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16. ABSTRACT

There has been an increasing trend towards the introduction of commercially and privately provided roads for the expansion of transportation systems around the world. The important questions being asked about the competitive provision of road infrastructures include the following: what capacities a private road should provide and how much toll should it charge? What will happen to the capacities and tolls when multiple privately built and operated roads compete with each other under various ownership regimes in a road network? How do these factors change the efficiency/inefficiency envelop when profit-seeking behavior substitutes for government regulation?

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Provision of Road Capacity through Privately Built Roads: Capacity, Pricing and Competition issues

Final Report

January 14, 2010

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Abstract

In this study, we develop strategic gaming models to study various BOT schemes with non-identical travelers and multiple agents in a one-O/D, two-road parallel network and analyze how value-of-time and market structure affect the outcome of social welfare, firm profitability, and efficiency gain/loss. Such study can be applied in predicting the possible outcomes of different government policies on a BOT project.

Keywords: Build-Operate-Transfer (BOT), economic games, social welfare, toll rate, regulation

Executive Summary

There has been an increasing trend towards the introduction of commercially and privately provided roads for the expansion of transportation systems around the world. The important questions being asked about the competitive provision of road infrastructures include the following: what capacities a private road should provide and how much toll should it charge? What will happen to the capacities and tolls when multiple privately built and operated roads compete with each other under various ownership regimes in a road network? How do these factors change the efficiency/inefficiency envelop when profit-seeking behavior substitutes for government regulation?

In the literature, the above issues were usually explored in the context of homogenous commuters. This research proposes to examine these issues under a Built-Operate-Transfer (BOT) contract considering commuters' differing value-of-time (VOT). In the first part, the model for one toll road competing with an alternative free road network is introduced. The efficiency loss and the road capacities and/or tolls set by private firms are investigated under different kinds of government regulatory. Continuously distributed VOT is assumed and integrated into the model. Due to the informational difficulty of accurately valuing the VOT over all the commuters, the rate-of-return regulation is examined and suggested to have attractive advantages for the regulatory authority. A numerical example is provided to illustrate the effect of VOT distribution on the social welfare, profit of the private firm and the resulting efficiency loss of the system. Then the SR91 express lane is chosen as the experiment site for the implementation of the study. The VOT distribution throughout the highway users is calibrated from the experimental data collected by PeMS. A relationship between the statistic characteristics of the VOT distribution and the economic efficiency of a BOT project is discovered. Finally, competitions between two parallel toll roads are also theoretically investigated. Two scenarios are investigated including the oligopolistic competition and the competition between private firm and the government.

The major findings of this study are:

1. In a BOT project, due to the profit-oriented behavior, the private firm tends to impose a much higher toll rate to the users and the capacity will be undersupplied if no intervention is made by the government. The level of service is also lower, which

makes the highway more congested. As a result, the monopolistic behavior of the private firm induces as high as 30.7% efficiency loss;

2. When the toll rate is regulated by the government to be at the level that maximizes social welfare, the private firm still intends to undersupply the capacity and the profit of the firm is greatly reduced. The toll-ceiling regulation induces even more efficiency loss than the government doing-nothing, although the resulting capacity is higher than the doing-nothing case. Under such a case, the efficiency loss will be high during the time period the private firm operates the highway,;
3. The investment-floor regulation forces the private firm to provide a much higher capacity than it wants. However, under this regulation, the private firm still intends to charge road users a higher toll rate to extract more revenue. However, the social welfare gain is close to the optimal level. When the highway is returned to the government after the franchising is over, the highway will be operated just to produce the maximum social welfare. Thus the investment-floor regulation performs better than the toll-ceiling regulation, no matter before or after the contract ends;
4. The rate-of-return regulation performs the best among the three regulations, not only because it induces the optimal results, but because it's very easy and simple to be implemented compared with the other two, since no information about the market, e.g. the cost structure of the private company and the VOT distribution of the population, is needed;
5. Road competition improves efficiency. The oligopolistic competition only has an efficiency loss of 9.3%. The traffic flow pattern does not change much under competition compared with the optimal one. And it's worth noting that the toll rates under competition are lower than the optimal level, which implies that the consumer surplus is even higher under road competition;
6. The existence of a publically controlled toll road competing with the BOT project will greatly lower the profit of the private firm. The efficiency is improved only little compared with the oligopoly competition, but the risks of greatly reduced profit makes the BOT project impalpable to the private firms.

1. Problem Background

Recently many countries have started massive highway franchising programs via the so-called Build-Operate-Transfer (BOT) contracts. Under such a contract, the private sector not only operates but also finances the highway and in turn receives the revenue from road toll charge for a long period. When the franchise ends, the road reverts to the government (Yang and Meng, 2000).

It is generally believed that introducing private provision of public roads is an efficient way to expand modern road systems. Private-sector participation in road construction and operations has the advantages of higher efficiency, private financing, and better identification of attractive investment projects. Private-sector participation in the form of BOT franchises has worked well in a number of projects such as road tunnels in Hong Kong. In mainland China, many local, mainly municipally affiliated companies have undertaken the development of toll roads in recent years, often in joint ventures with Hong Kong investors. The Guangzhou-Shenzhen super-highway conceived and developed by a Hong Kong entrepreneur is a good example of a BOT project.

In the United States, relatively few such projects are known. However, as the highway trust fund is expected to face a shortfall in 2009 (Fox News: *A cash crunch is fast approaching for the government trust fund that pays to build and repair highways and bridges.* <http://www.foxnews.com/story/0,2933,274113,00.html>), and a renewed emphasis on clean fuel and energy efficiency are likely to place a further dent on the cash flow of the highway trust fund past 2009, the provision of road capacity through private financing in forms such as BOT can be an attractive alternative to keep the nation's infrastructure functional and safe.

Once road provision moves into the market economy, there are many intriguing issues to be addressed. From the viewpoint of private investors, the profitability of a road project is of great concern; while from the public side, improving social welfare should be the main goal. It is thus imperative to assess whether or not the construction of a toll road will give a positive welfare increment if it is profitable, compared with the do-nothing alternative, and vice versa whether or not a toll road, which adds to welfare, will be profitable and hence can be provided by private firm(s). Obviously, answers to these questions depend crucially upon the supply decisions made by individual firms in terms of capacity choice and pricing, which directly affect the cost of the road project and its attractiveness to potential users. