4 girders: As an estimate, assuming the combined width of the overhangs is approximately equal to a bay width, S~W/4. Therefore S_4 = 58.83/4 = 14.71 ft. Since S_4 = 14.71 > S_{max} = 13.50 ft, an extra girder should be added to the typical section.
- Try 5 girders: S_5 = 58.83/5 = 11.77 ft. Since S_5 = 11.77 < S_{max} = 13.50 ft, 5 girders should be used to develop the typical section. Using 5 girders will improve shear resistance, provide one more girder stem for placing P/S ducts, and keep the overhang width less than 6 feet. With 5 girders use an exterior girder spacing of 11 ft -11 in. and an interior girder spacing of 12 feet.

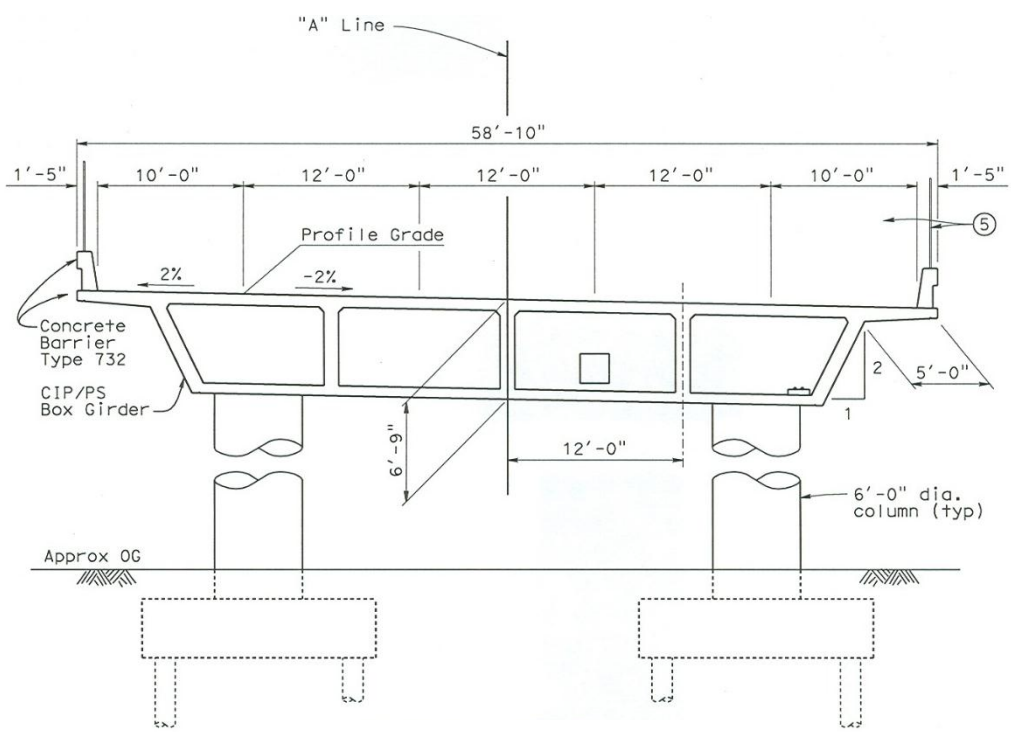


Figure 7.12.3-1 Typical Section View of Example Bridge

- The column diameter D_c = 6 ft
- 2:1 Sloped exterior girders used for aesthetic purposes
- Assuming no other issues, the distance as measured perpendicular to "*A*" line from "*A*" line to centerline of column is 16 ft-7 in.
- Bent cap width is D_c+2 ft = 8 ft
- Overhang thickness varies from 12 in. at exterior girder to 8 in. at Edge of Deck (*EOD*)
- Girders are 12 in. wide to accommodate concrete vibration (Standard Plans – Sheet B8-5)
- Exterior girders flared to 18 in. minimum at abutment diaphragms to accommodate prestressing hardware (Standard Plans – Sheet B8-5)
- Soffit flares to 12 in. at face of bent caps for seismic detailing and to optimize prestress design. Length of flares approximately 1/10 Span Length
- All supports are skewed 20° relative to centerline of bridge. Based on guidance material in MTD 11-28 (Caltrans, 2010), use abutment diaphragm thickness of 3 ft -3 in.
- Four inches fillets are to be located between perpedicular surfaces except for those adjoining the soffit

7.12.4 Determine Basic Design Data

Section Properties

Prismatic Section (midspan):
- Area (A_g) = 103 ft^2
- Moment of Inertia (I_g) = 729 ft^4
- Bottom fiber to C.G. (y_b) = 3.80 ft

Flared Section (bent):
- Area (A_g) = 115 ft^2
- Moment of Inertia (I_g) = 824 ft^4
- Bottom fiber to C.G. (y_b) = 3.50 ft

Loads

DC = Dead load of structural components and non-structural attachments (AASHTO Article 3.3.2)
- Unit weight of concrete (ω_c) = 0.15 kcf (AASHTO Article 3.5.1)
- Includes the weight of the box girder structural section
- Type 732 Barrier rail on both sides (0.4 klf each)

DW = Dead load of wearing surfaces and utilities (AASHTO Article 3.3.2)
– 3 in. Asphault Concrete (A.C.) overlay (3 in. thick of 0.140 kcf A.C.) = 0.035 ksf

*HL*93, which includes the design truck plus the design lane load (AASHTO Article 3.6.1.2)

California long-deck *P*15 (CA Amendements to AASHTO LRFD Section 3.6.1.8, Caltrans, 2014)

Slab design: Design is based on the approximate method of analysis – strip method requirements from Article 4.6.2.1 (AASHTO, 2012), and slab is designed for strength, service, and extreme event Limit Statess Article 9.5 (AASHTO, 2012). Caltrans Memo to Designers 10-20 (Caltrans, 2008b) provides deck thickness and reinforcement.

7.12.5 Design Deck Slab and Soffit

Deck Slab: Refer to MTD 10-20 Attachment 2. Enter centerline girder spacing into design chart and read the required slab thickness and steel requirements. In this example, the centerline spacing of girders is a maximum of 12 feet. Using the chart, the deck slab thickness required is 9 inches.

Table 7.12.5-1 LRFD Deck Design Chart, taken from MTD 10-20, Attachment 2 (Caltrans, 2008)

CIP PRESTRESSED BOX, PRECAST-I, & STEEL GIRDERS w/ flange width ≥ 24″						
"S"	"t"	Dimension	Transverse Bars		"D" Bars	"G" Bars
Girder CL to CL Spacing	Top Slab Thickness	"F"	Size	Spacing	#5 Bars	#4 Bars
11′-9″	8 7/8″	1′-5″	#6	11″	12	5
12′-0″	9″	1′-5″	#6	11″	12	5
12′-3″	9 1/8″	1′-6″	#6	11″	12	5

Soffit Slab: Refer to MTD 10-20 Attachment 3 (formerly BDD 8-30.1). Enter effective girder spacing into the design chart. Read the required slab thickness and steel requirements. In this example, the effective spacings for interior and exterior bays are 11 ft and 8 ft – 4 in., respectively.

Use a constant soffit thickness of 8.25 in. and "*E*" bar spacing based on 11 ft, and "*H*" bar spacing based on individual bay widths.

Table 7.12.5-2 LRFD Soffit Design Chart, taken from MTD 10-20, Attachment 3 (Caltrans, 2008)

S_{cir} Clear Span	"T_o" Min. Bottom slab thick	"E" Bar Spacing	"H" Bars Continuous Reinforced and Prestressed Concrete
8' - 6"	6 3/8"	12"	6 - #6
10' - 9"	8 1/8"	15"	8 - #7
11' - 0"	8 1/4"	15"	
11' - 3"	8 1/2"	14"	
11' - 6"	8 5/8"	14"	

7.12.6 Select Prestressing Cable Path

In general, maximum eccentricities (vertical distance between the C.G.s of the superstructure concrete and the *P*/*S* steel) should occur at the points of maximum gravity moment. These points usually occur at the maximum negative moment near the bent cap, or at the maximum positive moment regions near midspan. In order to define the prestressing path for this frame, we need to get an estimate of the jacking force, P_j. An estimate of P_j will aid us in determining how much vertical room is needed to physically fit the strands/ducts in the girders, and will help optimize the prestress design.

Estimate pounds of *P*/*S* steel per square meter of deck area using the chart found on the next page.

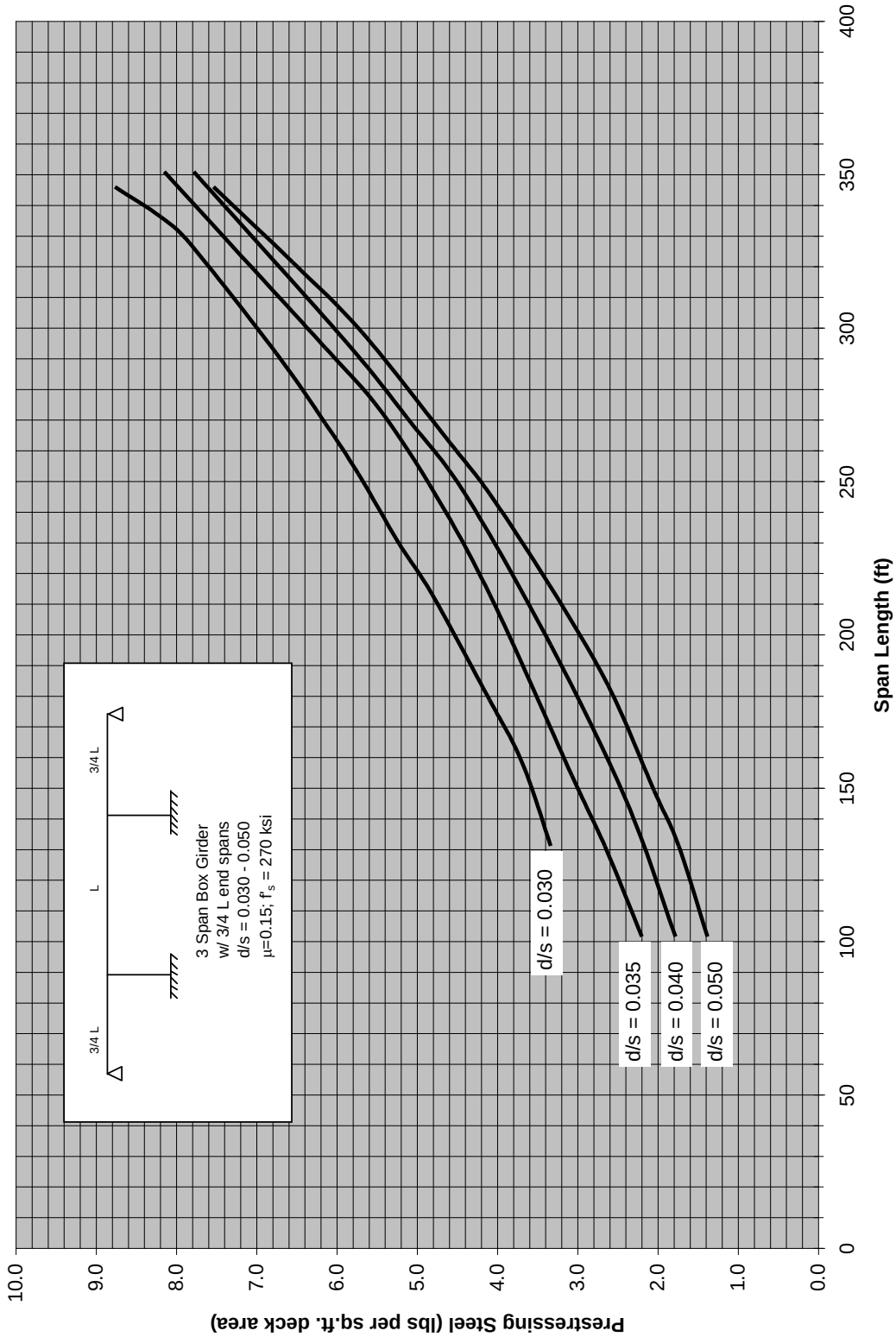


Figure 7.12.6-1 Estimate of Required P_j from the Modified for use with English Units BDA Chapter 11-15, (Caltrans, 2005)

With a length of span = 168 ft, and a depth of span ratio of 0.040, read 2.75 lb per ft^2 deck area off the chart shown below:

Estimate P_j using the Equation on BDA, page 11-66 (Caltrans, 2005).

Total weight of prestressing steel

= (2.80 lb/ft^2)(deck area)

= (2.80 lb/ft^2) (412.0 ft)(58.83 ft) = 67,870 lb

Re-arranging the English equivalent of the equation found on BDA, page 11-66, (Caltrans, 2005) to solve for P_j results in the following:

$$P_j = \frac{W \times 202.5}{L_{frame} \times 3.4} \qquad (7.12.6\text{-}1)$$

where:

P_j	=	force in prestress strands before losses (ksi)
L_{frame}	=	length of frame to be post-tensioned (ft)
W	=	weight of prestressing steel established by BDA, page 11-66 (lb)

$$P_j = \frac{67,870 \text{ lb} \times 202.5}{412 \text{ ft} \times 3.4} = 9,810 \text{ kips}$$

Develop preliminary maximum eccentricities at midspan and bent cap using MTD 11-28 (Caltrans, 2010) found on the next page.

Determine "*D*" value based on estimate of P_j :

- P_j /girder = 9,810 kips/5 girders = 1,962 kips/girder.
- Enter "*D*" chart for cast-in-place girders (MTD 11-28 Attachment 2, Caltrans 2010), and record a value of "*D*" as 5 inches. This chart accounts for the "*Z*" factor, which considers the vertical shift of the tendon within the duct, depending on whether you are at midspan, or at the centerline of bent. The "*D*" values produced in this chart are conservative, and the designer may choose to optimize the prestressing path by using an actual shop drawing to compute a "*D*" value.

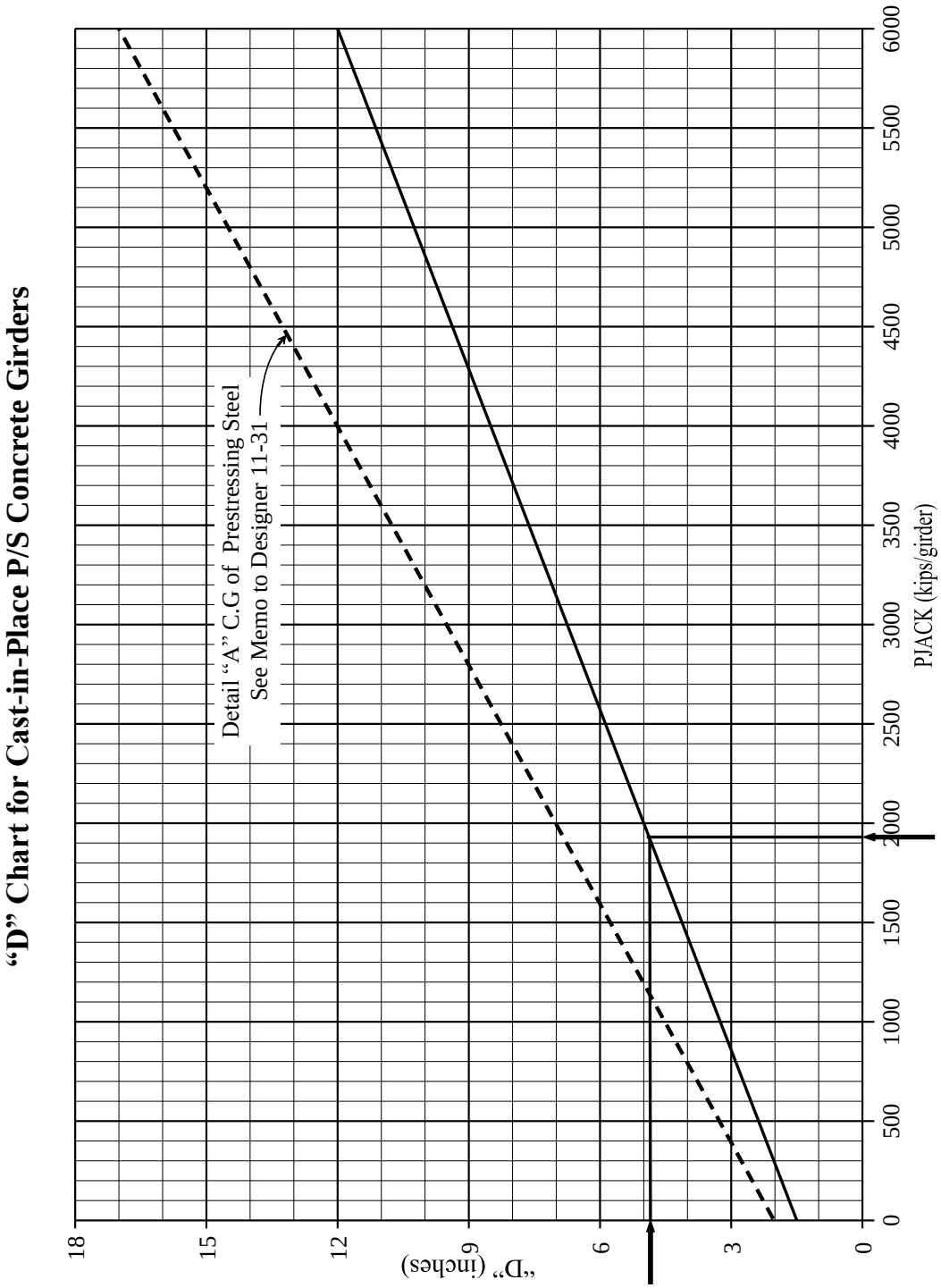


Figure 7.12.6-2 MTD 11-28 Attachment 2 "*D*" Chart used to Optimize Prestressing Tendon Profile (Caltrans, 2010)

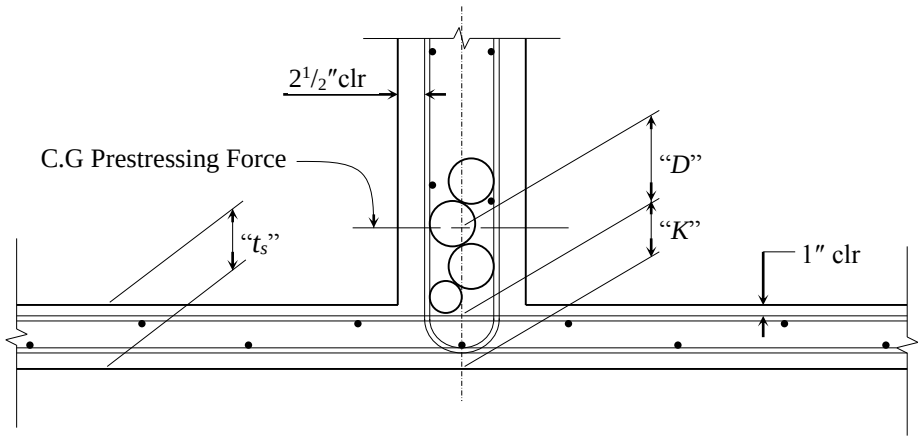


Figure 7.12.6-3 MTD 11-28 Attachment 1 Tendon Configuration at Low Point of Tendon Profile (Caltrans, 2013)

The value *K* for a bridge with a skew of less than 20° is the distance from the top mat of steel in the soffit to the bottom mat of steel in the soffit. Therefore, for this bridge, *K* is:

$$K = t - clr_{int} \tag{7.12.6-2}$$

where:

K = distance to the closest duct to the bottom of the soffit or top of the deck (in.)

t = thickness of soffit or deck (in.)

clr_{int} = clearance from interior face of bay to the first mat of steel in the soffit or deck (usually taken as 1 in.) (in.)

$$\delta_{lp} = t_s - 1 + "D" \tag{7.12.6-3}$$

where:

δ_{lp} = offset from soffit to centroid of duct (in.)

t_s = thickness of soffit (in.)

Using Figure 7.12.6-2 and Equation 7.12.6-3, determine offset from bottom fiber to the *C.G.* of the *P/S* path at the low point:

$$\delta_{lp} = t_{soffit} - 1 + "D" = 8.25 - 1 + 5 = 12.25 \text{ in.}$$

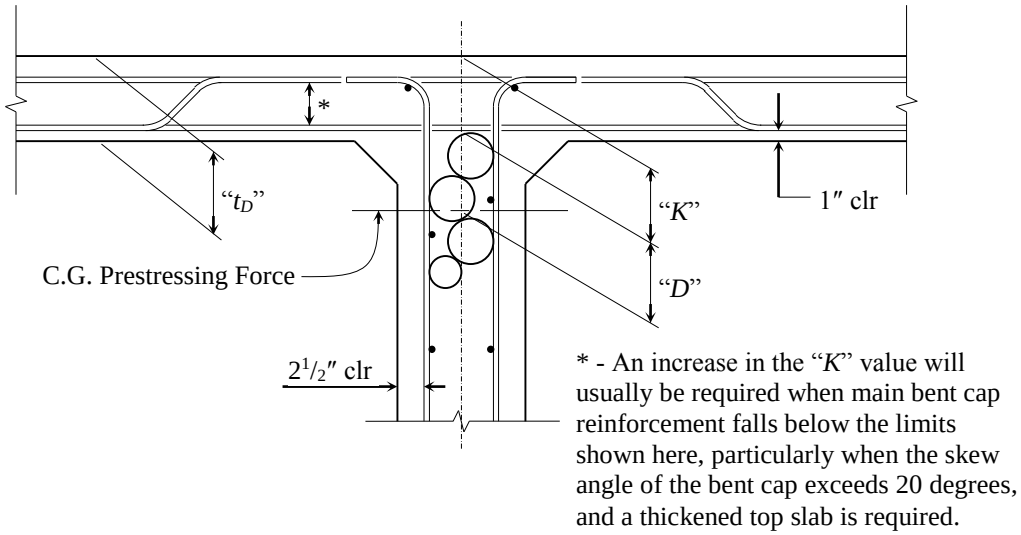


Figure 7.12.6-4 MTD 11-28 Attachment 1 Tendon Configuration at High Point of Tendon Profile (Caltrans, 2013)

$$\delta_{hp} = t_d - 1 + "D" \tag{7.12.6-4}$$

where:

δ_{hp} = offset from deck to centroid of duct (in.)

t_d = thickness of deck (in.)

Using Figure 7.12.6-4 and Equation 7.12.6-4, determine offset from top fiber to the C.G. of the *P/S* path at the high point:

$$\delta_{lp} = t_{deck} - 1 + "D" = 9 - 1 + 5 = 13 \text{ in.}$$

Another method to optimize the prestressing path is to use an actual set of *P/S* shop drawings to find "*D*". Both post-tensioning subcontractors as of the publication of this material use 27 tendon - 0.6 in. diameter strand systems, with a maximum capacity of 27 strands @ 44 kips/strand = 1188 kips. The calculation of "*D*" is as follows:

The equation for P_j in BDA, page 11-66 gave us an estimate of 1962 kips/girder. Assuming 0.6 in. diameter strands, with P_j per strand = 44 kips, the number of strands per girder is as given in Equation 7.12.6-5:

$$\frac{\text{strands}}{\text{girder}} = \frac{\text{force per girder}}{\text{force per strand}} \tag{7.12.6-5}$$

$$\frac{\text{strands}}{\text{girder}} = \frac{\text{force per girder}}{\text{force per strand}} = \frac{1,927 \text{ kips/girder}}{44 \text{ kips/strand}} = 43.79$$

Use: 44 Strands

- Assume 22 strands in duct *A*, and 22 strands in duct *B*.
- Find *"D"* based on the Equation 7.12.6-6

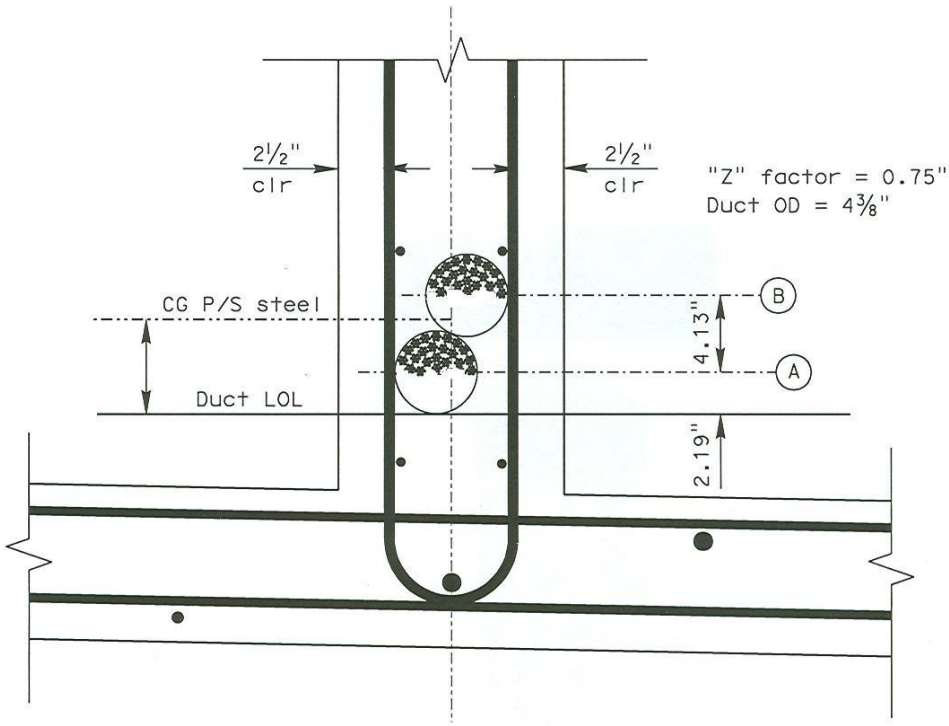


Figure 7.12.6-4 Example of Sub-contractor Tendon Layout for a Two-Duct per Girder Configuration

$$"D" = \frac{\sum_{i=1}^{n}(n_i \times d_i)}{\sum_{i=1}^{n} n_i} + Z \qquad (7.12.6\text{-}6)$$

where:

n_i = number of strands in the ith duct

d_i = distance between C.G. of ith duct and the ith duct *LOL* (See Figure 7.12.6-4) (in.)

Z = C.G. tendon shift within duct (in.)

$$"D" = \frac{\sum_{i=1}^{n}(n_i \times d_i)}{\sum_{i=1}^{n} n_i} + Z = \frac{(n_a \times d_a) + (n_b \times d_b)}{n_a + n_b} + Z$$

$$= \frac{(22 \times 2.19) + [22 \times (2.19 + 4.13)]}{22 + 22} + 0.75 \text{ in.}$$

$$"D" = 5.0 \text{ in.}$$

Final design offsets using Equations 7.12.6-3 and 7.12.6-4:

Values from Tendon Layout:

$$\delta_{lp} = t_{soffit} - 1 + "D" = 8.25 - 1 + 5.0 = 12.25 \text{ in.}$$

$$\delta_{hp} = t_{deck} - 1 + "D" = 9.0 - 1 + 5.0 = 13.0 \text{ in.}$$

Values using MTD 11-28 "*D*":

$$\delta_{lp} = 12.25 \text{ in.}$$

$$\delta_{hp} = 13.0 \text{ in.}$$

It is noted that for each change in P_{jack} there is an accompanying change in "*D*" value. Changes in "*D*" values results in rerunning a model, and can change other portions of the design. Because of this, it is recommended to use a conservative value that will result in stable "*D*" values. However, this is an area where P_j can be decreased through iteration, if necessary. In this case, these values are equal, use a minimum value of 12.5 in. at the soffit and of 13.5 in. at the deck.

Once the high and low points of the tendon path are established, the locations of the inflection points can be obtained. Locating the inflection points at the 10% span length locations on either side of the bent cap not only delivers adequate moment resistance to this zone, but provides a smooth path that allows for easy tendon installation. The vertical position of these inflection points lie on a straight line between the high and low points of the tendon path, at the 0.1*L* locations. Similar triangles can be used to find the vertical location of the inflection point: (See Figures 7.12.6-5 and 7.12.6-6)

Spans 1 and 3:

- y_{BD} = 81– 13.5– 12.5 = 55 in.
- Similar Triangles: $\frac{55}{0.5 + 0.1} = \frac{y_{BC}}{0.5}$
- Rearranging yields: $y_{BC} = \frac{55 \text{ in.} \times 0.5}{0.6} = 45.83 \text{ in.}$

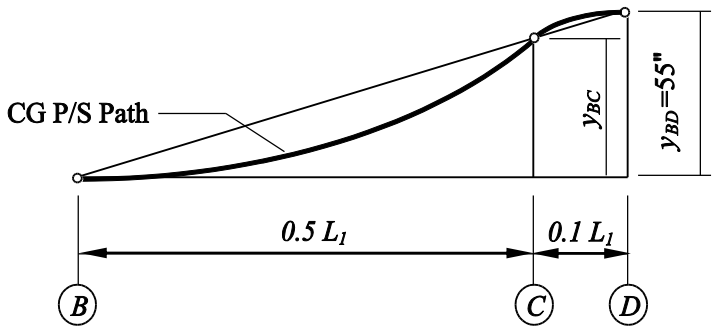


Figure 7.12.6-5 Spans 1 and 3 Inflection Point Sketch

Span 2:

- y_{FH} = 81– 13.5– 12.5 = 55 in.
- Similar Triangles: $\frac{55}{0.4+0.1} = \frac{y_{FG}}{0.4}$
- Rearranging yields: $y_{FG} = \frac{55 \times 0.4}{0.5} = 44 \text{ in.}$

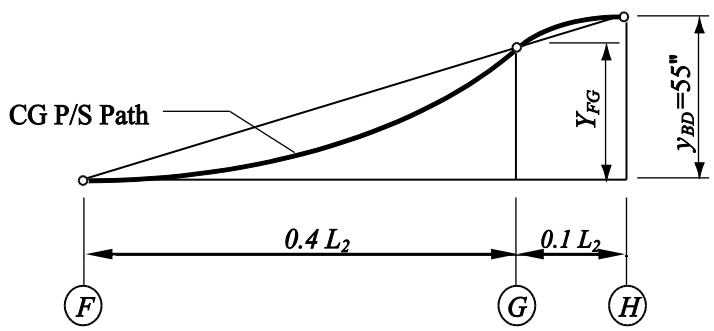


Figure 7.12.6-6 Span 2 Inflection Point Sketch. Middle Spans of Frames Typically have Inflection Points at Mid-Span

The final cable path used for design is shown in Figure 7.12.6-7 on the following page. The y_b = 45.625 in. values at the abutment diaphragms is the distance to the C.G. of the Concrete Box Section, with 6 in. of tolerance (up or down) to allow for constructability issues.

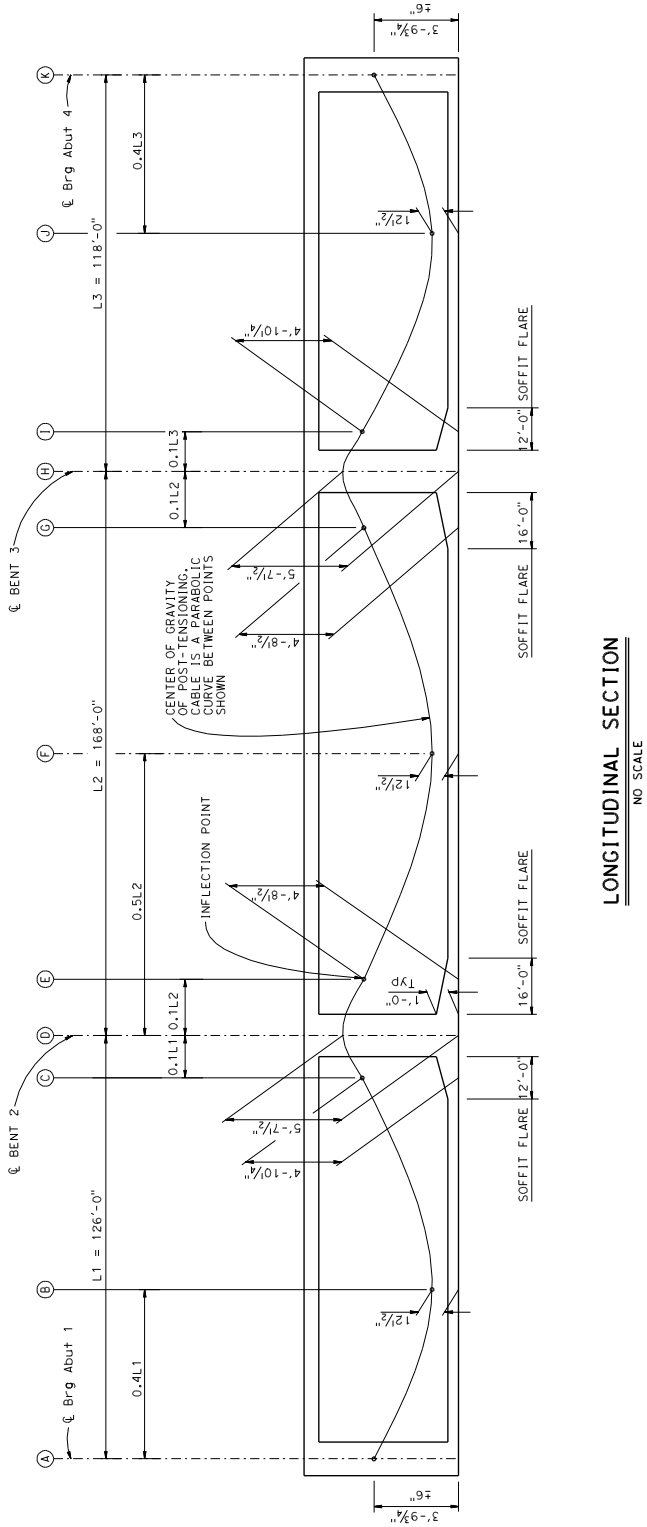


Figure 7.12.6-7 Final Cable Path as it would Appear on the Plans

One vs. Two-End Stressing: According to MTD 11-3, "One-end stressing is considered economical when the increase in P_j does not exceed 3%" when compared to two end stressing. An increase in P_j corresponds to an equivalent increase in materials, and 3% is the breakeven point between cost of materials vs. the cost of time and labor in moving the stressing operation to the opposite end of the frame. Two-end stressing controls the design of most 3+ span frames, and we will assume it controls in this example problem. Two-end stressing is used as a means to control friction loss.

7.12.7 Post Tensioning Losses

7.12.7.1 Friction Loss

Angle change of *P/S* path: The cumulative angle change of the *P*/*S* path must be found in order to find the friction loss. Each individual parabola (10 in. this example) must be isolated so that angle change (y_{ij}) can be calculated for each segment. Friction losses have a cumulative effect, and increase as you get further away from the jacking end. Use the following formula to solve for angle change in each parabolic segment:

$$\alpha_{ij} = \frac{2y_{ij}}{l_{ij}}(rad) \qquad (7.12.7.1\text{-}1)$$

where:

y_{ij} = height of individual parabola (in.)

l_{ij} = length of individual parabola (in.)

Find the angle change in segment *BC* in span 1:

y_{BC} = 4.861 – 1.042 = 3.82 ft

l_{BC} = 0.5L_1 = (0.5)(126) = 63 ft

$$\alpha_{ij} = \frac{2y_{BC}}{l_{BC}} = \frac{2 \times 3.82}{63} = 0.121(rad)$$

The table shown on the next page includes a summary of values that will be used to calculate initial friction losses:

Note in Table 7.12.7.1-1 that the prestressing cable is located at the exact neutral axis of the member (3.813 in.). However, for all future equations the rounded number 3.8 in. will be used.

Table 7.12.7.1-1 Summary of *P/S* Path Angle Changes used in Friction Loss Calculations

Segment	y_{ij} Calculation (ft)	y_{ij} (ft)	x_{ij} Calculation (ft)	x_{ij} (ft)	Left-end $\Sigma\, x_{AK}$ (ft)	Right-end $\Sigma\, x_{KA}$ (ft)	α_{ij} (rad)	Left-end $\Sigma\, \alpha_{AK}$ (rad)	Right-end $\Sigma\, \alpha_{KA}$ (rad)
AB	3.813 - 1.042	2.77	(0.4)(126)	50.4	50.4	412.0	0.110	0.110	1.166
BC	4.861 - 1.042	3.82	(0.5)(126)	63.0	113.4	361.6	0.121	0.231	1.056
CD	5.625 - 4.861	0.76	(0.1)(126)	12.6	126.0	298.6	0.121	0.352	0.934
DE	5.625 - 4.708	0.92	(0.1)(168)	16.8	142.8	286.0	0.109	0.462	0.813
EF	4.708 - 1.042	3.67	(0.4)(168)	67.2	210.0	269.2	0.109	0.571	0.704
FG	4.708 - 1.042	3.67	(0.4)(168)	67.2	277.2	202.0	0.109	0.680	0.595
GH	5.625 - 4.708	0.92	(0.1)(168)	16.8	294.0	134.8	0.109	0.789	0.486
HI	5.625 - 4.861	0.76	(0.1)(118)	11.8	305.8	118.0	0.130	0.919	0.376
IJ	4.861 - 1.042	3.82	(0.5)(118)	59.0	364.8	106.2	0.130	1.048	0.247
JK	3.813 - 1.042	2.77	(0.4)(118)	47.2	412.0	47.2	0.117	1.166	0.117
				412.0			1.166		

Initial Friction Coefficient: The percent in decimal form of jacking stress remaining in the *P/S* steel after losses due to friction, FC_{pF}, can now be calculated based on the cumulative l_{ij} and α_{ij} computed above. Using Equation 7.5.1.2-1, modified Equation 5.9.5.2.2b-1 (AASHTO, 2012) from Article 7.5.1.2 above:

$$FC_{pF} = \frac{\Delta f_{pF}}{f_{pj}} = e^{-(Kx+\mu\alpha)}$$

(7.5.1.2-1)

Inputting values from CA Amendments to AASHTO LRFD Table 5.9.5.2.2b-1 (Caltrans, 2014)

- K = 0.0002 per ft
- μ = 0.15 (L_{frame} < 600 ft)

Find the percent in decimal form of P_j remaining after the effects of friction loss at the 0.1 point in span 2.

- At pt. E: $FC_{pF} = e^{-(Kx+\mu\alpha)}$
  - x_E (left-end) = 142.8 ft
  - α_E (left-end) = 0.4616 rad
- FC_{pF} (@E) = $e^{-[(0.0002)(142.8) + (0.15)(0.462)]}$ = 0.907

Table 7.12.7.1-2 includes a summary of values of initial friction losses:

Table 7.12.7.1-2 Summary of Cumulative Angle Change for Two-End Stressing

	Left-end Stressing			Right-end Stressing		
Location	Σx_{AK} (ft)	$\Sigma\alpha_{AK}$ (rad)	$e^{-(Kx_{AK}+\mu\alpha_{AK})} = FC_{pF}$ (decimal %)	Σx_{KA} (ft)	$\Sigma\alpha_{KA}$ (rad)	$e^{-(Kx_{KA}+\mu\alpha_{KA})} = FC_{pF}$ (decimal %)
A	0.0	0.000	1.000	412.0	1.166	0.773
B	50.4	0.110	0.974	361.6	1.056	0.794
C	113.4	0.231	0.944	298.6	0.934	0.819
D	126.0	0.352	0.925	286.0	0.813	0.836
E	142.8	0.462	0.907	269.2	0.704	0.853
F	210.0	0.571	0.880	202.0	0.594	0.878
G	277.2	0.680	0.854	134.8	0.486	0.905
H	294.0	0.789	0.837	118.0	0.376	0.923
I	305.8	0.919	0.819	106.2	0.247	0.943
J	364.8	1.048	0.794	47.2	0.117	0.973
K	412.0	1.166	0.773	0.0	0.000	1.000

7.12.7.2 Anchor Set Losses

The method used for determining losses due to anchor set is based on "similar triangles". The assumption is that the effects of friction is the same whether a tendon is being stressed, or released back into the duct to seat the wedges. Most of the time, the end of the influence length of anchor set, x_{pA}, lies between the high and low inflection points from the jacking end in a multi-span frame. The equations that solve for ΔFC_{pA} and x_{pA} are shown on the next page:

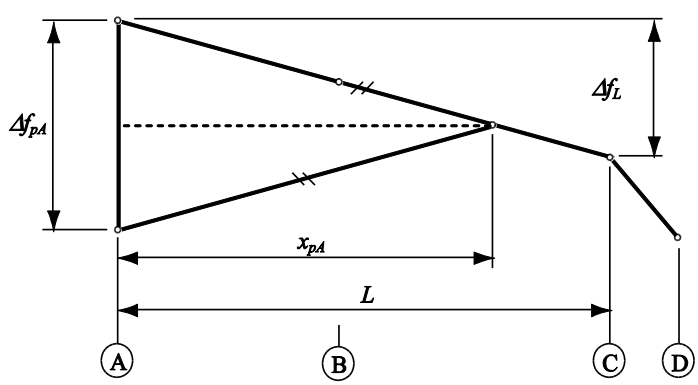


Figure 7.12.7.2-1 Typical Anchor Set Loss

$$\Delta FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2(\Delta f_L)x_{pA}}{L(f_{ps})} \tag{7.5.1.1-1}$$

$$x_{pA} = \sqrt{\frac{E_p(\Delta_{Aset})L}{12\Delta f_L}} \tag{7.5.1.1-2}$$

Define the anchor set loss diagram by finding x_{pA} and ΔFC_{pA} in span 1 due to the left-end stressing operation:

Given:

E_p	=	28,500 ksi
Δ_{Aset}	=	0.375 in.
L	=	Distance from point A to point C = 0.9 L_1 = (0.9) (126 ft) = 113.4 ft
f_{pj}	=	0.75
f_{pu}	=	(0.75)(270 ksi)
	=	202.5 ksi (Table 5.9.3-1 AASHTO, 2012)
Δf_L	=	[1 - ΔFC_{pF} (@ point C)](202.5 ksi)
	=	[1 – 0.944](202.5) = 11.34 ksi

Solving for x_{pA} and ΔFC_{pA} at the intersection of initial losses and anchor set (the location x_{pA} away from the anchor).

$$x_{pA} = \sqrt{\frac{E_p(\Delta_{Aset})L}{12\Delta f_L}} = \sqrt{\frac{28,500(0.375)113.4}{12(11.34)}} = 94.37\,\text{ft}$$

$$\Delta FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2(\Delta f_L)(x_{pA})}{L(f_{ps})} = \frac{2(11.34)(94.37)}{113.4(202.5)} = 0.093$$

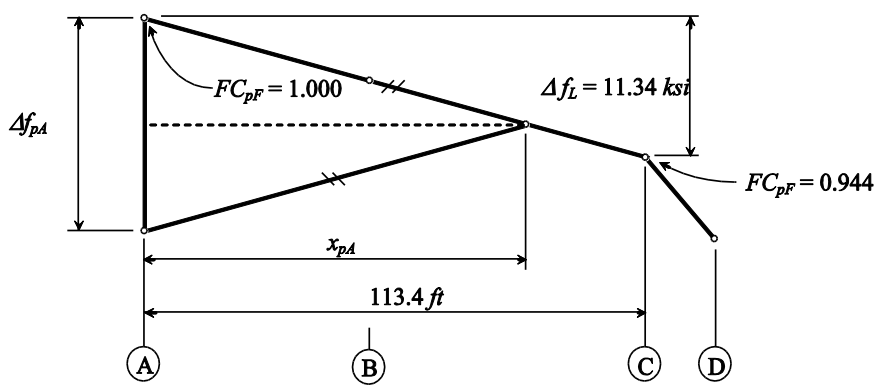


Figure 7.12.7.2-2 Bridge Specific Anchor Set Loss

The anchor set loss diagram is found in a similar manner in Span 3 due to the second (right) end stressing operation.

Given:

E_p = 28,500 ksi

Δ_{Aset} = 0.375 in.

L = Distance from point K to point I = 0.9 L_3 = (0.9)(118 ft) = 106.2 ft

f_{pj} = $0.75f_{pu}$ = (0.75)(270ksi) = 202.5 ksi (Table found in commentary to AASHTO Article 5.9.3-1)

Δf_L = [1 - ΔFC_{pF} (@ point I)] (202.5 ksi)

= [1 – 0.943] (202.5) = 11.54 ksi

Solving for x_{pA} and ΔFC_{pA} at the intersection of initial losses and anchor set (the location x_{pA} away from the anchor).

$$x_{pA} = \sqrt{\frac{E_p(\Delta_{Aset})L}{12\Delta f_L}} = \sqrt{\frac{28{,}500(0.375)106.2}{12(11.54)}} = 90.53\text{ ft}$$

$$\Delta FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2(\Delta f_L)(x_{pA})}{L(f_{ps})} = \frac{2(11.54)(90.53)}{106.2(202.5)} = 0.097$$

7.12.7.3 Elastic Shortening

Losses due to elastic shortening are usually assumed at the beginning and then checked once more for convergent numbers for M_{DL}, P_j, and e have been found. Therefore, based on experience, we will assume a realistic and typically conservative value of Δf_{pES} = 3 ksi for this practice problem.

To turn this into a Force Coefficient:

$$\Delta FC_{pES} = \frac{\Delta f_{pES}}{f_{ps}} = \frac{3\text{ ksi}}{202.5\text{ ksi}} = 0.015$$

7.12.7.4 Approximate Estimate of Time Dependent Long-Term Loss

The long-term change in prestressing steel stress due to creep of concrete, shrinkage of concrete, and relaxation of P/S steel occur over time and begin immediately after stressing.

$$\Delta f_{pSH} = 12.0(1.7 - .01H)\frac{5}{(1+f'_{ci})} \qquad (7.5.2.1\text{-}1)$$

$$\Delta f_{pCR} = 10.0\frac{f_{pi}A_{ps}}{A_g}(1.7 - .01H)\frac{5}{(1+f'_{ci})} \qquad (7.5.2.2\text{-}1)$$

$$\Delta f_{pR} = 2.4\text{ ksi} \quad \text{for lo-lax strands} \qquad (\text{AASHTO } 5.9.5.3\text{-}1)$$

Most of the research done on time-dependant losses considered precast concrete girders, without much consideration given to continuous, CIP PT box girder structures. Ongoing research, indicate time-dependant losses as high as 30 ksi may be appropriate for cast-in-place, post-tensioned structures. This portion of the code has undergone revision from 2008 to 2014.

$$\Delta f_{pLT} = 20 \text{ ksi} \qquad \text{(CA Amendments 5.9.5.3)}$$

For this example, let's use 25 ksi. This is a reasonable value and was the value used in the 2008 CA Amendments.

Converting to a Force Coefficient:

$$\Delta FC_{pLT} = \frac{\Delta f_{pR}}{f_{ps}} = \frac{25}{202.5} = 0.123$$

7.12.7.5 Total Loss of Prestress Force

As stated in section 7.5 the loss of force in the prestressing steel is cumulative.

In lieu of more detailed analysis, prestress losses in members constructed and prestressed in a single stage, relative to the stress immediately before transfer, may be taken as:

$$FC = (1 - \frac{\sum \Delta f_i}{f_{ps}}) \qquad (7.5\text{-}1)$$

A summary of our calculated immediate and total prestress stress remaining along the prestressing path is included in Table 7.12.7.5-1.

Table 7.12.7.5-1 Summary of Cumulative Prestress Loss

	Percentage of Cumulative Prestress Stresses Remaining After All Losses							
	Left-end Stressing				Right-end Stressing			
Location	FC_{pF}	FC_{pA}	FC_{pES}	FC_{pLT}	FC_{pF}	FC_{pA}	FC_{pES}	FC_{pLT}
A	1.000	0.907	0.893	0.770	0.773	0.773	0.758	0.635
B	0.974	0.933	0.917	0.794	0.794	0.794	0.779	0.656
C	0.944	0.944	0.929	0.806	0.819	0.819	0.804	0.681
D	0.925	0.925	0.910	0.787	0.836	0.836	0.821	0.698
E	0.907	0.907	0.892	0.769	0.853	0.853	0.838	0.715
F	0.880	0.880	0.865	0.742	0.878	0.878	0.863	0.740
G	0.854	0.854	0.839	0.716	0.905	0.905	0.890	0.767
H	0.837	0.837	0.822	0.699	0.923	0.923	0.908	0.785
I	0.819	0.819	0.804	0.681	0.943	0.943	0.928	0.805
J	0.794	0.794	0.779	0.656	0.973	0.930	0.915	0.792
K	0.773	0.773	0.758	0.635	1.000	0.903	0.890	0.767
			FC_{pI}	FC_{pT}			FC_{pI}	FC_{pT}

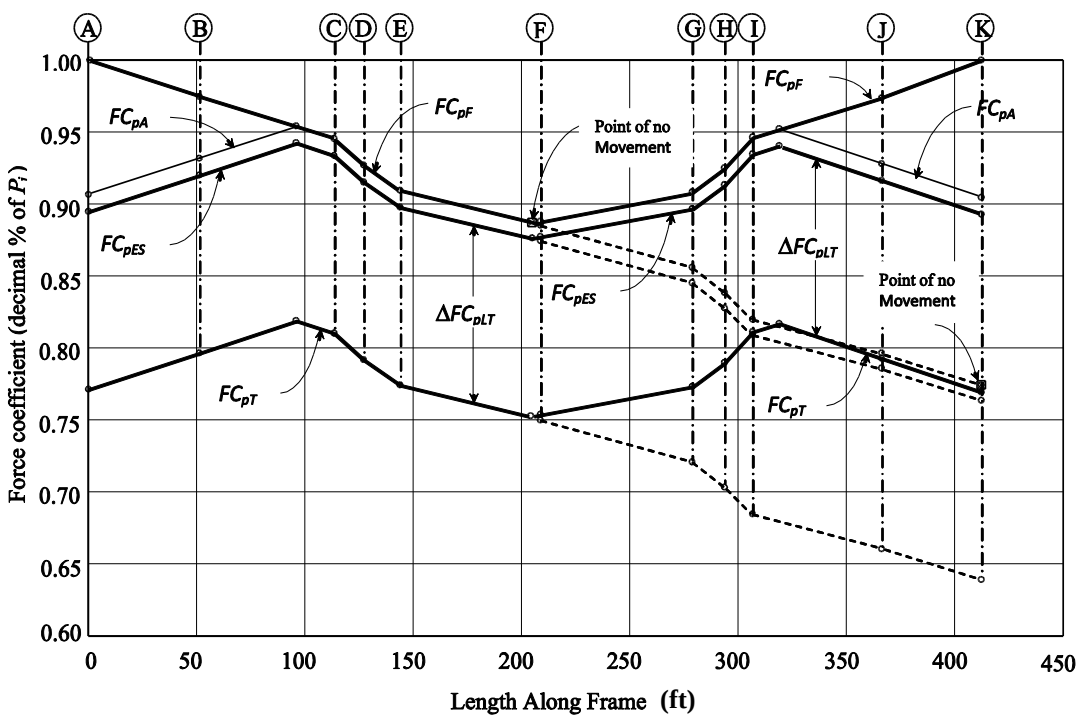


Figure 7.12.7.5-1 Summary of Prestress Losses for Two-End Stressing

7.12.8 Cable Path Eccentricities

In order to design the jacking force, and track stresses in both top and bottom fibers, cable path eccentricities must be calculated at the 10th points of each span in the frame. Derived from the equation of a parabola, the diagram and formula shown below can be used to compute eccentricities for parabolic *P*/*S* paths:

$$e_x = -\left[y_{ij} \left(1 - \frac{x^2}{x_{ij}^{\ 2}} \right) \right] + c \qquad (7.12.8\text{-}1)$$

where:

e_x = eccentricity as a function of x along parabolic segment (ft)

y_{ij} = height of the individual parabola (ft)

x = location along parabolic segment where eccentricity is calculated (percent of span L)

x_{ij} = length of parabolic segment under consideration (must originate at vertex) (percent of span L)

c = shifting term to adjust eccentricities when y_{ij} does not coincide with the C.G. of concrete

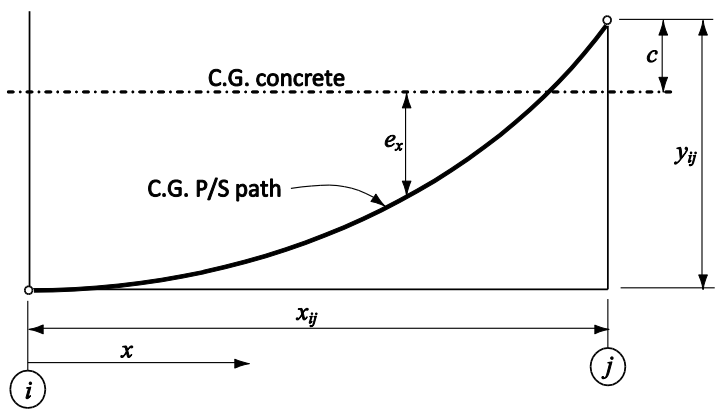


Figure 7.12.8-1 Cable Path Calculation Diagram

Find the cable path eccentricities in Span 1:

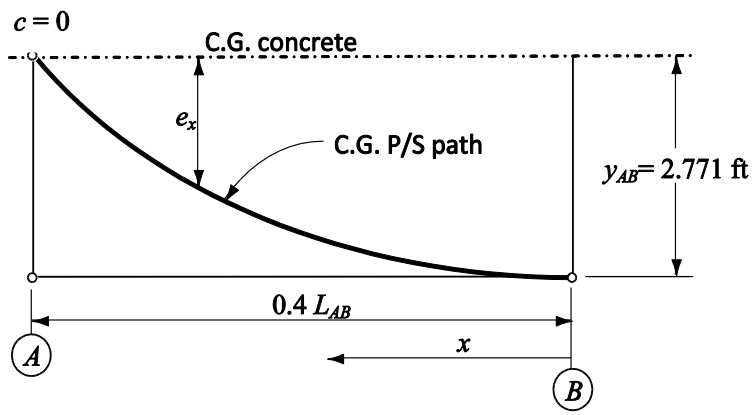


Figure 7.12.8-2 Cable Path Calculation Diagram from Abut 1 to the Low Point of Span 1

$$e_{0.0L1} = -\left[2.771\left(1-\frac{0.4^2}{0.4^2}\right)\right]+0 = 0 \text{ ft}$$

$$e_{0.1L1} = -\left[2.771\left(1-\frac{0.3^2}{0.4^2}\right)\right]+0 = -1.212 \text{ ft}$$

$$e_{0.2L1} = -\left[2.771\left(1-\frac{0.2^2}{0.4^2}\right)\right]+0 = -2.078 \text{ ft}$$

$$e_{0.3L1} = -\left[2.771\left(1-\frac{0.1^2}{0.4^2}\right)\right]+0 = -2.598 \text{ ft}$$

$$e_{0.4L1} = -\left[2.771\left(1-\frac{0.0^2}{0.4^2}\right)\right]+0 = -2.771 \text{ ft}$$

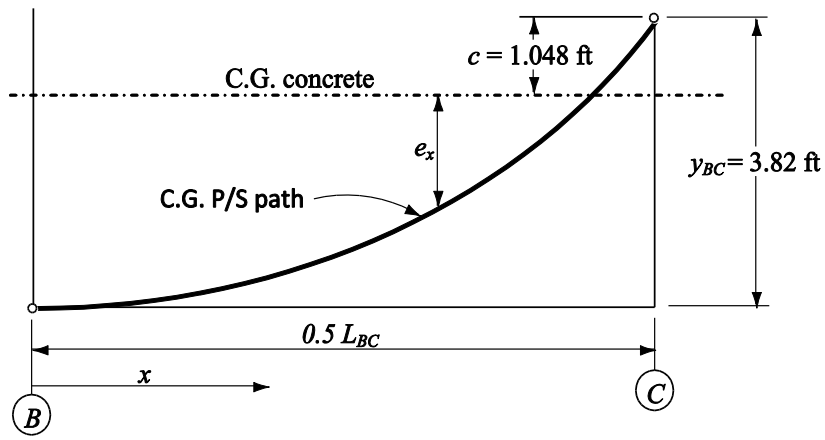


Figure 7.12.8-3 Cable Path Calculation Diagram from the Low Point of Span 1 to the Inflection Point Near Bent 2

$$e_{0.4L1} = -\left[3.819\left(1-\frac{0.0^2}{0.5^2}\right)\right]+1.048 = -2.771 \text{ ft}$$

$$e_{0.5L1} = -\left[3.819\left(1-\frac{0.1^2}{0.5^2}\right)\right]+1.048 = -2.618 \text{ ft}$$

$$e_{0.6L1} = -\left[3.819\left(1-\frac{0.2^2}{0.5^2}\right)\right]+1.048 = -2.160 \text{ ft}$$

$$e_{0.7L1} = -\left[3.819\left(1-\frac{0.3^2}{0.5^2}\right)\right]+1.048 = -1.396 \text{ ft}$$

$$e_{0.8L1} = -\left[3.819\left(1-\frac{0.4^2}{0.5^2}\right)\right]+1.048 = -0.327 \text{ ft}$$

$$e_{0.9L1} = -\left[3.819\left(1-\frac{0.5^2}{0.5^2}\right)\right]+1.048 = 1.048 \text{ ft}$$

The eccentricity at the centerline of Bent 2 must be calculated using the section properties that include the soffit flare:

$e_{1.0L1} = y_{CD} + c + \Delta_{yb} = 0.764 + 1.048 + 0.310 = 2.122$ ft

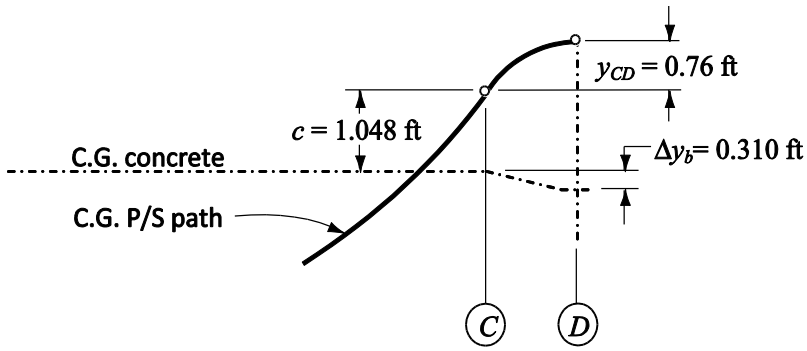


Figure 7.12.8-4 Cable Path Calculation Diagram from the Inflection Point Near Bent 2 to *CL* of Bent 2

Cable path eccentricities for all three spans are summarized in the Figure 7.12.8-5 below:

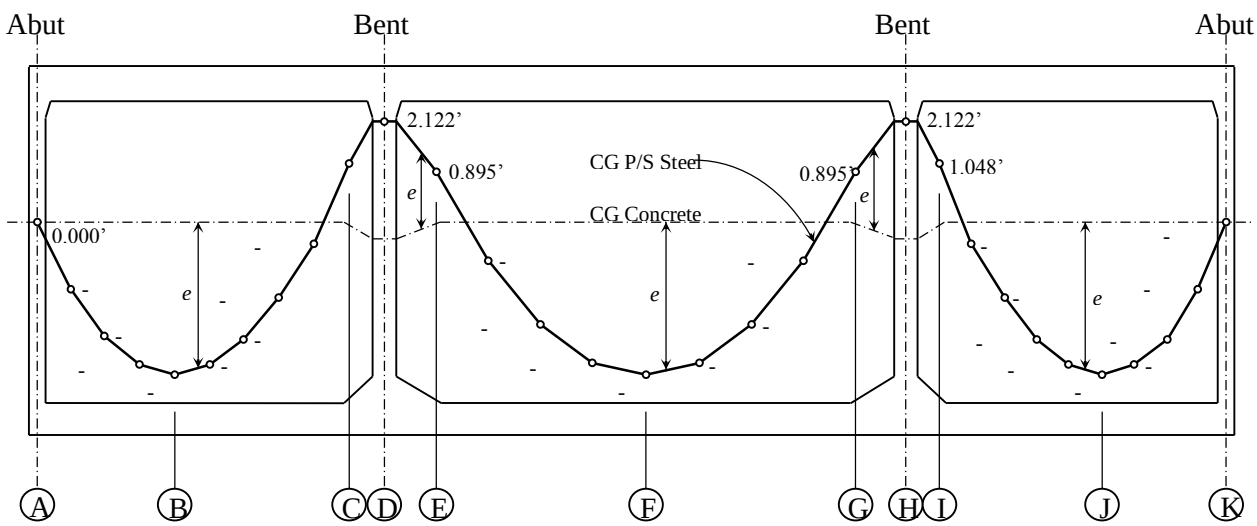


Figure 7.12.8-5 Cable Path Eccentricities

7.12.9 Moment Coefficients

Now that we've identified both the eccentricity and the percentage of prestressing force present at each tenth point along each span in the frame, we can now find the moment coefficients. The moment coefficients will help us solve for P_j, as well as compute flexural stresses in the concrete for determining f'_c and f'_{ci}. The total moment coefficient consists of two parts; primary and secondary moment coefficients.

Primary Moment Coefficient: The primary moment coefficient at any location along the frame is simply defined as the total force coefficient (FC_{pT}) multiplied by the eccentricity (e_x).

Find the primary moment coefficients at each tenth point in Span 1:

$$MC_p = (FC_{pT})(e_x) \tag{7.12.9-1}$$

where:

MC_p = primary moment force coefficient for loss (ft)
FC_{pT} = total force coefficient for loss
e_x = eccentricity as a function of x along parabolic segment (ft)

@ 0.0 L_1: (0.770) (0) = 0 ft
@ 0.1 L_1: (0.776) (-1.212) = -0.941 ft
@ 0.2 L_1: (0.782) (-2.078) = -1.625 ft
@ 0.3 L_1: (0.788) (-2.598) = -2.047 ft
@ 0.4 L_1: (0.794) (-2.771) = -2.200 ft
@ 0.5 L_1: (0.800) (-2.618) = -2.094 ft
@ 0.6 L_1: (0.806) (-2.160) = -1.741 ft
@ 0.7 L_1: (0.812) (-1.396) = -1.134 ft
@ 0.8 L_1: (0.812) (-0.327) = -0.266 ft
@ 0.9 L_1: (0.806) (1.048) = 0.845 ft
@ 1.0 L_1: (0.787) (2.122) = 1.670 ft

Secondary Moment Coefficient: Prestress secondary moments occur in multi-span post-tensioned concrete frames where the superstructure is "fixed" to the column. The result of this fixity is an indeterminate structure. Prestress secondary moments are made up of two components:

- Distortions due to primary prestress moments, $MC_P = (\Delta FC_{pT})(e_x)$, generate fixed-end moments at rigid column supports. These fixed-end moments are always positive due the the geometry of the cable path and always enhance the effects of prestressing at the bent caps. On the other hand, this component of prestress secondary moment always reduces the flexural effects of the prestressing force near midspan.
- Prestress shortening of superstructure between rigid supports generates moments in the columns, which result in fixed-end moments in the superstructure. This component of secondary prestress moment occurs in frames with three or more spans. Long frames with short columns result in larger secondary prestress moments, which can be a significant factor in the design of the superstructure.
- There are several analysis methods a designer can use to find the prestress secondary moments for a given frame. In stiffness based frame analysis software packages, the forces generated by prestressing the concrete are

replicated with a series of uniform and point loads. In other words, primary "internally applied" moments and axial loads are converted into "externally applied" loads. The drawback of this method is that it is extremely difficult to do by hand, especially in multi-span frames.

- Each span within the frame is transformed into a simple span so that the ends can rotate freely.
- Create an MC_P/EI diagram, as the applied prestress moments are simply the prestress force times eccentricity.
- Using conjugate beam theory, sum moments about one end of the beam to solve for the rotation at the opposite end. The moment needed to rotate the end of the beam back to zero is the "fixed-end" secondary moment due to ΔMC_P distortion.
- When a frame is three spans or longer, secondary prestress deflections are generated in the column supports. The resulting column moments are a result of prestress shortening of the superstructure between the interior spans of the frame.
- The fixed-end moments of the two components of prestress secondary moments are then combined; with the use of moment distribution, these fixed-end moments are distributed to both the superstructure and columns based on the relative stiffness of each member.
- Because P_j is still unknown, the prestress secondary moments must be solved for in terms of coefficient (MC_S).

The secondary prestress moment coefficients used in this example problem are a result of the conjugate beam and moment distribution methods of analysis. The primary (MC_P) and secondary (MC_S) are then added together algebraically resulting in the total moment coefficient (MC_{PT}). A summary of MC_P, MC_S, and MC_{PT} are summarized in the following diagram:

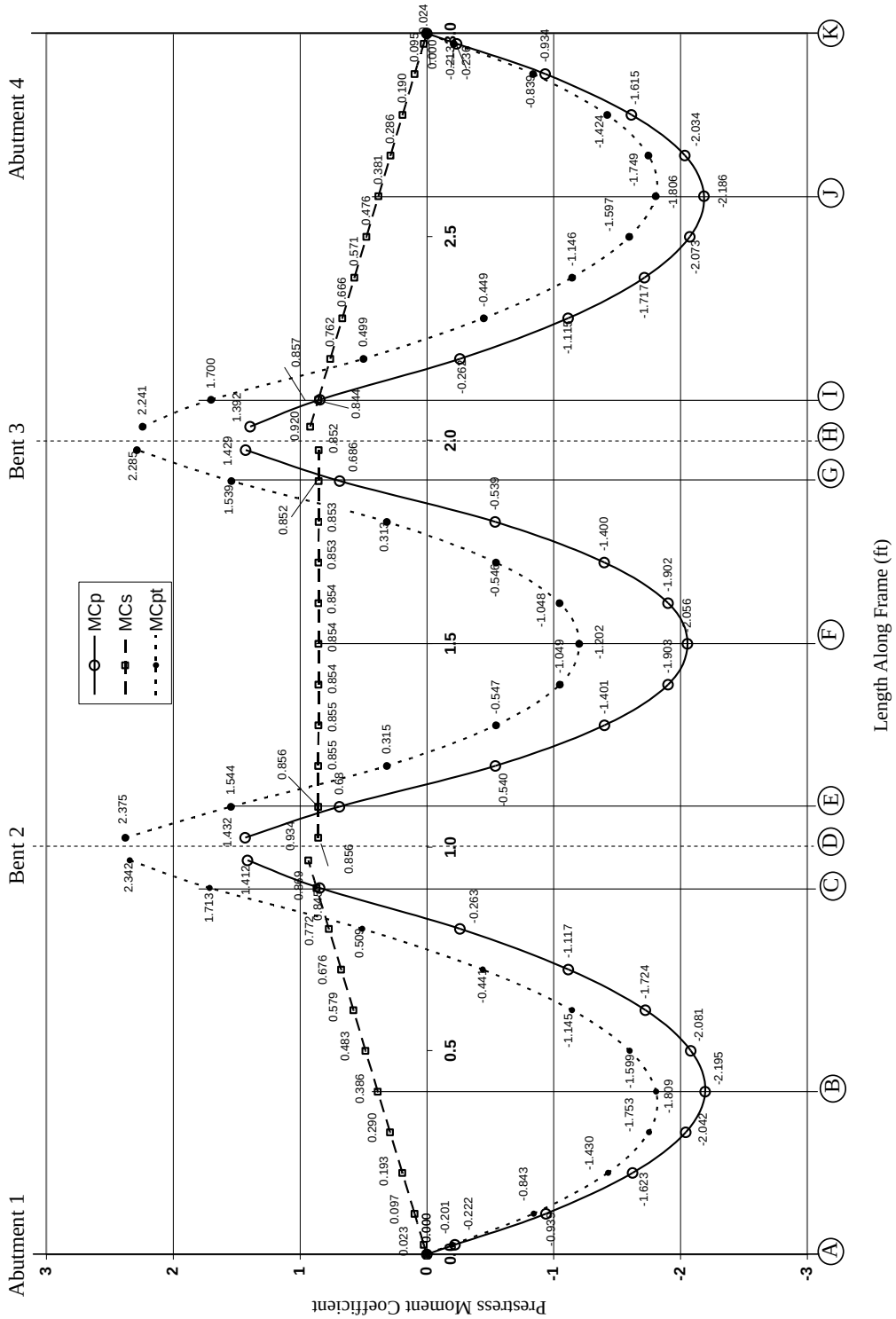


Figure 7.12.9-1 Moment Coefficients

7.12.10 Gravity Loads

DC = Dead load of structural components and nonstructural attachments.

- Includes the self weight of the box section itself, assuming a unit weight of concrete (w_c) = 0.15 (kcf) (Round up from 0.145) Table 3.5.1-1(AASHTO, 2012)
- Type 732 barrier rail: (2 barriers) (0.4 klf ea.) = 0.80 klf

DW = Dead load of wearing surface and utilities.

- 3 in. A.C. Overlay: (56 ft) (0.035 ksf) = 1.96 klf

Vehicular Live load (*LL*): The application of vehicular live loads on the superstructure shall be calculated separately for flexure and shear design. In each case, live load distribution factors shall be calculated for an interior girder, then multiplied by the total number of girders in the cross-section. Treating all girders as interior is justifiable because exterior girders become interior when bridges are widened.

Distribution of live load per lane for moment in interior beams, with two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} \qquad \text{(AASHTO Table 4.6.2.2.2b-1)}$$

where:

N_c = number of cells in a concrete box girder ($N_c \geq 3$)

S = spacing of beams or webs (ft) ($6.0 \leq S \leq 13.0$)

L = individual span length (ft) ($60 \leq L \leq 240$)

(Note: if L varies from span to span in a multi-span frame, so will the distribution factors)

Distribution of live load per lane for shear in interior beams, with two or more design lanes loaded:

$$\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1} \qquad \text{(AASHTO Table 4.6.2.2.3a-1)}$$

where:

S = spacing of beams or webs (ft) ($6.0 \leq S \leq 13.0$)

L = individual span length (ft) ($20 \leq L \leq 240$)

(Note: if L varies from span to span in a multi-span frame, so will the distribution factors)

d = depth of member (in.) ($35 \le d \le 110$)

Calculate the number of live load lanes for both moment and shear design for Span 1:

7.12.10.1 Live Load Lanes for Moment

Using an equation from Table 4.6.2.2.2b-1:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{4}\right)^{0.3}\left(\frac{12}{5.8}\right)\left(\frac{1}{126}\right)^{0.25} = 0.879 \text{ lanes/girder}$$

Number of live load moment lanes (Span 1) = (0.879 lanes/girder)(5 girders)
= 4.395 lanes

7.12.10.2 Live Load Lanes for Shear

Using an equation from Table 4.6.2.2.3a-1:

$$\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1} = \left(\frac{12}{7.3}\right)^{0.9}\left(\frac{81}{(12.0)(126)}\right)^{0.1} = 1.167 \text{ lanes/girder}$$

Number of live load shear lanes (Span 1) = (1.167 lanes/girder)(5 girders)
= 5.835 lanes

Notes:

- Assuming a constant structure width, the only term that will probably vary within a frame is the span length, *L*. Therefore, the number of live load lanes for both moment and shear will vary from span to span. Article 4.6.2.2.1 (Caltrans, 2014*)*.
- In negative moment regions, near interior supports, between points of *DC* flexural contraflexure, the *"L"* used to calculate negative moment is the average length of the two adjacent spans.
- The Dynamic Load Allowance Factor (*IM*) (in the LFD code known as Impact) is applied to the design and permit trucks only, not the design lane load. Table 3.6.2.1-1 (Caltrans, 2014) summarizes the values of *IM* for various components and load cases.
- The results of a gravity load analysis, including the unfactored moment envelopes of the (*DC*), (*DC* + *DW*) and (*DC* + *DW*+ *HL93*) load cases, are summarized in the diagram on the next page:

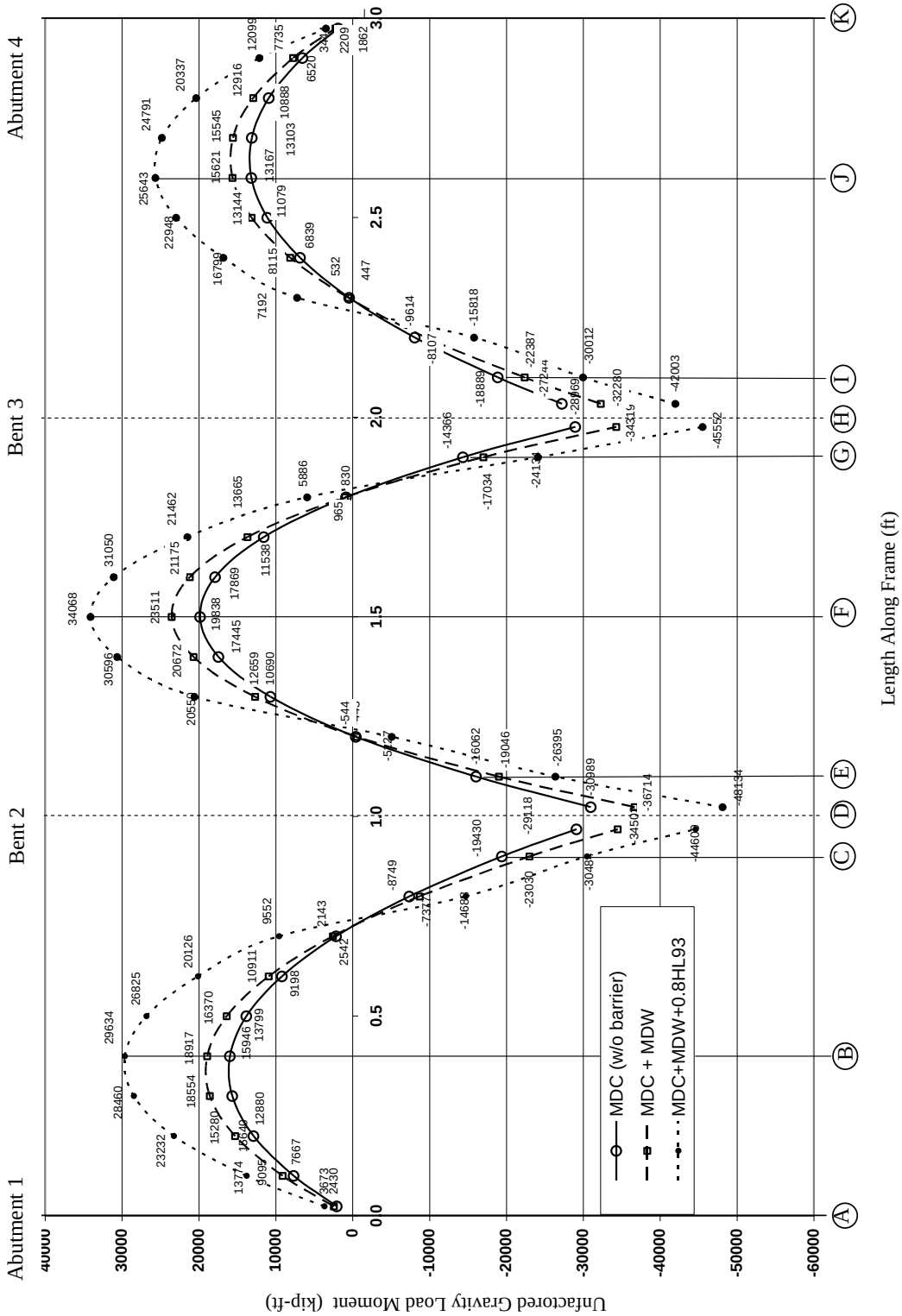


Figure 7.12.10-1 Gravity and Service Load Moment Envelopes for Design of Prestressing

7.12.11 Determine the Prestressing Force

The design of the prestressing force, P_j, is based on the Service III Limit States. The Service III Limit States is defined as the "Load combination for longitudinal analysis relating to tension in prestressed concrete superstructures with the objective of crack control…" Article 3.4.1 (AASHTO, 2012). The following loads, and corresponding load factors, shall be considered in the design of P_j:

DC = Dead load of structural components and non-structural attachments, (γ = 1.0) Article 3.3.2 (AASHTO, 2012)

DW = Dead load of wearing surface and utilities, (γ = 1.0) Article 3.3.2 (AASHTO, 2012)

*HL*93 = Service live load, (γ = 0.8) Section 3.3.2 (AASHTO, 2012)

Service III Limit States load cases: CA Amendments Table 5.9.4.2.2 (Caltrans, 2014).

Case 1: No tension allowed for components with bonded prestressing tendons or reinforcement, subjected to permanent loads (*DC*, *DW*) only.

$$\frac{M_{DC+DW}y}{I} + \frac{FC_{pT}(P_j)}{A} + \frac{MC_{pT}(P_j)y}{I} = 0 \qquad \text{(7.12.11-1, modified 7.1-1)}$$

Case 2: Allowable tension $0.19\sqrt{f'_c}$ ksi are for components subjected to the Service III Limit States (*DC*, *DW*, (0.8) *HL*93), and subjected to not worse than moderate corrosion conditions, located in Environmental Areas I or II. Allowable tension $0.0948\sqrt{f'_c}$ ksi are for components subjected to severe corrosion condions located in Enviromental Area III.

$$\frac{M_{DC+DW+0.8HL93}y_{t(bent)}}{I} + \frac{FC_{pT}(P_j)}{A} + \frac{MC_{pT}(P_j)y}{I} = 0.19\sqrt{f'_c}\ or\ 0.0948\sqrt{f'_c}$$

(7.12.11-2, modified 7.1-1)

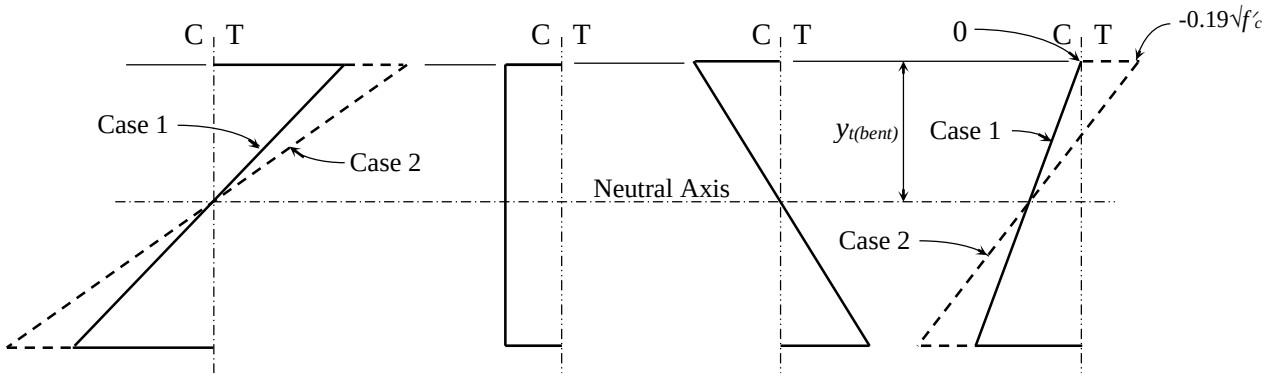
The design of the jacking force usually controls at locations with the highest demand moments within a given frame. Upon inspection of the demand moment diagram plotted earlier in this example, the design of P_j will control at one of two locations:

- The right face of the cap at Bent 2 (top fiber)
- Mid-span of Span 2 (bottom fiber)

Load cases 1 and 2 must be applied at both the right face of the cap at Bent 2, and at midspan of Span 2, with the overall largest P_j controlling the design of the entire frame.

Solve for the jacking force based on two-end stressing data gathered earlier in the example problem:

Right face of the cap at Bent 2 (top fiber):



$$\text{Case 1: } \frac{M_{DC+DW}\,y_{t(bent\ face)}}{I_{(bent\ face)}} + \frac{FC_{pT}P_j}{A_{(bent\ face)}} + \frac{MC_{pT}P_j\,y_{t(bent\ face)}}{I_{(bent\ face)}} = 0$$

$$\text{Case 2: } \frac{M_{DC+DW+0.8HL93}\,y_{t(bent)}}{I_{(bent)}} + \frac{FC_{pT}P_j}{A_{(bent\ face)}} + \frac{MC_{pT}P_j\,y_{t(bent\ face)}}{I_{(bent\ face)}} = 0.19\sqrt{f_c'}$$

Figure 7.12.11-1 Elastic Stresses in an Uncracked Prestress Beam. Effects of Prestress by Component at Top of the Beam

Reading Figure 7.12.13-1
M_{DC+DW} = -36,713 kip-ft
$M_{DC+DW+0.8HL93}$ = -48,134 kip-ft

Interpolating the Force Coefficent from Table 7.12.7.5-1 between points *D* and *E*
$FC_{@pt\ D}$ = 0.787
$FC_{@pt\ E}$ = 0.769
Span 2 Length = 168 ft
Distance from *CL* of column to face of cap (pt *D*) = 4 ft
Distance from *CL* of column to location of first inflection point of Span 2 (pt *E*) = 16.8 ft

$$\left(\frac{0.787-0.769}{0-16.8}\right)(4-0)+0.787=0.783$$

Reading Figure 7.12.9-1 for MC_{PT} @ the face of cap at Bent 2: MC_{PT} = 2.375

Rearranging terms of 7.12.11-1 and 7.12.11-2 to solve for P_j:

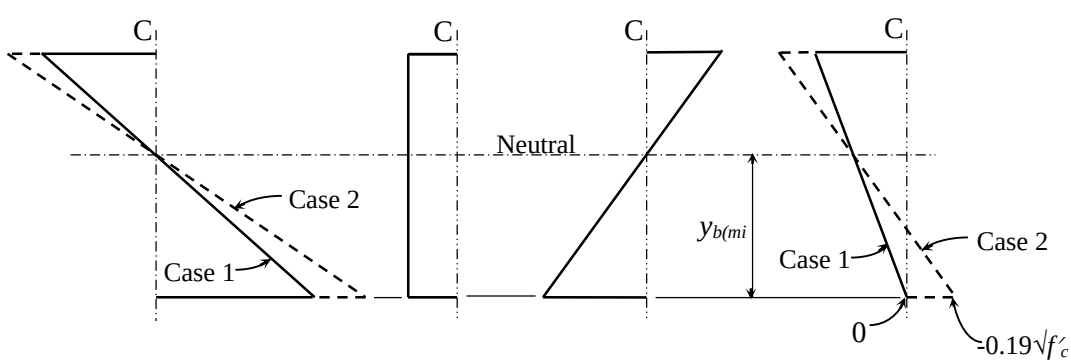
Case 1: $$P_j = -\frac{\dfrac{M_{DC+DW}\,y_{t(bent\ face)}}{I_{(bent)}} + 0}{\dfrac{FC_{pT}}{A_{(bent\ face)}} + \dfrac{MC_{pT}\,y_{t(bent\ face)}}{I_{(bent\ face)}}}$$

$$P_j = -\frac{\dfrac{(-36,713)(3.25)}{824} + 0}{\dfrac{0.783}{115} + \dfrac{(2.375)(3.25)}{824}} = 8,957 \text{ kips}$$

Case 2: $$P_j = -\frac{\dfrac{M_{DC+DW+0.8HL93}\,y_{t(bent\ face)}}{I_{(bent\ face)}} - (0.19)\sqrt{f'_c}\,(144)}{\dfrac{FC_{pT}}{A_{(bent\ face)}} + \dfrac{MC_{pT}\,y_{t(bent)}}{I_{(bent\ face)}}}$$

$$P_j = -\frac{\dfrac{(-48,134)(3.25)}{824} - (0.19)\sqrt{4}(144)}{\dfrac{0.783}{115} + \dfrac{(2.375)(3.25)}{824}} = 8,359 \text{ kips}$$

Mid-span of Span 2 (bottom fiber):



Case 1: $$\frac{M_{DC+DW}\,y_{b(mid)}}{I_{(mid)}} + \frac{FC_{pT}P_j}{A_{(mid)}} + \frac{MC_{PT}P_j\,y_{b(mid)}}{I_{(mid)}} = 0$$

Case 2: $$\frac{M_{DC+DW+0.8HL93}\,y_{b(mid)}}{I_{(mid)}} + \frac{FC_{pT}P_j}{A_{(mid)}} + \frac{MC_{pT}P_j\,y_{t(bent)}}{I_{(bent)}} = -0.19\sqrt{f'_c}$$

Figure 7.12.11-2 Elastic Stresses in an Uncracked Prestress Beam. Effects of Prestress by Component at the Bottom of the Beam

Reading the Force Coefficent from Table 7.12.7.5-1 at point *F*
$FC_{@\text{pt } F} = 0.742$

Reading Figure 7.12.9-1 for MC_{PT} @ the *CL* of Span 2: $MC_{PT} = -1.202$

Reading Figure 7.12.10-1
M_{DC+DW} = 23,511 kip-ft
$M_{DC+DW+0.8HL93}$ = 34,068 kip-ft

Case 1: $$P_j = -\frac{\dfrac{M_{DC+DW}\,y_{b(mid)}}{I_{(mid)}} + 0}{\dfrac{FC_{pT}}{A_{(mid)}} + \dfrac{(MC_{pT})\,y_{b(mid)}}{I_{(mid)}}}$$

$$P_j = -\frac{\dfrac{(23,511)(-3.8)}{729} + 0}{\dfrac{0.742}{103} + \dfrac{(-1.202)(-3.8)}{729}} = 9,100 \text{ kips}$$

Case 2: $$P_j = -\frac{\dfrac{M_{DC+DW+0.8HL93}\,y_{b(mid)}}{I_{(mid)}} + (0.19)\sqrt{f_c'}}{\dfrac{FC}{A_{(mid)}} + \dfrac{MCy_{b(mid)}}{I_{(mid)}}}$$

$$P_j = -\frac{\dfrac{(34,068)(-3.8)}{729} + (0.19)(\sqrt{4})(144)}{\dfrac{0.742}{103} + \dfrac{(-1.202)(-3.8)}{729}} = 9,124 \text{ kips}$$

Therefore, P_j = 9,124 kips, round to the nearest 10 kips, P_j = 9,120 kips

Notes: The overall largest P_j was calculated at the midspan of Span 2 under the Case 2 load condition. Two observations can be made:

Now that we have a P_j we can check our elastic shortening assumption using CA Amendments 5.9.5.2.3b-1 and Equation 7.5.1.3-1:

$$f_{cgp} = f_g + f_{ps} = \frac{M_{DL}e}{I_g} - \left(\frac{P_j}{A_g} + \frac{P_j e^2}{I_g}\right) \qquad (7.5.1.3\text{-}1)$$

$$\Delta f_{pES} = 0.5\frac{E_p}{E_{ct}} f_{cgp} \qquad \text{(CA Amendments 5.9.5.2.3b-1)}$$

Table 7.12.11-1 Values of f_{cgp} and Δf_{pES}

Loc	f_{cgp}	Δf_{pES}	Loc	f_{cgp}	Δf_{pES}	Loc	f_{cgp}	Δf_{pES}
0.024	-0.615	-2.285	1.024	-1.546	-5.745	2.034	-1.424	-5.292
0.100	-0.770	-2.863	1.100	-0.847	-3.147	2.100	-0.934	-3.470
0.200	-1.170	-4.348	1.200	-0.655	-2.433	2.200	-0.594	-2.208
0.300	-1.579	-5.870	1.300	-1.137	-4.227	2.300	-0.791	-2.940
0.400	-1.772	-6.585	1.400	-1.677	-6.232	2.400	-1.187	-4.412
0.500	-1.682	-6.252	1.500	-1.903	-7.071	2.500	-1.538	-5.716
0.600	-1.357	-5.043	1.600	-1.689	-6.277	2.600	-1.694	-6.297
0.700	-0.929	-3.453	1.700	-1.155	-4.293	2.700	-1.586	-5.894
0.800	-0.632	-2.349	1.800	-0.665	-2.471	2.800	-1.246	-4.629
0.900	-0.797	-2.964	1.900	-0.830	-3.083	2.900	-0.832	-3.091
0.968	-1.281	-4.761	1.976	-1.504	-5.590	2.975	-0.615	-2.285
							Average	-4.351

At this point, we would rerun our numbers using the 4.4 ksi value for Δf_{pES}. However, for this example we will choose not to rerun the numbers. The 1.4 ksi difference between assumed and calculated result in about a 3% increase in P_j.

7.12.12 Determine the Required Concrete Strength

Now that the jacking force has been calculated for this structure, we can determine the stresses in the concrete due to prestressing. Prestress stresses need to be computed in order to determine the initial (f'_{ci}) and final (f'_c) concrete strengths required. The design of f'_{ci} is based on concrete stresses present at the time of jacking, which includes the initial prestress stress, f_{pi}. The initial prestress stress considers losses due to friction (ΔFC_{pF}) only. The design of f'_c is based on service level concrete stresses that occur after a period of time, which includes the effective prestress stress, f_{pe}. The effective (total) prestress stress considers the effects of all prestress losses (ΔFC_{pT}). The equations for concrete stresses due to prestressing are as follows:

$$f_{pi} = \frac{P_j FC_F}{A_g} + \frac{P_j (e)(FC_F) y_b}{I_g} \tag{7.12.12-1}$$

$$f_{pe} = \frac{P_j FC_{pT}}{A_g} + \frac{P_j (MC_{pT}) y_b}{I_g} \tag{7.12.12-2}$$

where:

A_g = gross area of section (in.^{2})

FC = force coefficient for loss

FC_{pF} = force coefficient for loss from friction

e	=	eccentricity of the anchorage device or group of devices with respect to the centroid of the cross section. Always taken as a positive (ft)
f'_{pe}	=	effective stress in the prestressing steel after losses (ksi)
f'_{pi}	=	initial stress in the prestressing steel after losses, considering only the effects of friction loss. No other P/S losses have occurred (ksi)
I_g	=	moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})
MC_{PT}	=	total moment force coefficient for loss (ft)
P_j	=	force in prestress strands before losses (kip)

Both initial and final stresses for the concrete top and bottom fibers due to the effects of prestressing have been calculated, and are shown on the following two diagrams.

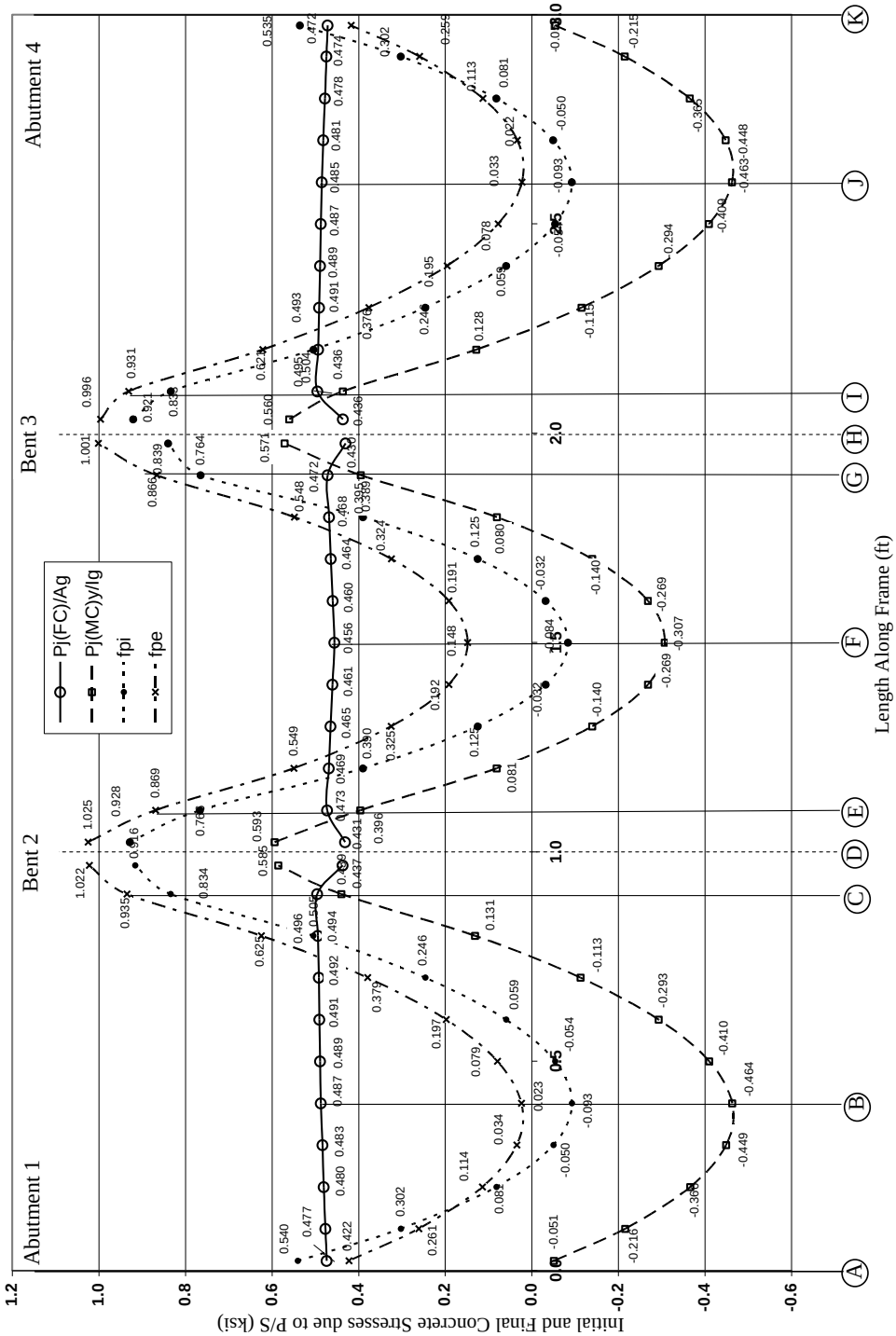


Figure 7.12.12-1 Top Fiber Concrete Stresses Due to Prestressing

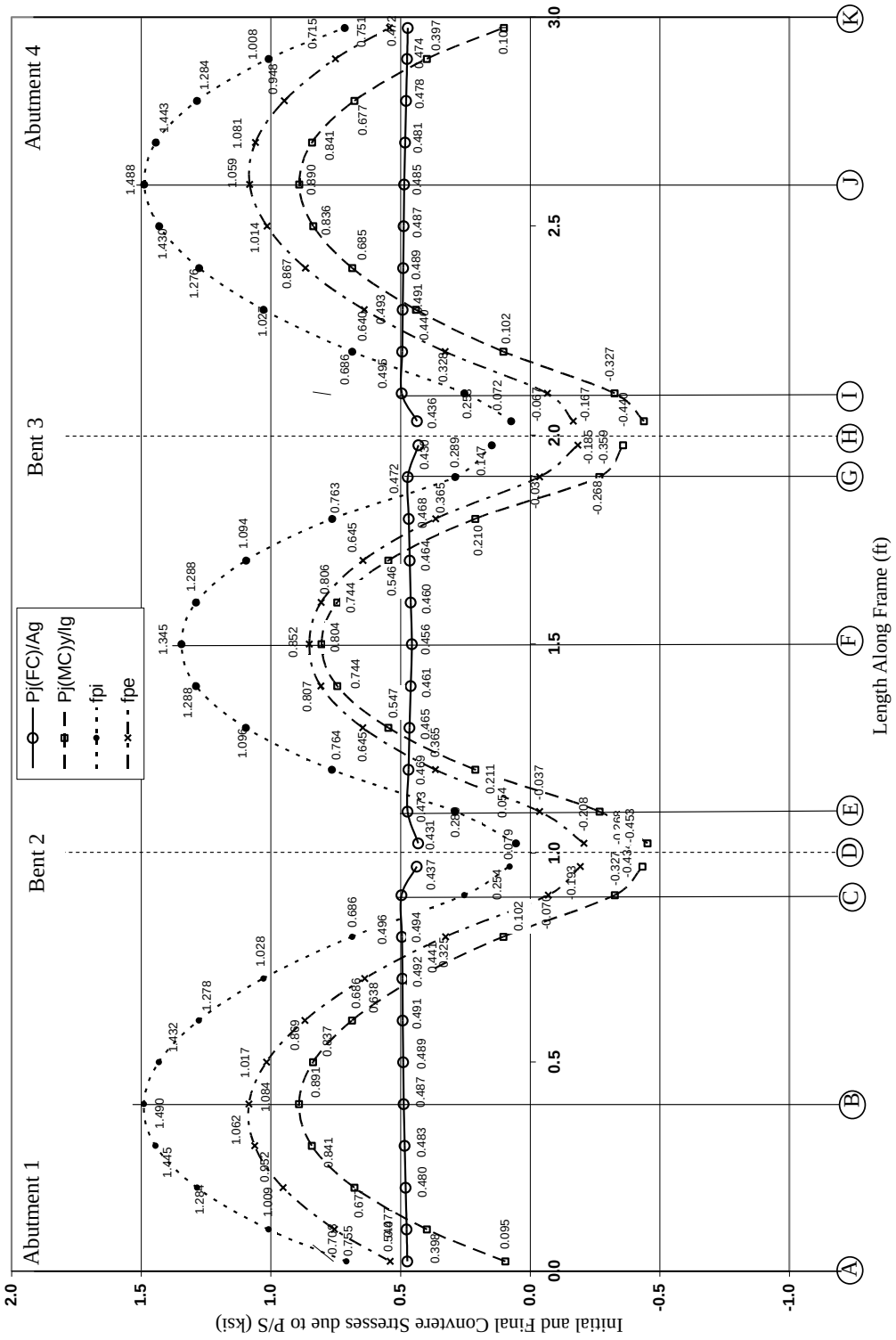


Figure 7.12.12-2 Bottom Fiber Concrete Stresses Due to Prestressing

Design of both initial and final concrete strengths required are governed by the Service I load case. This load case is defined as a load combination relating to the normal operational use of the bridge with a 55 mph wind and all loads taken at their nominal value (γ = 1.0) Article 3.4.1(AASHTO, 2012).

7.12.12.1 Design of f'_c

The definition of f'_c is "the specified strength concrete for use in design" Article 5.3 (AASHTO, 2012). Article 5.4.2.1 (AASHTO, 2012) specifies the compressive strength for prestressed concrete and decks shall not be less than 4.0 ksi.

Additionally, Table 5.9.4.2.1-1 (AASHTO, 2012*)* lists compressive stress limits for prestressed components at the service Limit States after all losses as:

- In other than segmentally constructed bridges due to the sum of the effective prestress and permanent loads, the concrete has a compressive stress limit of 0.45 f'_c (ksi).

$$f'_c \geq \frac{f_{pe} + f_{DC+DW}}{0.45} \text{ (ksi)} \qquad (7.12.12\text{-}3)$$

where:

f_{DC+DW} = stress in bridge from *DC* and *DW* load cases (ksi)

- Due to the sum of effective prestress, permanent loads and transient loads, the concrete has a compressive stress limit of 0.60 f'_c (ksi).

$$f'_c \geq \frac{f_{pe} + f_{DC+DW} + f_{HL93}}{0.60} \text{ (ksi)} \qquad (7.12.12\text{-}4)$$

Solving for f'_c at face of support at Bent 2 (0.0 L_2 point).

Using Equations 7.12.12-2 and 7.12.12-3 for only the *DL* case.

At the cap face for Bent 2 the deck is in tension under service loads, therefore the prestressing steel is close to the deck to pull the section together, and resist tension. The controlling concrete strength demand will be opposite of the tension where the service loads act in compression, and the prestressing force acts in tension.

$$f_{pi} = \frac{P_j FC_{pT}}{A_g} + \frac{P_j (MC_{pT}) y_b}{I_g}$$

$$= \left[\frac{9{,}120\,(0.783)}{115} + \frac{(9{,}120)(2.375)(-3.5)}{824} \right] = -30.0 \text{ ksf}$$

Notice that the stress due to the prestressing steel is negative. This means that the prestressing steel is pulling together the face of the concrete that is in tension, causing the side opposite side of the prestressing steel to be in tension.

$$f_{pe} = -30.0\,\text{ksf}\,\frac{1}{144} = -0.21\,\text{ksi}$$

$$f_{DC+DW} = \frac{M_{DC+DW}(y_b)}{I_g} = \frac{-36,713(-3.5)}{824} = 156.0\ \text{ksf}$$

$$f_{DC+DW} = 156.0\ \text{ksf}\,\frac{1}{144} = 1.08\ \text{ksi}$$

$$f_c' \geq \frac{f_{pe} + f_{DC+DW}}{0.45} = \frac{-0.21 + 1.08}{0.45} = 1.9\ \text{ksi}$$

Using Equation 7.12.12-4 for only the *LL* case.

$$f_{HL93} = \frac{M_{HL93}(y_b)}{I_g} = \frac{-14,275\ \text{kip-ft}(-3.5\,\text{ft})}{824\ \text{ft}^4} = 60.6\ \text{ksf}$$

$$f_{HL93} = 60.6\ \text{ksf}\,\frac{1}{144} = 0.42\ \text{ksi}$$

Using Equation 7.12.12-5 for only the service load case.

$$f_c' \geq \frac{f_{pe} + f_{DC+DW} + f_{HL93}}{0.60} = \frac{-0.21 + 1.08 + 0.42}{0.60} = 2.16\ \text{ksi}$$

Therefore the minimum f'_c controls from Article 5.4.2.1 (AASHTO, 2012):
f'_c = 4.0 ksi

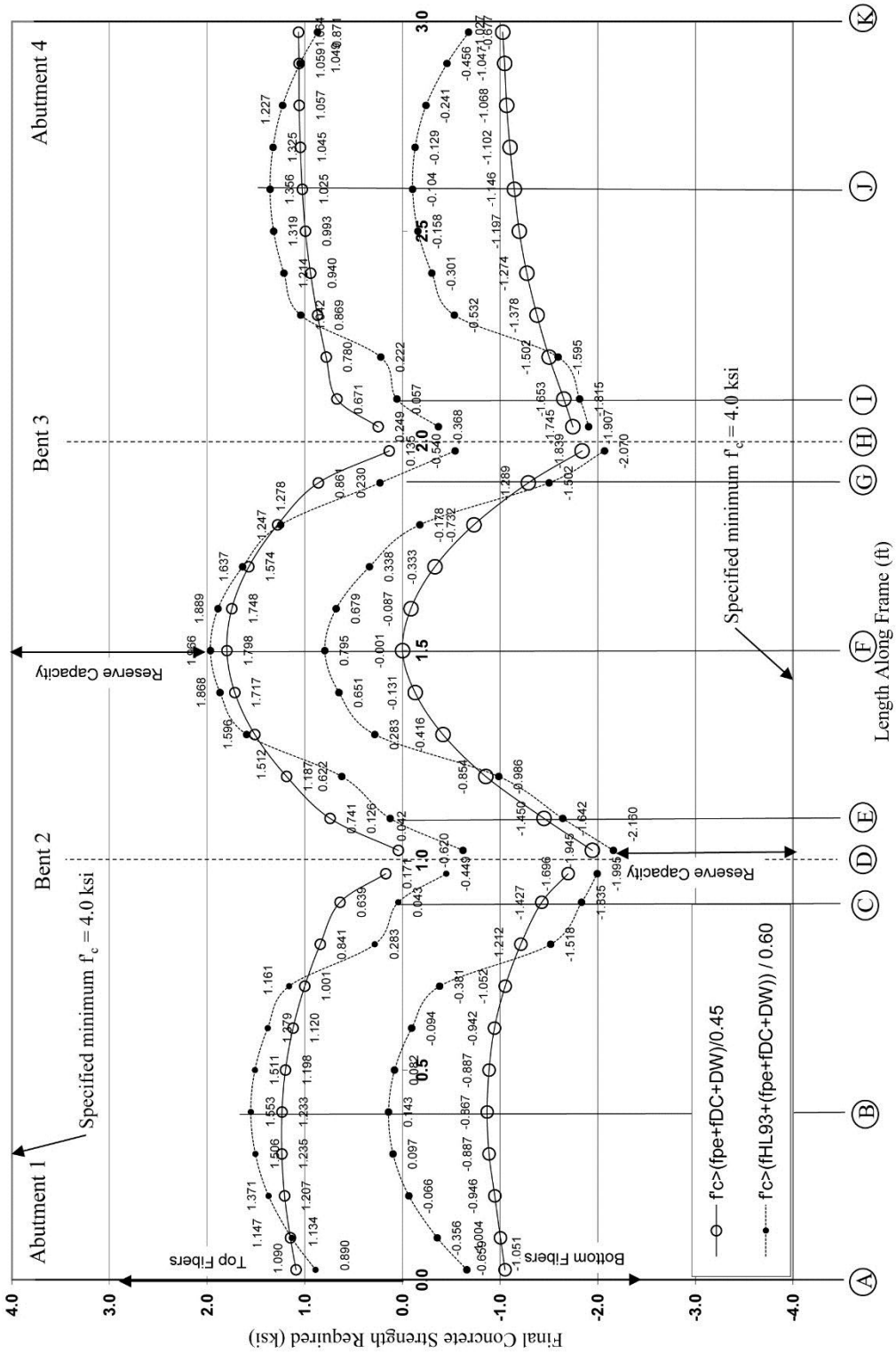


Figure 7.12.12-3 Determining Final Concrete Strength Required

7.12.12.2 Design of f'_{ci}

The definition of f'_{ci} is "the specified compressive strength of concrete at time of initial loading of prestressing" (AASHTO Article 5.3). There are two criteria used to design f'_{ci}, and they are as follows:

The compressive stress limit for pretensioned and post-tensioned concrete components, including segmentally constructed bridges, shall be 0.60 f'_{ci} (ksi). Only the stress components present during the time of prestressing shall be considered (AASHTO Article 5.9.4.1.1).

$$f'_{ci} \geq \frac{f_{pi} + f_{DC\,w/o\ b}}{0.60} \text{ (ksi)} \qquad (7.12.12\text{-}6)$$

where:

$f_{DC\,w/o\,b}$ = stress in concrete due to the Dead Load of the structural section only (ksi)

The specified initial compressive strength of prestressed concrete shall not be less than 3.5 ksi (MTD 11-3).

Solving for f'_{ci} at the cap face Bent 2 (0.02 L_2 point).

Interpolating the Force Coefficent due to Friction from Table 7.12.7.5-1 between points *D* and *E*

$FC_{pF@\text{pt }D}$ = 0.925
$FC_{pF@\text{pt }E}$ = 0.907
Span 2 Length = 168 ft
Distance from *CL* of column to face of cap (pt *D*) = 4 ft
Distance from *CL* of column to location of first inflection point of Span 2 (pt *E*) = 16.8 ft

$$\left(\frac{0.925 - 0.907}{0 - 16.8}\right)(4 - 0) + 0.925 = 0.921$$

Interpolating the eccentricities from Figure 7.12.8-5 between points *D* and *E*

$e_{@\text{pt }D}$ = 2.122 ft
$e_{@\text{pt }E}$ = 0.896
Span 2 Length = 168 ft
Distance from *CL* of column to face of cap (pt *D*) = 4 ft
Distance from *CL* of column to location of first inflection point of Span 2 (pt *E*) = 16.8 ft

$$\left(\frac{2.122 - 0.896}{0 - 16.8}\right)(4 - 0) + 2.122 = 1.830$$

Using Equations 7.12.12-1 and 7.12.12-5

$$f_{pi} = \frac{P_j FC_F}{A_g} + \frac{P_j(e)(FC_F)y_b}{I_g}$$

$$= \left[\frac{9{,}120\,(0.921)}{115} + \frac{9{,}120\,(1.830)\,(0.921)\,(-3.5)}{824}\right] = 7.75\,\text{ksf}$$

$$f_{pi} = 7.5\frac{1}{144} = 0.054\,\text{ksi}$$

Using Equation 7.12.12-6

$$f_{DCw/oB} = \frac{M_{DCw/ob}(y_b)}{I_g} = \frac{-30{,}990\,(-3.5)}{824} = 68.2\,\text{ksf}$$

$$f_{DCw/oB} = 68.2\text{ ksf} = 0.474\,\text{ksi}$$

Using Figure 7.12.12-4 the minimum f'_c controls from MTD 11-3: f'_{ci} = 3.5 ksi

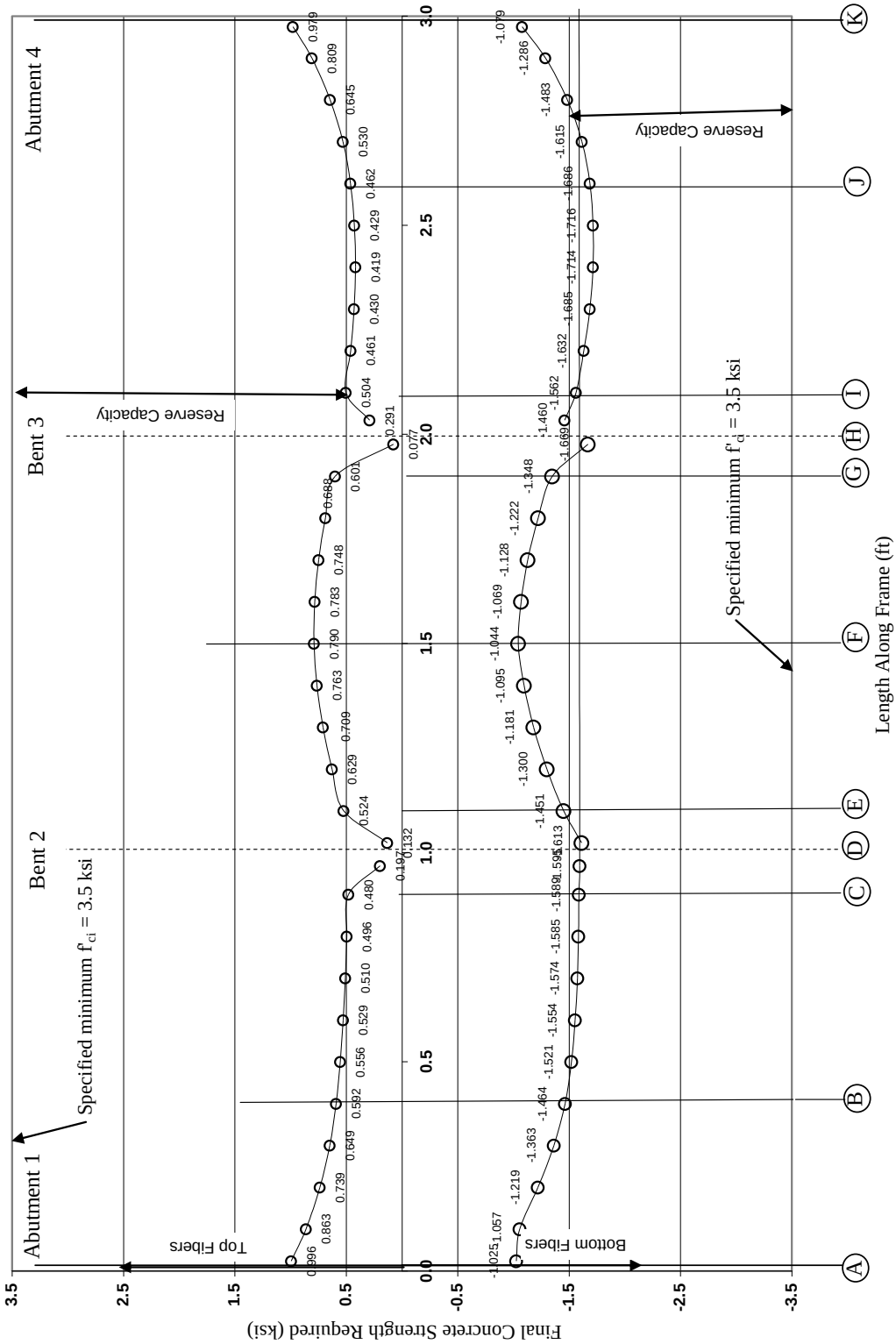


Figure 7.12.12-4 Determining Initial Concrete Strength Required

7.12.13 Design of Flexural Resistance

For rectangular or flanged sections subjected to flexure about one axis where the approximate stress distribution as specified in Article 5.7.2.2 (AASHTO, 2012) is used and for which f_{pe} is not less than 0.5 f_{pu}, the average stress in the prestressing steel, f_{ps}, may be taken as:

$$f_{ps} = f_{pu}\left(1 - \frac{kc}{d_p}\right) \qquad \text{(AASHTO 5.7.3.1.1-1)}$$

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) \qquad \text{(AASHTO 5.7.3.1.1-2)}$$

Alternatively, the stress in the prestressing steel may be determined by strain compatibility (see AASHTO 5.7.3.2.5).

For rectangular section behavior, the distance between the neutral axis and the compressive face can be represented as:

$$c = \frac{A_{ps} f_{pu} + A_s f_s + A_s' f_s'}{0.85 f_c' \beta_1 b + k A_{ps} \dfrac{f_{ps}}{d_p}} \qquad \text{(AASHTO 5.7.3.1-4)}$$

Substitute f_{ps} for f_{pu} after solving Equation 5.7.3.1.1-1 (AASHTO, 2012)

The factored resistance M_r shall be taken as:

$$M_r = \phi M_n \qquad \text{(AASHTO 5.7.3.2.1-1)}$$

For flanged sections subjected to flexure about one axis and for biaxial flexure with axial load as specified in Article 5.7.4.5, (AASHTO, 2012), where the approximate stress distribution specified in Article 5.7.2.2 (AASHTO, 2012) is used and the tendons are bonded and where the compression flange depth is less than $a = \beta_1 c$, as determined in accordance with Equation 5.7.3.1.1-3, the nominal flexural resistance may be taken as:

$$M_n = \underbrace{A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)}_{\substack{\text{Prestressing}\\ \text{Steel only}}} + \underbrace{A_s f_s\left(d_s \frac{a}{2}\right)}_{\substack{\text{Additional}\\ \text{Flexural Steel}}} - \underbrace{A_s' f_s'\left(d_s' - \frac{a}{2}\right)}_{\substack{\text{Compression}\\ \text{Steel}}} + \underbrace{0.85 f_c'(b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)}_{\substack{\text{Flanged Section Component}\\ a = \beta_1 c \le h_f}}$$

$$\frac{f_{ps}}{f_{pu}} = 0.9 \qquad \text{(AASHTO C5.7.3.1.1-1)}$$

$$k = 0.28 \qquad \text{(AASHTO C5.7.3.1.1-1)}$$

$$A_{ps} = \frac{P_j}{0.75 f_{pu}} \qquad (7.12.13\text{-}1)$$

where:

A_{ps} = area of prestressing steel (in.^{2})

A_s = area of non-prestressed tension reinforcement (in.^{2})

b = width of the compression face of a member (in.)

c = distance from extreme compression fiber to the neutral axis (in.)

d_p = distance from extreme compression fiber to the centroid of the prestressing tendons (in.)

f'_c = specified compressive strength of concrete used in design (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

f_{pu} = specified tensile strength of prestressing steel (ksi)

M_r = factored flexural resistance of a section in bending (kip-in.)

M_n = nominal flexure resistance (kip-in.)

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength Limit States to the depth of the actual compression zone

ϕ = resistance factor

The factored ultimate moment, M_u, shall be taken as the greater of the following two Strength I and II Limit Statess as defined in California Amendments Article 3.4.1 and Table 3.4.1-1 (Caltrans, 2014).

Strength I: $M_{u(HL93)} = 1.25\ (M_{DC}) + 1.50\ (M_{DW}) + 1.75\ (M_{HL93}) + 1.00\ (M_{P/S\ s})$

Strength II: $M_{u(P\text{-}15)} = 1.25\ (M_{DC}) + 1.50\ (M_{DW}) + 1.35\ (M_{P\text{-}15}) + 1.00\ (M_{P/S\ s})$

The largest value of M_u indicated the governing Limit States at a given location. It is possible to have different Limit Statess at different locations.

Unless otherwise specified, at any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, $M_{r\ (min)}$, at least equal to the lesser of:

1.33 M_u as defined in Section 5.7.3.3.2 (AASHTO, 2012)

$$f_r = 0.24\sqrt{f_r} \qquad \text{(AASHTO 5.4.2.6)}$$

$$M_{cr} = \gamma_3 \left[(\gamma_1 f_r + \gamma_2 f_{cpe}) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right] \qquad \text{(AASHTO 5.7.3.3.2-1)}$$

The second part of the equation is crossed out because this is not a composite section.

Article 5.7.3.3.2 defines:

γ_1 = 1.6 for super structures that are not precast segmental

γ_2 = 1.1 for bonded tensions

γ_3 = 0.75 if additional mild reinforcement is A 706, grade 60 reinforcement

where:

f_r = modulus of rupture of concrete (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

M_{cr} = cracking moment (kip-in.)

S_c = section modulus for the extreme fiber of the composite sections where tensile stress is caused by externally applied loads (in.^{3})

γ_1 = flexural cracking variability factor

γ_2 = prestress variability factor

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of reinforcement

The prestressing steel present in the section by itself may be enough to resist the applied factored moment. However, additional flexural steel may have to be added to provide adequate moment resistance for the Strength I and II Limit Statess. Flexural steel provided for seismic resistance can be relied upon for Strength Limit Statess.

AASHTO Article 5.7.3.3, defines a limit on tension steel to prevent over-reinforced sections, has been eliminated in the 2006 interims. The current approach involves reducing the flexural resistance factor when the tensile strain in the reinforcement falls below 0.005. In other words, over-reinforced sections are allowed by the code, but a more conservative resistance factor is applied. Conventional designs will likely result in tensile strains greater than 0.005. The tensile strain can be determined using the c/d_e ratio. From a simple plane strain diagram assuming concrete strain of 0.003, a c/d_e ratio of 0.375 corresponds to a tensile strain of 0.005. If the c/d_e ratio exceeds 0.375, then a reduced ϕ must be used as defined in Article 5.5.4.2 (Caltrans, 2014).

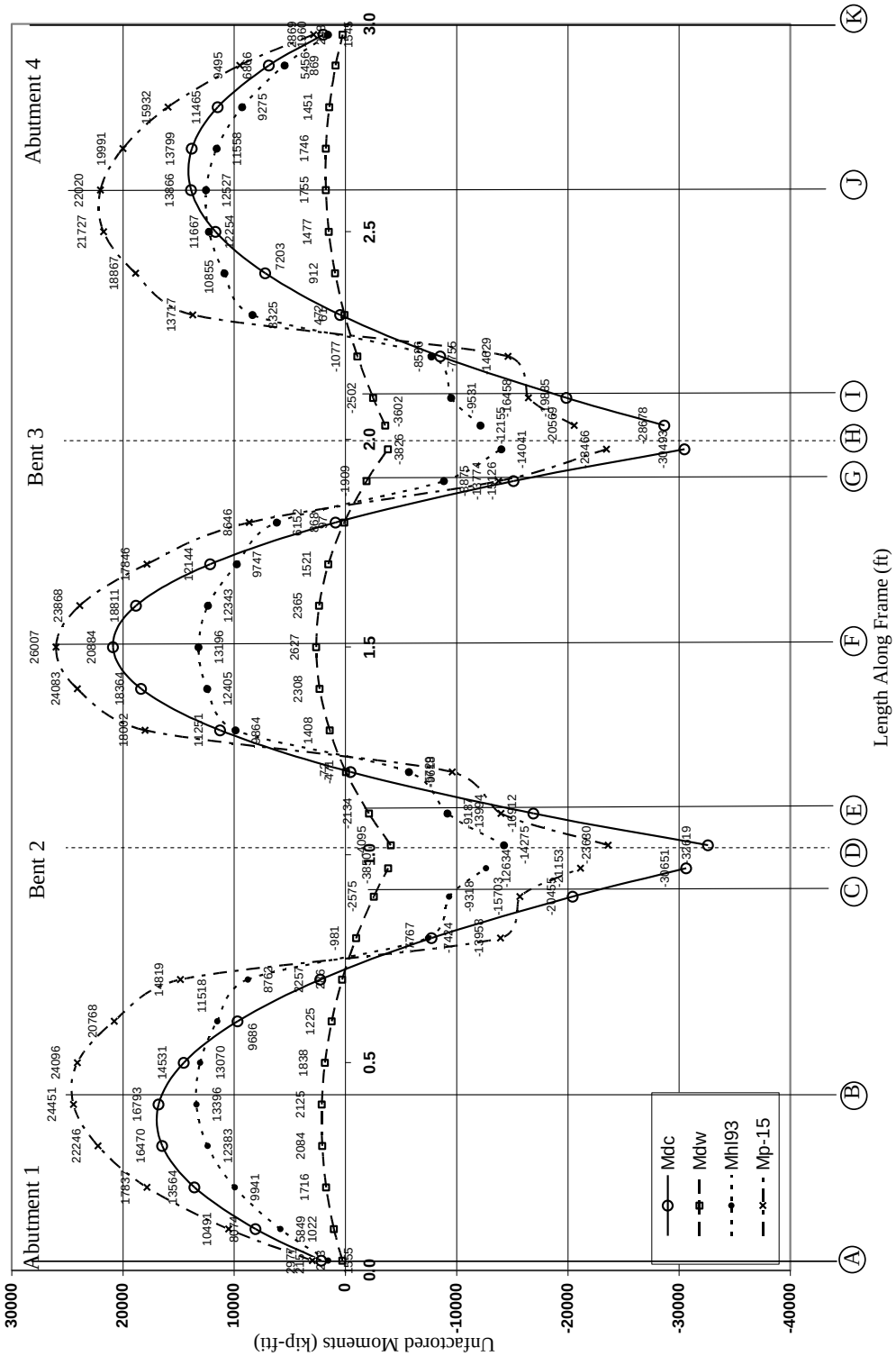


Figure 7.12.13-1 Gravity Load Moment Envelopes for Design of Flexural Resistance

Find the flexural resistance of the section at the right face of cap at Bent 2 considering the Area of *P*/*S* steel only. If required, find the amount of additional flexural steel needed to resist the factored nominal resistance ϕM_n.

$$M_{P/S\,s} = P_j\,(MC_s) \qquad (7.12.13\text{-}2)$$

where:

$M_{P/S\,s}$ = moment due to the secondary effects of prestressing (k-ft)

Step 1: Determine the controlling Strength Limit State used to determine the factored ultimate moment, M_u:

Strength I:

(MC_s) = 0.856 ft (Figure 9.12.12-1)

$M_{P/S\,s}$ = (9,120 kips) (0.856 ft) = 7,810 kip-ft

$M_{u(HL93)}$ = 1.25 (M_{DC}) + 1.50 (M_{DW}) + 1.75 (M_{HL93}) + 1.00 ($M_{P/S\,s}$)

$M_{u(HL93)}$ = 1.25 (-32,619) + 1.50 (-4,095) + 1.75 (-14,275) + 1.00 (7,810)

= -64,090 kip-ft

Strength II:

$M_{u(P\text{-}15)}$ = 1.25 (M_{DC}) + 1.50 (M_{DW}) + 1.35 ($M_{P\text{-}15}$) + 1.00 ($M_{P/S\,s}$)

$M_{u(P\text{-}15)}$ = 1.25 (-32,619) + 1.50 (-4,095) + 1.35 (-23,630) + 1.00 (7,810)

= -71,010 kip-ft

The Strength II Limit State controls, M_u = -71,010 kip-ft

Step 2: Compute M_{cr} to determine which criteria governs the design of the factored resistance, M_r (1.33M_u or using AASHTO 5.7.3.3.2-1).

$$S_c = \frac{I_{bent\ face}}{y_{bent\ face}} = \frac{(824\ \text{ft}^4)12^4}{3.25\ \text{ft}(12)} = 438{,}100\ \text{in.}^3$$

$$f_r = 0.24\sqrt{f'_c} = 0.37\sqrt{4} = 0.48\,\text{ksi}$$

f_{cpe} = 1.025 ksi (from plot of *P*/*S* stresses)

$$M_{cr} = \gamma_2\left[(\gamma_1 f_r + \gamma_2 f_{cpe})\,S_c - M_{drc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$$

$$= 0.75\left\{\left[1.6(0.48) + 1.1(1.024)\right]438{,}100 - 0\right\}$$

$$= 622{,}700\ \text{kip-in.}$$

M_{cr} = 622,700 kip-in. = 51,900 kip-ft

$M_{r\ (min)}$ = the lesser of:

M_{cr} = (51,900) = 51,900 kip-ft

$1.33M_u$ = 1.33(71,010) = 94,440 kip-ft

Therefore, $M_{r\ (min)}$ = 51,900 kip-ft

Step 3: Compute the nominal moment resistance of the section based on the effects of the prestressing steel using AASHTO 5.7.3.1.1-4 only and substituting out A_{ps}:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A_s' f_s'}{0.85 f_c' \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}} = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{pu} + A_s f_s - A_s' f_s'}{0.85 f_c' \beta_1 b + k \dfrac{P_j}{0.75 f_{pu}} \dfrac{f_{pu}}{d_p}}$$

Assuming no compression or tension resisting mild steel, A_s and A'_s both equal zero.

b	=	The soffit width = overall width – overhang width (including slope)
b	=	58(12)+10 – 2[5(12)+10.5] = 517 in.
d_p	=	structure depth – prestressing force distance to deck (interpolated between points *D* and *E*)
d_p	=	81 – 17.0 = 64.0 in.

$$c = \frac{\dfrac{9{,}120}{0.75} + 0 + 0}{0.85(4)(0.85)(517) + 0.28\left(\dfrac{9{,}120}{0.75(64)}\right)} = 7.9 \text{ in.}$$

7.9 in. < h_{soffit} = 12 in.; therefore, rectangular section assumption is satisfied

Using AASHTO Equation 5.7.3.1.1-1:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) = 270\left(1 - 0.28\frac{7.9}{64.0}\right) = 260.7 \text{ ksi}$$

Modifying AASHTO Equation 5.7.3.2.2-1 for rectangular sections produces Equation 7.12.13-3:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) \qquad (7.12.13\text{-}3)$$

From Article 5.7.2.2 (AASHTO, 2012):

$$a = \beta_1 c \tag{7.12.13-4}$$

$$a = \beta_1 c = 0.85(7.9) = 6.7 \text{ in.}$$

$$A_{ps} = \frac{P_j}{0.75 f_{pu}} = \frac{9{,}120}{0.75(270)} = 45.0 \text{ in.}^2$$

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) = 45.0(260.7)\left(64.0 - \frac{6.7}{2}\right) = 711{,}500 \text{ kip-in.}$$

M_n = 711,500 kip-in. = 59,300 kip-ft

Calculating ϕM_n = 0.95 (59,300) = 56,350 kip-ft > 751,900 kip-ft shows that no additional flexural steel is required. However, for illustrative purposes, let's determine A_s based on M_r = 77,300 kip-ft.

Step 4: Compute the area of mild steel required to increase ϕM_n to resist the full factored resistance, M_r:

Rearranging AASHTO 5.7.3.1.1-4 and Equation 7.12.13-4 and substituting out the values for f_{ps} and $A_{ps,}$ results in Equation 7.12.13-5.

$$a = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{pu} + A_s f_s + A_s' f_s'}{0.85 f_c' b + \dfrac{k P_j f_{pu}}{\beta_1 \left(0.75 f_{pu}\right) d_p}} \tag{7.12.13-5}$$

$$\frac{a}{2} = \frac{\dfrac{9{,}120}{0.75} + (60)A_s + 0}{2\left(0.85(4)(517) + \dfrac{0.28(9{,}120)}{(0.85)(0.75)(64.0)}\right)} = 3.35 + 0.0166 A_s$$

Note that we will assume $f_s = f_y$. This assumption is valid if the reinforcement at the extreme steel tension fiber fails. We can check this by measuring ε_t at the end of the calculation.

Modifying AASHTO Equation 5.7.3.2.2-1 for rectangular sections produces Equation 7.12.13-6.

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y \left(d_s - \frac{a}{2}\right) \tag{7.12.13-6}$$

where:

d_s = distance from extreme compression fiber to the centriod of the nonprestressed tensile reinforcement (in.)

d_s = $d - 0.5(h_{deck}) = 81 - 0.5(9.125) = 76.4$ in.

$$\left(\frac{77,300(12)}{0.95}\right) = (45.0)(260.6)(64.0 - (3.35 + 0.0166A_s)) + \ldots$$

$$\ldots + 60\,A_s\,(76.4 - (3.5 + 0.0166\,A_s))$$

$$0.99\,A_s^2 - 4{,}190\,A_s + 264{,}990 = 0$$

Solving the quadratic equation: A_s = 64.2 in.^{2} (65 # 9 bars A_s = 65 in.^{2})

Step 5: Verify Assumptions – Two assumptions were made in the determination of A_s. The first was that the mild steel would yield and we could use f_y for f_s. The validity of this assumption can be checked by calculating the ε_t. According to the California Amendments (Caltrans, 2014) and Figure 7.12.13-2 if ε_t is > 0.005 then the section is tension controlled and ϕ = 1.00, which is more conservative than the 0.95 we used initially. These values can be easily obtained with a simple strain diagram setting the concrete strain to 0.003.

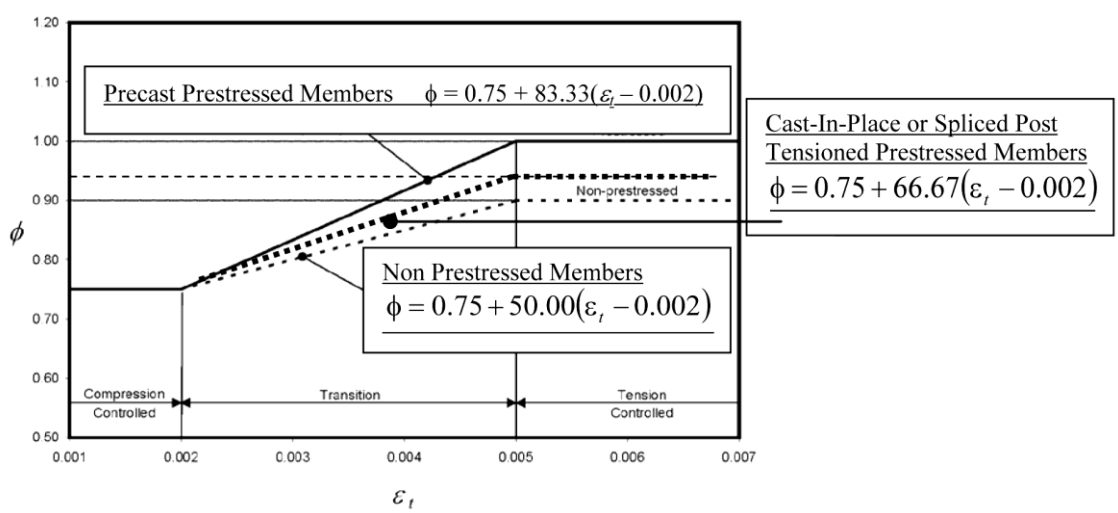


Figure 7.12.13-2 Variation of ϕ with Net Tensile Strain ε_t for Grade 60 Reinforcement and Prestressed Members (California Amendments Figure C5.5.4.2.1-1, 2014)

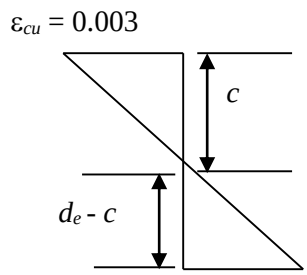


Figure 7.12.13-3 Strain Diagram

where:

c = distance from extreme compression fiber to the neutral axis (in.)

d_e = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.)

ε_{cu} = failure strain of concrete in compression (in./in.)

ε_t = net tensile strain in extreme tension steel at nominal resistance (in./in.)

Having just calculated A_s, it is possible to calculate both c and d_e using AASHTO 5.7.3.1.1-4 and substituting out f_{ps} and A_{ps}:

$$c = \frac{\dfrac{P_j}{0.75} + A_s f_s - A_s' f_s'}{0.85 f_s' \beta_1 b + k \dfrac{P_j}{0.75} \dfrac{1}{d_p}}$$

$$= \frac{\dfrac{9{,}120}{0.75} + (64.2)(60)}{(0.85)(4)(0.85)(517) + (0.28)\dfrac{9{,}120}{0.75}\left(\dfrac{1}{64.0}\right)} = 10.35 \text{ in.}$$

Therefore c = 10.35 in. < 12 in., the rectangular section assumption satisfied.

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}$$

$$= \frac{(45.02)(260.6)(64.2) + (64.2)(60)(76.50)}{(45.02)(260.6) + (64.2)(60)}$$

d_e = 67.1 in.

By similar triangles: $\frac{\varepsilon_c}{c} = \frac{\varepsilon_t}{d_e - c}$:

$$\varepsilon_t = \frac{\varepsilon_c}{c}\left(d_e - c\right) = \frac{0.003}{10.35}\left(67.1 - 10.35\right) = 0.016 > 0.005$$

Therefore, the mild steel yields and $\phi = 1.00$

Find the flexural resistance of the section at midspan of Span 2 considering the area of *P*/*S* steel only. If required, find the amount of additional flexural steel needed to resist the factored nominal resistance, ϕM_n:

Step 1: Determine the controlling Strength Limit State used to determine the factored ultimate moment, M_u:

Strength I:

$M_{P/S\,s}$ = (9,120) (0.854) = 7,790 kip-ft

$M_{u(HL93)}$ = 1.25 (M_{DC}) + 1.50 (M_{DW}) + 1.75 (M_{HL93}) + 1.00 ($M_{P/S\,s}$)

$M_{u(HL93)}$ = 1.25 (20,884) + 1.50 (2,627) + 1.75 (13,196) + 1.00 (7,790)

= 60,930 kip-ft

Strength II:

$M_{u(P\text{-}15)}$ = 1.25 (M_{DC}) + 1.50 (M_{DW}) + 1.35 ($M_{P\text{-}15}$) + 1.00 ($M_{P/S\,s}$)

$M_{u(P\text{-}15)}$ = 1.25 (20,884) + 1.50 (2,627) + 1.35 (26,007) + 1.00 (7,790)

= 72,940 kip-ft

The Strength II Limit State controls, M_u = 72,940 kip-ft

Step 2: Compute M_{cr} to determine which criteria governs the design of the factored resistance, M_r (1.33M_u or using AASHTO 5.7.3.3.2-1).

$$S_c = \frac{I_{mid}}{y_{b-mid}} = \frac{(728.94)}{3.8}\,\text{ft}^3 = 331,475\ \text{in.}^3$$

$$f_r = 0.24\sqrt{f_c'} = 0.24\sqrt{4} = 0.48\,\text{ksi}$$

f_{cpe} = 0.851 ksi (from plot of *P*/*S* stresses)

$$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\right]$$
$$= 0.75\left[\left(1.6\right)\left(0.48\right) + \left(1.1\right)\left(0.851\right)\times 331,500\right] = 423,800\ \text{kip-in.}$$

M_{cr} = 423,800 kip-in. = 35,300 kip-ft

$M_{r\,(min)}$ = the lesser of:

$1.33M_u = 1.33(72{,}940) = 97{,}000$ kip-ft

Therefore $M_{r\,(min)}$ = 35,300 kip-ft

Step 3: Compute the nominal moment resistance of the section based on the effects of the prestressing steel using AASHTO 5.7.3.1-4 only and substituting out f_{ps} and A_{ps}:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A_s' f_s'}{0.85 f_c' \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}} = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{pu} + A_s f_s - A_s' f_s'}{0.85 f_c' \beta_1 b + k \dfrac{P_j}{0.75 f_{pu}} \dfrac{f_{pu}}{d_p}}$$

Assuming no compression or tension resisting mild steel, A_s and A'_s both equal zero.

b = compression (top) flange width = 58 ft –10 in. = 706 in.

d_p = structure depth – prestressing force distance to soffit

d_p = 81– 12.5 = 68.5 in.

$$c = \frac{\dfrac{9{,}120}{0.75} + 0 - 0}{0.85(4)(0.85)(706) + 0.28\left(\dfrac{9{,}120}{0.75}\right)\dfrac{1}{68.5}} = 5.8\,\text{in.}$$

5.8 in. < h_{deck} = 9.0 in., therefore, rectangular section assumption satisfied.

$$f_{ps} = f_{pu}\left(1 - \frac{kc}{d_p}\right) = 270\left(1 - 0.28\frac{5.8}{68.5}\right) = 263.6 \text{ ksi}$$

Using Equations 7.12.13-3 and 7.12.13-4:

$$a = \beta_1 c = 0.85(5.8) = 4.9\,\text{in.}$$

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) = (45.0)(263.6)\left(68.5 - \frac{4.9}{2}\right) = 783{,}500 \text{ kip-in.}$$

M_n = 783,500 kip-in. = 65,300 kip-ft

Calculating ϕM_n = 0.95 (65,300) = 62,000 kip-ft > 33,500 kip-ft shows that no additional flexural steel is required. At this point, the calculation should stop, and we would not include any mild steel for the bottom of the superstructure at midspan. However, to illustrate this example we will continue by setting M_r = 75,000 kip-ft.

Step 4: Compute the area of mild steel required to increase ϕM_n to resist the full factored, M_r.

Using Equation 7.12.13-5:

$$a = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{pu} + A_s f_s - A_s' f_s'}{0.85 f_c' b + \dfrac{k}{\beta_1} \dfrac{P_j}{0.75 f_{pu}} \dfrac{f_{pu}}{d_p}}$$

$$\frac{a}{2} = \frac{\dfrac{9{,}120}{0.75} + (60) A_s - 0}{2\left(0.85(4)(706) + \dfrac{0.28}{0.85}\left(\dfrac{9{,}120}{0.75}\right)\dfrac{1}{68.5}\right)} = 2.48 + 0.012 A_s$$

Again assume $f_s = f_y$. This assumption is valid if the reinforcement at the extreme steel tension fiber fails. We can check this by measuring ε_t at the end of the calculation.

Using Equation 7.12.13-6:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y \left(d_s - \frac{a}{2}\right)$$

$$\left(\frac{((75{,}000)(12))}{0.95}\right) = (45.0)(263.6)(68.6 - (2.48 + 0.012 A_g)) + \ldots$$

$$\ldots + 60 A_g (76.44 - (2.48 + 0.012 A_g))$$

$0.73A_s^2 - 4{,}292A_s + 164{,}257 = 0$

Solving the quadratic equation: A_s = 38.520 in.^{2} (39 # 9 bars A_s = 39.00 in.^{2})

Step 5: Verify the two assumptions that were made in the determination of A_s. The first was that the mild steel would yield and we could use f_y for f_s. The validity of this assumption can be checked by calculating the ε_t. According to the California Amendments (Caltrans, 2014) and Figure 7.12.13-2 if ε_t is > 0.005 then the section is tension controlled and ϕ = 1.0. These values can be easily obtained with a simple strain diagram setting the concrete strain to 0.003.

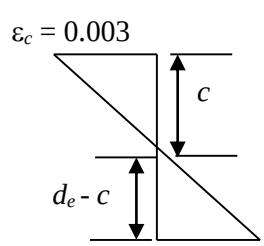


Figure 7.12.13-4 Strain Diagram

Having just calculated A_s., it is possible to calculate both c and d_e using AASHTO 5.7.3.1.1-4 and substituting out f_{ps} and A_{ps}:

$$c = \frac{\frac{P_j}{0.75} + A_s f_s - A_s' f_s'}{0.85 f_c' \beta_1 b + k \frac{P_j}{0.75} \frac{1}{d_p}} = \frac{\frac{9{,}120}{0.75} + (39.0)(60)}{(0.85)(4)(0.85)(706) + (0.28)\frac{9{,}120}{0.75}\left(\frac{1}{68.5}\right)} = 6.9 \text{ in.}$$

t_s = 8.25 in.

c = 6.9 in. < 8.25 in., therefore, the rectangular section assumption is satisfied.

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = \frac{(45.04)(263.6)(68.5) + (39.00)(60)(76.44)}{(45.04)(263.6) + (39.00)(60)}$$

d_e = 69.8 in.

By similar triangles: $\frac{\varepsilon_c}{c} = \frac{\varepsilon_t}{d_e - c}$ therefore

$$\varepsilon_t = \frac{\varepsilon_c}{c}\left(d_e - c\right) = \frac{0.003}{6.9}\left(69.8 - 6.9\right) = 0.027 > 0.005$$

Therefore, the mild steel yields and ϕ = 1.00.

Table 7.12.13-1 Additional Mild Steel Required per Location of Bridge

Loc	A_s req'd	M_n	Loc	A_s req'd	M_n	Loc	A_s req'd	M_n
0.024	0.0	35492	1.024	65.0	81785	2.034	64.0	80527
0.100	0.0	46543	1.100	0.0	51660	2.100	20.0	60549
0.200	7.0	59535	1.200	0.0	33687	2.200	0.0	38316
0.300	0.0	63394	1.300	0.0	54413	2.300	0.0	48864
0.400	0.0	65317	1.400	0.0	62681	2.400	0.0	58036
0.500	0.0	63592	1.500	0.0	65317	2.500	0.0	63592
0.600	0.0	58036	1.600	0.0	62681	2.600	0.0	65317
0.700	0.0	48864	1.700	0.0	54413	2.700	0.0	63394
0.800	0.0	38316	1.800	0.0	40528	2.800	0.0	57031
0.900	15.0	58814	1.900	0.0	51660	2.900	0.0	46543
0.968	66.0	81518	1.976	62.0	80767	2.975	0.0	35699

7.12.14 Design for Shear

Where it is reasonable to assume that plane sections remain plane after loading, regions of components shall be designed for shear using either the sectional method as specified in Article 5.8.3 (AASHTO, 2012), or the strut-and-tie method as specified in Article 5.6.3 (AASHTO, 2012). When designing for nominal shear resistance in box-girders, it is appropriate to use the sectional method.

In the sectional design approach, the component is investigated by comparing the factored shear force and the factored shear resistance at a number of sections along its length. Usually, this check is made at the tenth point of the span and at locations near the supports.

Where the reaction force in the direction of the applied shear introduces compression into the end region of a member, the location of the critical section for shear shall be taken as d_v from the internal face of support.

Figure CB5.2-5 shown on the next page, illustrates the shear design process by means of a flow chart. This Figure is based on the simplified assumption that 0.5 cot θ =1.0.

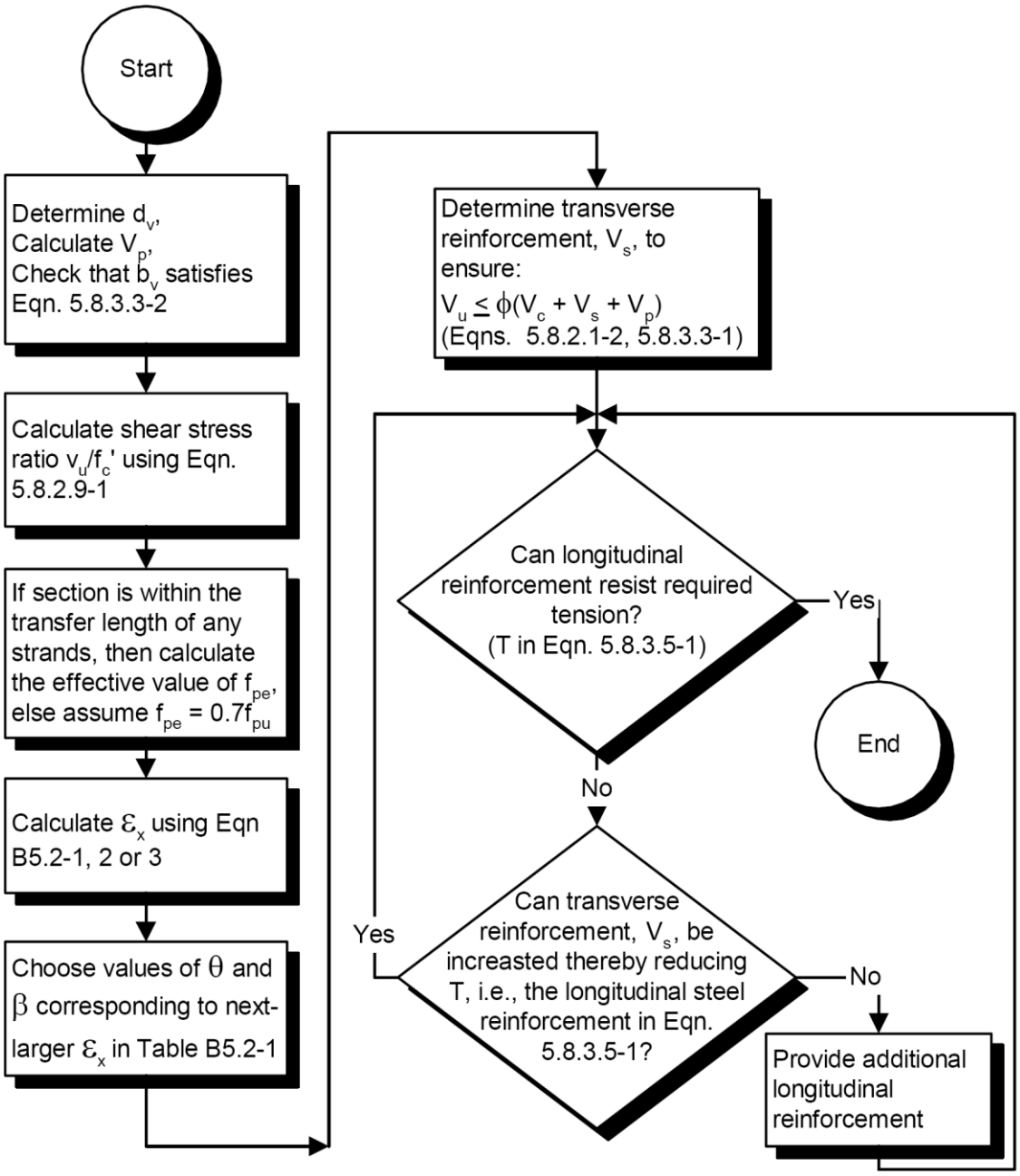


Figure CB5.2-5 Flow Chart for Shear Design Containing at Least Minimum Transverse Reinforcement (AASHTO, 2012)

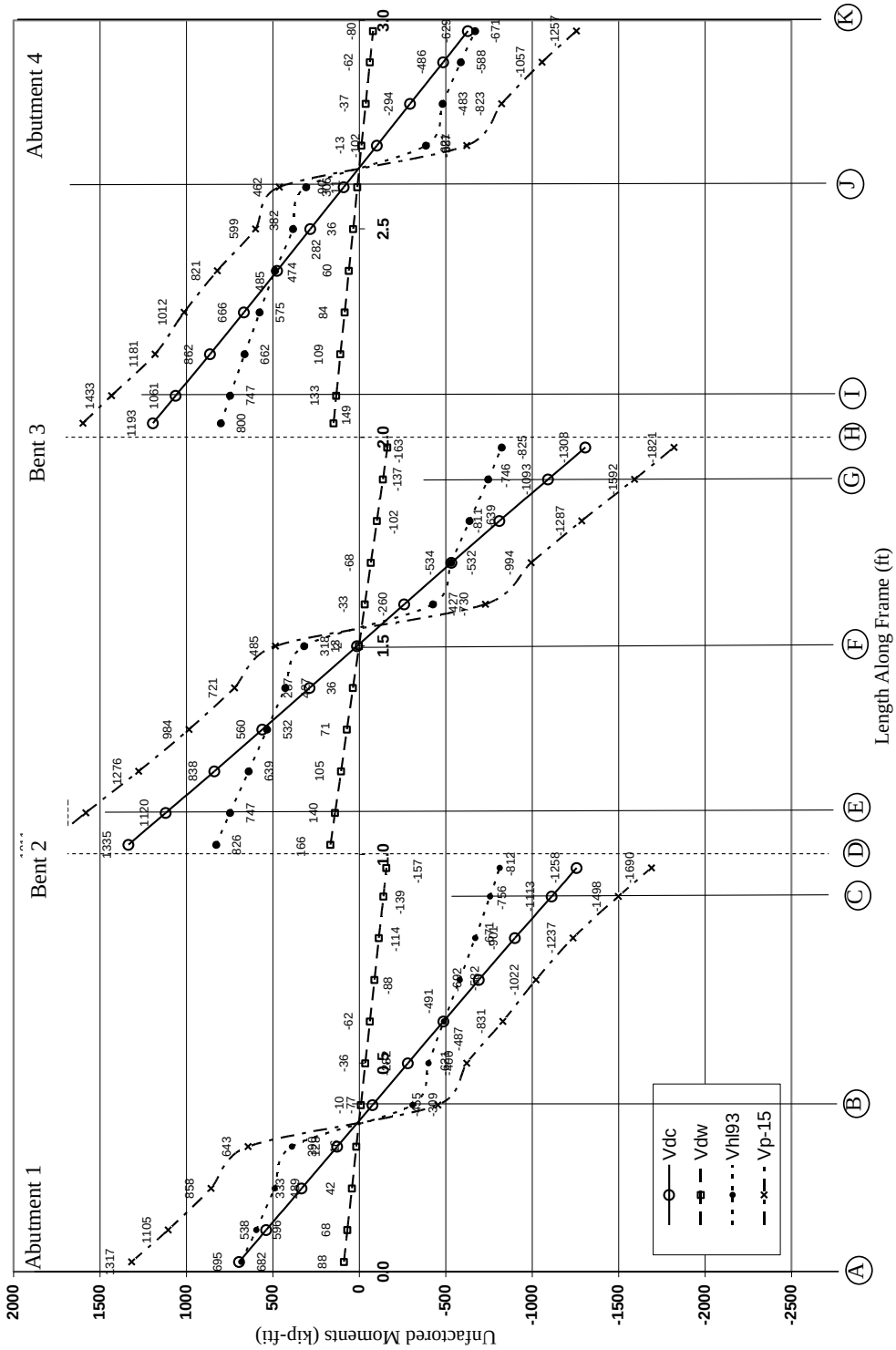


Figure 7.12.14-1 Gravity Load Shear Envelopes for Design of Shear Resistance

Design the interior girders at the right cap face at Bent 2 to resist the ultimate factored shear demand. Use Figure 7.12.14-1 as a guide, and take advantage of the reduction in demands by using the critical section for shear as d_v from the internal face of support.

Step 1: Determine d_v, calculate V_p. Check that b_v satisfies. Equation 5.8.3.3-2. (AASHTO, 2012)

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}} \qquad (7.12.14\text{-}1)$$

where:

A_{ps} = area of prestressing steel (in.^{2})

A_s = area of non-prestressed tension reinforcement (in.^{2})

d_v = the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

f_y = yield strength of mild steel (ksi)

M_n = nominal flexure resistance (kip-in.)

At the right cap face of Bent 2, d_v becomes:

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}} = \frac{81,785}{64.00(60) + 45.04(260.7)}$$

$$= 5.25 \text{ ft} \qquad = 63.0 \text{ in.}$$

Article 5.8.2.9 (AASHTO, 2012) states that d_v should not be less than the greater of $0.9d_e$ or $0.72h$. Therefore:

d_e = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.)

d_e (min) = 0.9(67.0 in.) = 60.3 in.

d_v (min) = 0.72(81 in.) = 58.32 in.

d_v = 63.0 in. > 60.3 in. > 58.3 in.

Finding V_p after establishing Equation 7.12.14-2:

$$V_p = P_j\,(\alpha) \text{ kips} \qquad (7.12.14\text{-}2)$$

where:

P_j = force in prestress strands before losses (kip)

V_p = the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip).

α = total angular change of prestressing steel path from jacking end to a point under investigation (rad)

Since the angle that is formed between the tangent of the parabola and the horizontal changes as the location on the parabola changes, we will take two α's at each point. One looking back to the previous point, and the other looking forward to the next point.

Given such a short distance along the larger parabolas, a triangle can be used to approximate angle change of the much smaller parabola segments:

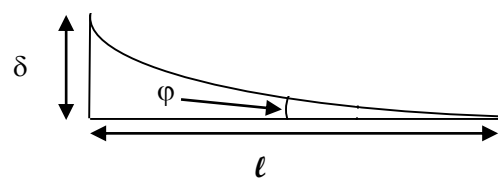


Figure 7.12.14-2 Components of Alpha

The general equation of a parabola: $y = ax^2$

For any parabola with side ℓ and δ: $a = \dfrac{\delta}{l^2}$

The angle change at any given point on the parabola is its first derivative:

$$\phi(\text{rads}) = \frac{dy}{dx} = \frac{2\delta}{l^2}x \qquad (7.12.14\text{-}3)$$

ϕ at a distance d_v from the face of cap at Bent 2

$$\phi(\text{rads}) = \frac{2\left[(2.12+3.5)-(0.90+3.8)\right]}{(0.1(168))^2}\left(4+\frac{63.0}{12}\right) = 0.06$$

$V_p = P_j\,(\alpha)$ kips = 9,120 (0.06 rad) = 547 kips

V_n is the lesser of:

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3-1)

$V_n = 0.25\, f'_c b_v d_v + V_p$ (AASHTO 5.8.3.3-2)

where:

b_v = effective web width taken as the minimum web width, measured parallel to the neutral axis, between resultants of the tensile and compressive forces due to flexure. This value lies within the depth d_v (in.)

d_v = the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in.)

For now we will use AASHTO 5.8.3.3-2, and return to AASHTO 5.8.3.3-1 later.

$$b_v\,(req'd) = \frac{V_n - V_p}{0.25\, f'_c d_v} \qquad (7.12.14\text{-}4)$$

where:

V_n = the nominal shear resistance of the section considered (kip)

V_p = the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip)

$$V_n = \frac{V_u}{\phi} \qquad (7.12.14\text{-}5)$$

Using Figure 7.12.14-1 to compare Strength I and Strength II results of V_u at the Bent 2 cap face:

$$V_n\ @\,\text{face}\ (Str\ I) = \frac{V_u}{\phi} = \frac{1.25(1,334) + 1.50(166) + 1.75(826)}{0.90} = 3,736\ \text{kips}$$

$$V_n\ @\,\text{pt}\ E\ (Str\ I) = \frac{V_u}{\phi} = \frac{1.25(1,120) + 1.50(140) + 1.75(745)}{0.90} = 3,238\ \text{kips}$$

$$V_n\ @\,\text{face}\ (Str\ II) = \frac{V_u}{\phi} = \frac{1.25(1,334) + 1.50(166) + 1.35(1,810)}{0.90} = 4,844\ \text{kips}$$

$$V_n\ @\,\text{pt}\ E\ (Str\ II) = \frac{V_u}{\phi} = \frac{1.25(1,120) + 1.50(140) + 1.35(1,582)}{0.90} = 4,146\ \text{kips}$$

Strength II controls so now interpolate to find V_n a distance d_v from the face of Bent 2:

$$V_n @ d_v \text{ from face} = \frac{4{,}162 - 4{,}844}{0.1(168) - 4}\left(\frac{63.0}{12} + 4 - 4\right) + 4{,}844 = 4{,}564 \text{ kips}$$

From above f'_c = 4 ksi and now using Equation 7.14.12-3 to find b_v:

$$b_v(req'd) = \frac{V_n - V_p}{0.25 f'_c d_v} = \frac{4{,}564 - 547}{0.25(4)63.0} = 64.0 \text{ in.}$$

This results in b_v ~12.8 in. per girder. Flare the interior girders at bent faces to 13 in. for added capacity in future calculations.

$$b_v(13 \text{ in. flare}) = (5 \text{ girders})(13 \text{ in.}) = 65 \text{ in.}$$

Step 2: Calculate shear stress ratio v_u / f'_c using Equation 5.8.2.9-1. (AASHTO, 2012)

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} \qquad \text{(AASHTO 5.8.2.9-1)}$$

where:

v_u = average factored shear stress on the concrete (ksi) (5.8.2.7) (5.8.2.9)

V_u @ d_v from face = 0.90 (4564) = 4,108 kips

ϕV_p = 0.9(547) = 492 kips

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} = \frac{|4{,}108 - 492|}{0.9(65)63.0} = 0.981 \text{ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.981}{4} = 0.245$$

Step 3: If section is within the transfer length of any strands, then calculate the effective value of f_{po}, else assume f_{po} = 0.7 f_{pu}. This step is necessary for members without anchorages.

f_{po} = 0.7 f_{pu} = 189 ksi

Step 4: Calculate ε_x using Equations B5.2-1, B5.2-2 or 5.2-3 (AASHTO, 2012)

Assuming the section meets the requirements specified in Article 5.8.2.5 (AASHTO, 2012)

$$\varepsilon_x = \frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})} \qquad \text{(AASHTO B5.2-1)}$$

where:

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)

N_u = applied factored axial force taken as positive if tensile (kip)

ε_x = longitudinal strain in the web reinforcement on the flexural tension side of the member (in/in)

θ = angle of inclination of diagonal compressive stresses (degrees)

M_u @ face = 1.25(-32,619)+1.50(-4,095)+1.35(-23,620)+0.856(9,120)
= -70,997 kip – ft

M_u @ *pt E* = 1.25(-16,910)+1.50(-2,130)+1.35(-14,000)+0.856(9,120)
= -35,426 kip – ft

By interpolation, find M_u a distance d_v from the face of Bent 2:

$$M_u @ d_v \text{ fromface} = \frac{-35{,}426 + 70{,}997}{0.1(168) - 4}\left(\frac{63.0}{12} + 4 - 4\right) - 70{,}997$$

= -56,407 kip – ft = 676,884 kip – in.

Now using AASHTO B5.2-1 with N_u = 0, begin with cot θ = 1, and values calculated earlier

$$\varepsilon_x = \frac{\frac{|-676{,}884|}{63.0} + 0.5\,(0) + 0.5\,|\,4{,}108 - 547\,|\,(1) - 45.04\,(189)}{2\left[29{,}000\,(64) + 28{,}500\,(45.04)\right]} = 0.000639$$

Step 5: Choose values of θ and β corresponding to next-larger ε_x from AASHTO Table B5.2-1 (California Amendments to AASHTO, 2012).

Based on calculated values of $\frac{v_u}{f'_c} = 0.245$; ε_x = 0.000639, we obtain:

θ = 34.3° and β = 1.58.

Table B5.2-1 Values of θ and β for Sections with Transverse Reinforcement (AASHTO, 2012)

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

Step 6: Determine transverse reinforcement, V_s, to ensure: $V_u \leq \phi\,(V_c + V_s + V_p)$ Equations 5.8.2.1-2, 5.8.3.3-1 (AASHTO, 2012). First we must introduce the concrete component of shear resistance:

$$V_c = 0.0316\,\beta\sqrt{f'_c}\,b_v d_v \qquad \text{(AASHTO 5.8.3.3-3)}$$

where:

V_c = nominal shear resistance provided by tensile stresses in the concrete (kip)

β = factor relating effect of longitudinal strain on the shear capacity of concrete as indicated by the ability of diagonally cracked concrete to transmit tension

$$V_c = 0.0316\,\beta\sqrt{f'_c}\,b_v d_v = 0.0316(1.58)\sqrt{4}(65)(63.0) = 409 \text{ kips}$$

$$V_p = 547 \text{ kips}$$

$$V_u = 4{,}108 \text{ kips}$$

Combining Equations 5.8.2.1-2 and 5.8.3.3-1 (*AASHTO*, 2012) results in design equation as follows:

$$V_u \leq \phi\,(V_c + V_s + V_p)$$

Rearranging and solving for V_s:

$$V_s \geq \frac{V_u}{\phi} - V_c - V_p = \frac{4{,}108}{0.90} - 409 - 547 = 3{,}608 \text{ kips}$$

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} \qquad \text{(AASHTO 5.8.3.3-4)}$$

where:

A_v = area of transverse reinforcement within distance s (in^2.)

α = angle of inclination of transverse reinforcement to longitudinal axis (º)

s = spacing of reinforcing bars (in.)

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by AASHTO 5.8.3.3-4, except V_s shall not be taken greater than V_u/ϕ (kip)

Then α = 90º since stirrups in this bridge are perpendicular to the deck. Thus Equation 5.8.3.3-4 reduces to:

$$V_s = \frac{A_v f_y d_v (\cot\theta)}{s} \qquad \text{(AASHTO C5.8.3.3-1)}$$

Rearranging terms to solve for s

$$s \le \frac{A_v f_y d_v (\cot\theta)}{V_s} \qquad (7.12.14\text{-}6)$$

Assume 2 - #5 legs per stirrup ×5 girders, A_v = 2(5)(0.31) = 3.10 in.^{2}

$$s \le \frac{A_v f_y d_v\ (\cot\theta)}{V_s} = \frac{3.10\,(63.0)\,(\cot 34.3°)}{3,608} = 4.76\text{ in.}$$

Use # 5 stirrups, s = 4 in. spacing

Step 7: Use Equation 5.8.3.5-1 (AASTHO, 2012) to check if the longitudinal reinforcement can resist the required tension.

$$A_{ps} f_{ps} + A_s f_y \ge \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_c} - V_p\right| - 0.5V_s\right)\cot\theta \qquad \text{(AASHTO 5.8.3.5-1)}$$

Breaking into parts and solving both sides of Equation 5.8.3.5-1 results in:

45.0(260.2) + 64.0(60) = 15,549 kips

$$\frac{|-56,407(12)|}{63.0(0.9)} + 0 + \left(|4,564 - 547| - 0.5(3,520)\right)\cot 36.0 = 15,044\text{ kips}$$

Since 15,549 is greater than 15,044 therefore the shear design is complete. Had the left side been smaller than the right side we would use the following procedure to determine A_s. If the conditions of Equation 5.8.3.5-1 (AASHTO, 2012) were met then the shear design process is complete.

Step 8: If the right side of AASHTO 5.8.3.5-1 was greater than the left side, we would need to solve Equation 5.8.3.5-1 to increase A_s to meet the minimum requirements of 5.8.3.5-1.

$$A_s = \frac{\frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_c} - V_p\right| - 0.5V_s\right)\cot\theta - A_{ps} f_{ps}}{f_y} \qquad \text{(AASHTO 5.8.3.5-1)}$$

Design the exterior girders right cap face at Bent 2 to resist the ultimate factored shear demand. Use Figure 7.12.14-1 as a guide, and take advantage of the reduction in demands by using the critical section for shear as d_v from the internal face of support. In this example, all values will be per girder, since only the exterior girder is affected by this analysis. Use the modification chart found in BDA 5-32 to amplify values of V_u.

Step 1: Determine d_v, calculate V_p. Check that b_v satisfies Equation 5.8.3.3-2. (AASHTO, 2012)

From above:

d_v = 63.0 in.

V_p = 547 kips or 110 kips/girder

V_n is the lesser of AASHTO 5.8.3.3-1 and AASHTO 5.8.3.3-2:

$V_n = V_c + V_s + V_p$

$V_n = 0.25\, f'_c b_v d_v + V_p$

For now we will use AASHTO 5.8.3.3-2, and return to AASHTO 5.8.3.3-1 later.

Equations 7.12.14-4 and 7.12.14-5 respectively state:

$$b_v(req'd) = \frac{V_n - V_p}{0.25 f'_c d_v} \qquad (7.12.14\text{-}4)$$

$$V_n = \frac{V_u}{\phi} \qquad (7.12.14\text{-}5)$$

From above Strength II controls:

V_n @ d_v from face = 4,570 kips

We can now use BDA 5-32 (Figure 7.12.14-2) to amplify the obtuse exterior girders shear demand.

For an exterior girder with a 20º skew the chart reads that the modification factor is 1.4.

Now $V_{n(mod)}$ = $V_n(ext) = 1.4\frac{4564}{5} = 1278$ kips

From above f'_c = 4 ksi and now using Equation 7.12.12-3 to find b_v:

$$b_v(req'd) = \frac{V_n - V_p}{0.25 f'_c d_v} = \frac{1,278 - 110}{0.25(4)63.0} = 18.5 \text{ in.}$$

This results in b_v = 18.5 in. per girder. We will flare exterior girders at bent faces to 19 in. for added capacity in future calculations.

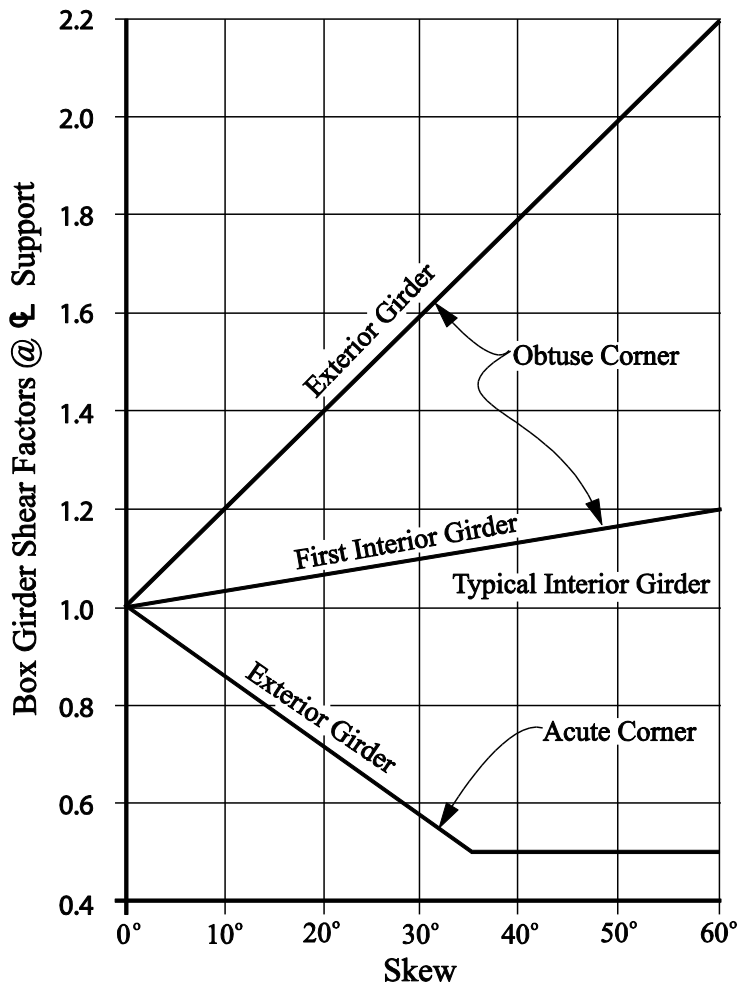


Figure 7.12.14-3 Shear Modification Factor Found in BDA 5-32 (Caltrans, 1990)

Step 2: Calculate shear stress ratio v_u / f'_c using Equation 5.8.2.9-1. (AASHTO, 2012)

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v}$$

V_u @ d_v from face = 0.90 (1,2780) = 1,150 kips

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} = \frac{|1{,}150 - 100|}{0.9\,(19)\,63.0} = 0.975 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.975}{4} = 0.244$$

Step 3: If the section is within the transfer length of any strands, then calculate the effective value of f_{po}, else assume $f_{po} = 0.7\ f_{pu}$. This step is necessary for members with no anchorage devices.

$f_{po} = 0.7\ f_{pu}$ = 189 ksi

Step 4: Calculate ε_x using Equations B5.2-1, B5.2-2 or B5.2-3 (AASHTO, 2012)

Assuming the section meets the requirements specified in Article 5.8.2.5 (AASHTO, 2012)

$$\varepsilon_x = \frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})}$$

From above and in terms of a single girder:

$$M_u @ d_v \text{ from face} = \frac{\text{-676,884 kip} - \text{in}}{5} = -135{,}877\,\text{kip} - \text{in}$$

Now using AASHTO 5.8.3.4.2-1 with N_u = 0, begin with cot θ = 1, and values calculated earlier…

$$\varepsilon_x = \frac{\frac{|-135{,}530|}{63.0} + 0.5\,(0) + 0.5\,|1{,}150 - 100|\,(1) - \frac{45.04}{5}(189)}{2\left[29{,}000\left(\frac{65}{5}\right) + 28{,}500\left(\frac{45.04}{5}\right)\right]} = 0.0012$$

Step 5: Choose values of θ and β corresponding to next-larger ε_x from AASHTO Table B5.2-1 (California Amendments to AASHTO, 2012).

Based on calculated values of

$\frac{v_u}{f'_c} = 0.244$ and ε_x = 0.0012, we obtain

θ = 35.8° and β = 1.50

Table B5.2-1 Values of θ and β for Sections with Transverse Reinforcement (AASHTO, 2012)

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

Step 6: Determine transverse reinforcement, V_s, to ensure: $V_u \leq \phi\,(V_c + V_s + V_p)$ Equations 5.8.2.1-2, 5.8.3.3-1 (AASHTO, 2012). First use AASHTO 5.8.3.3-3 to determine the concrete component of shear resistance.

$V_c = 0.0316\,\beta\sqrt{f'_c}\,b_v d_v$

$V_c = 0.0316\,\beta\sqrt{f'_c}\,b_v d_v = 0.0316\,(1.50)\sqrt{4}\,(19)(63.0) = 113.5$ kips

$V_p = 110$ kips

$V_u = 1{,}110$ kips

Combining Equations 5.8.2.1-2 and 5.8.3.3-1 (AASHTO, 2012) results in design equation as follows:

$V_u \leq \phi(V_c + V_s + V_p)$

Rearranging and solving for V_s:

$$V_s \geq \frac{V_u}{\phi} - V_c - V_p = \frac{1{,}110}{0.9} - 113.5 - 110 = 1{,}010 \text{ kips}$$

Then using Eq. 7.12.14-6

$$s \leq \frac{A_v f_y d_v (\cot\theta)}{V_s}$$

Assume 2 - #5 legs per stirrup × 1 girders, $A_v = 2\,(1)\,(0.31) = 0.62$ in.^{2}

$$s = \frac{A_v f_y d_v (\cot\theta)}{V_s} = \frac{(0.62)\,(60)\,(63.0)\,(\cot 35.8°)}{1{,}010} = 3.22 \text{ in.}$$

Use # 5 stirrups, s = 3 in. spacing

Step 7: Use Modified Equation 5.8.3.5-1 (AASTHO, 2012) to check if the longitudinal reinforcement per girder can resist the required tension.

$$\frac{A_{ps} f_{ps}}{5} + \frac{A_s f_y}{5} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_c} - V_p\right| - 0.5V_s\right)\cot\theta$$

Breaking into parts and solving both sides of Equation 5.8.3.5-1 results in:

$$\frac{45.04(260.2)}{5} + \frac{64(60)}{5} = 3{,}112 \text{ kips}$$

$$\frac{|-11{,}281(12)|}{63.0\,(0.9)} + 0 + \left(|1{,}278 - 110| - 0.5(1{,}039)\right)\cot 37.4 = 3{,}236 \text{ kips}$$

3,112 is not greater than 3,236. Use the following procedure to determine A_s.

Step 8: Solve Equation 5.8.3.5-1 to increase A_s to meet the minimum requirements of 5.8.3.5-1:

$$A_s = \frac{\frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_c} - V_p\right| - 0.5V_s\right)\cot\theta - A_{ps} f_{ps}}{f_y}$$

$$A_s = \frac{3{,}236 - \frac{45.04(260.2)}{5}}{60} = 14.87 \text{ in.}^2$$

If the right side of Equation 5.8.3.5-1 is equal to 3,290, the exterior 1/2 bay reinforcement should be increased from $A_s = \frac{64}{5} = 12.8 \text{ in.}^2$ to A_s = 15 in.^{2}

7.12.15 Calculate the Prestressing Elongation

Tendon elongation calculations are necessary to help ensure the proper jacking force is delivered to the superstructure. Elongation calculations are one way for construction field personnel to check the actual P_j force applied to tendons.

Since the structure has been designed for two-end stressing, both first and second end elongations need to be computed.

Figure 7.12.15-1 shows the information that should be included on the contract plans.

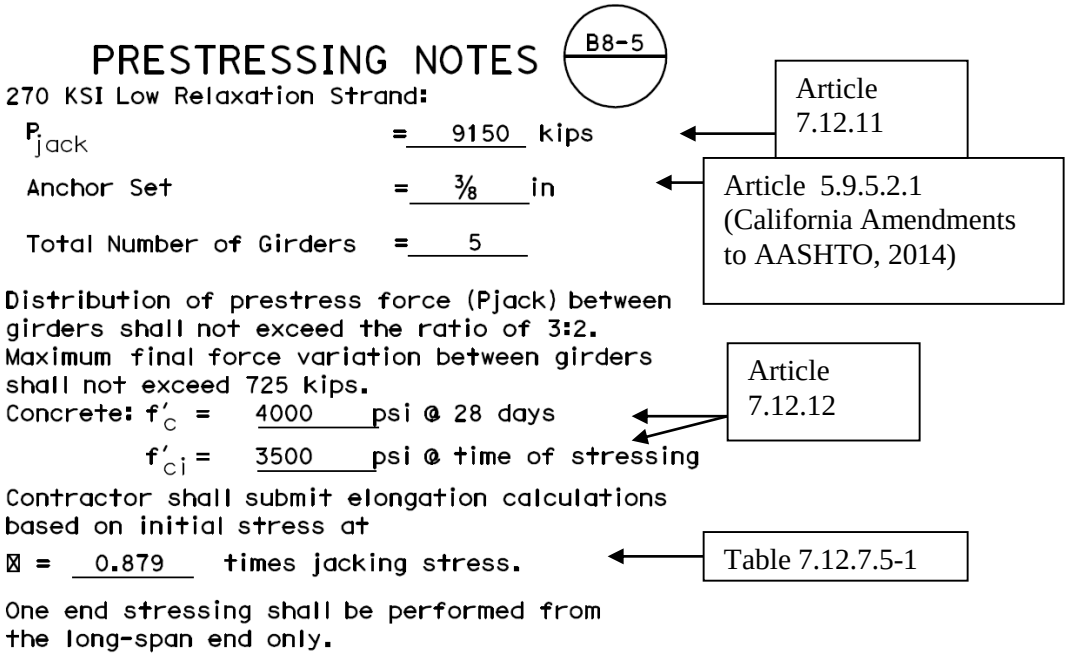


Figure 7.12.15-1 Decal Shown on Contract Plans

Based on the location and magnitude of f_{pF} (stress with friction losses) shown on the contract plans, the post-tensioning fabricator develops a simplified diagram, like the one shown in Figure 7.12.15-2.

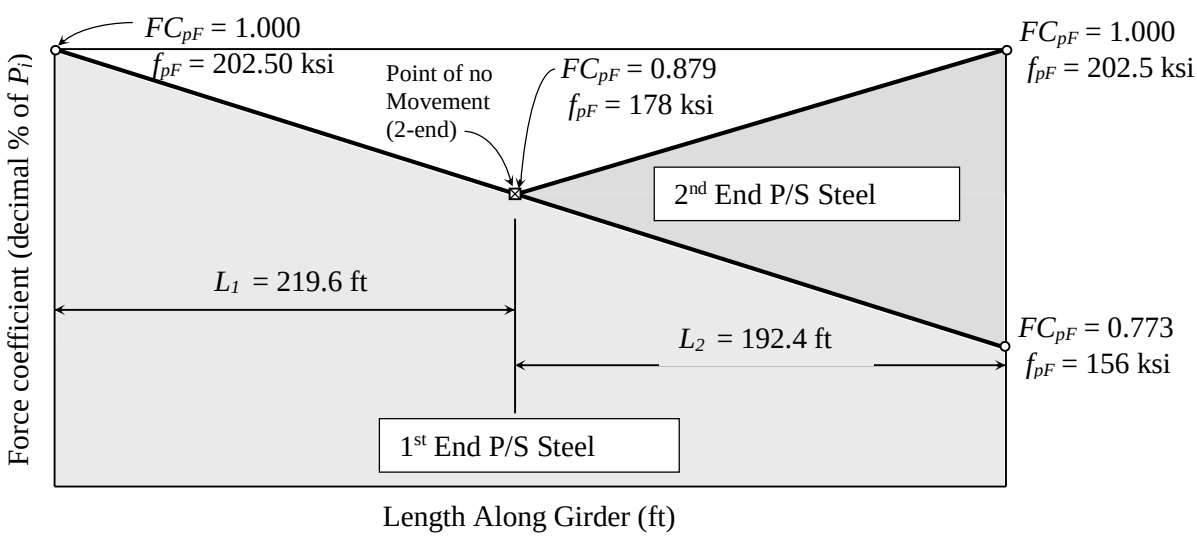


Figure 7.12.15-2 Simplified Diagram

The 1st end elongation calculation (from the 1st end of jacking side to the anchorage side, the entire length of the span):

The prestressing elongation is based on the stress-strain relationship and results in Equation 7.12.15-1:

$$\Delta_E = \frac{f_{avg} L_x}{E_p} \tag{7.12.15-1}$$

E_p = modulus of elasticity of prestressing tendon (ksi)

f_{avg} = average stress in the strand from jacking end to point of no movement (ksi)

Δ_E = change in length of prestressing tendons due to jacking (in.)

For one-end stressing, MTD 11-1 (Caltrans, 2013) provides the following formula:

$$\Delta_E = \frac{T_O(1+\otimes)(L+3.5')}{2E_p} \tag{7.12.15-2}$$

In Equation 7.12.15-2, the 3.5′ term is the expected length of jack.

For two end stressing, MTD 11-1 (Caltrans, 2013) provides the following formula:

$$\Delta_{1st} = \frac{T_O}{2E_p}\left[(1+\otimes)L_1 + (3\otimes -1)L_2\right] \tag{7.12.15-3}$$

$$\Delta_{2nd} = \frac{T_O(1+\otimes)L_2}{E_p} \tag{7.12.15-4}$$

where:

Δ_{1st} = elongation after stressing the first end (in)

Δ_{2nd} = elongation after stressing the second end (in)

T_o = steel stress at the jacking end before seating (generally 202.5 ksi) (ksi)

$\otimes$ = initial force coefficient at the point of no movement

L = Length of tendon (ft)

L_1 = Length of tendon from first stressing end to the point of no movement (ft)

L_2 = Length of tendon from point of no movement to second stressing end (ft)

For our bridge, let's use the values shown on the graph above and applying the 3 ft length of jack:

$$\Delta_{1st} = \frac{202.5}{(2)\ 28{,}500}\,[(1+0.879)(219.6)+((3\times 0.897)-1)(192.40)]=2.585 \text{ ft} \quad =31.02 \text{ in.}$$

When applying these measurements in the field, it is necessary to determine the "measureable" elongation. The measureable elongation includes the length of tendon in the contractor's jack (3 ft). It is been found that by applying 20% of P_j before monitoring elongations, the tendon is allowed to shift from it's resting place to it's final position. Therefore, only 80% of the elongation calculated above is measureable.

Δ_{1st} = (0.8)(31.02) = 24.81 in.

The 2nd end elongation calculation (from the 2nd end of jacking side to the point of no movement):

$$\Delta_{2nd} = \frac{202.5\,(1-0.879)\,(192.4)}{28{,}500} = 0.165 \text{ ft} = 1.98 \text{ in.}$$

NOTATION

A_g	=	gross area of section (in.^{2})
A_{ps}	=	area of prestressing steel (in.^{2})
A_s	=	area of non-prestressed tension reinforcement (in.^{2})
A'_s	=	area of compression reinforcement (in.^{2})
A_v	=	area of transverse reinforcement within distance s (in.^{2})
b	=	width of the compression face of a member (in.)
b_v	=	effective web width taken as the minimum web width, measured parallel to the neutral axis, between resultants of the tensile and compressive forces due to flexure. b_v lies within the depth d_v (in.)
b_w	=	web width (in.)
c	=	distance from extreme compression fiber to the neutral axis (in.)
clr_{int}	=	clearance from interior face of bay to the first mat of steel in the soffit or deck (Usually taken as 1 in.) (in.)
d	=	depth of member (in.)
d_e	=	defective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.)
d_i	=	distance between C.G. of ith duct and the ith duct *LOL* (See Figure 7.12.7-4) (in.)
d_p	=	distance from extreme compression fiber to the centroid of the prestressing tendons (in.)
d_s	=	distance from extreme compression fiber to the centriod of the nonprestressed tensile reinforcement (in.)
d_v	=	the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in.)
E_{ct}	=	modulus of elasticity of concrete at transfer or time of load application (ksi)
E_p	=	modulus of elasticity of prestressing tendons (ksi)
FC_{pA}	=	force coefficient for loss from anchor set
FC_{pES}	=	force coefficient for loss from elastic shortening
FC_{pF}	=	force coefficient for loss from friction
FC_{pT}	=	total force coefficient for loss
e	=	eccentricity of resultant of prestressing with respect to the centroid of the cross section. Always taken as a positive. (ft) The base of Napierian logarithms

e_x = eccentricity as a function of x along parabolic segment (ft)

f_{avg} = average stress in the strand from jacking end to point of no movement (ksi)

f'_c = specified compressive strength of concrete used in design (ksi)

f'_{ci} = specified compressive strength of concrete at time of initial loading or prestressing (ksi); nominal concrete strength at time of application of tendon force (ksi)

f'_{pe} = effective stress in the prestressing steel after losses (ksi)

f'_{pi} = initial stress in the prestressing steel after losses, considering only the effects of friction loss. No other P/S losses have occurred (ksi)

f_{cgp} = concrete stress at the center of gravity of prestressing tendons, that results from the prestressing force at either transfer or jacking and the self-weight of the member at maximum moment sections (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

f_{DC+DW} = stress in concrete from *DC* and *DW* load cases (ksi)

$f_{DCw/o\ b}$ = stress in concrete due to the Dead Load of the structural section only (ksi)

f_{pi} = prestressing steel stress immediately prior to transfer (ksi)

f_g = stress in the member from dead load (ksi)

f_{HL93} = stress in concrete from *HL*93 load cases (ksi)

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

f_{pu} = specified tensile strength of prestressing steel (ksi)

f_{py} = yield strength of prestressing steel (ksi)

f_r = modulus of rupture of concrete (ksi)

f_y = yield strength of mild steel (ksi)

f'_y = specified minimum yield strength of compression reinforcment (ksi)

H = average annual ambient mean relative humidity (percent)

h_f = compression flange depth (in.)

I_{cr} = moment of inertia of the cracked section, transformed to concrete (in.^{4})

I_e = effective moment of inertia (in.^{4})

I_g = moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

K = distance to the closest duct to the bottom of the soffit or top of the deck (in.)

k = wobble friction coefficient (per ft of tendon)

L = distance to a point of known stress loss (ft), individual span length (ft) ($60 \leq L \leq 240$)

L_{frame} = length of frame to be post-tensioned (ft)

M_a = maximum moment in a member at the stage which the deformation is computed (kip-in.)

M_{cr} = cracking moment (kip-in.)

M_n = nominal flexure resistance (kip-in.)

M_r = factored flexural resistance of a section in bending (kip-in.)

M_{DL} = dead load moment of structure (kip-in.)

$M_{P/S\,s}$ = moment due to the secondary effects of prestressing (k-ft)

MC_p = primary moment force coefficient for loss (ft)

MC_s = secondary moment force coefficient for loss (ft)

MC_{PT} = total moment force coefficient for loss (ft)

N = number of identical prestressing tendons

N_c = number of cells in a concrete box girder ($N_c \geq 3$)

N_u = applied factored axial force taken as positive if tensile (kip)

P_j = force in prestress strands before losses (kip)

l_{ij} = length of individual parabola (in.)

n_i = number of strands in the ith duct

S_c = section modulus for the extreme fiber of the composite sections where tensile stress is caused by externally applied loads (in.^{3})

s = spacing of reinforcing bars (in.)

t = thickness of soffit or deck (in.)

t_d = thickness of deck (in.)

t_s = thickness of soffit (in.)

V_c = nominal shear resistance provided by tensile stresses in the concrete (kip)

V_n = the nominal shear resistance of the section considered (kip)

V_p = the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip)

V_s	=	shear resistance provided by the transverse reinforcement at the section under investigation as given by AASHTO 5.8.3.3-4, except V_s shall not be taken greater than V_u/ϕ (kip)
v_u	=	average factored shear stress on the concrete (ksi)
W	=	weight of prestressing steel established by BDA page 11-66 (lb)
x	=	general distance along tendon (ft), location along parabolic segment where eccentricity is calculated (% span L)
x_{pA}	=	influence length of anchor set (ft)
y	=	general distance from the neutral axis to a point on member cross-section (in.)
y_{ij}	=	height of individual parabola (in.)
y_t	=	distance from the neutral axis to the extreme tension fiber (in.)
Z	=	C.G. tendon shift within duct (in.)
α	=	angle of inclination of transverse reinforcement to longitudinal axis (°) total angular change of prestressing steel path from jacking end to a point under investigation (rad)
β	=	factor relating effect of longitudinal strain on the shear capacity of concrete, as indicated by the ability of diagonally cracked concrete to transmit tension
β_1	=	ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone
δ_{hp}	=	offset from deck to Centroid of duct (in.)
δ_{lp}	=	offset from soffit to Centroid of duct (in.)
Δ_{Aset}	=	anchor set length (in.)
Δ_E	=	change in length of prestressing tendons due to jacking (in.)
Δf_i	=	change in force in prestressing tendon due to an individual loss (ksi)
Δf_L	=	friction loss at the point of known stress loss (ksi)
Δf_{pA}	=	jacking stress lost in the P/S steel due to anchor set (ksi)
Δf_{pCR}	=	change in stress due to creep loss
Δf_{pES}	=	change in stress due to elastic shortening loss
Δf_{pF}	=	change in stress due to friction loss
Δf_{pSR}	=	change in stress due to shrinkage loss
ε_{cu}	=	failure strain of concrete in compression (in./in.)
ε_s	=	net longitudinal tensile strain in section at the centroid of the tension reinforcement (in./in.)
ε_t	=	net tensile strain in extreme tension steel at nominal resistance (in./in.)

θ	=	angle of inclination of diagonal compressive stresses (degrees)
ϕ	=	resistance factor
μ	=	coefficient of friction
γ_1	=	flexural cracking variability factor
γ_2	=	prestress variability factor
γ_3	=	ratio of specified minimum yield strength to ultimate tensile strength of reinforcement

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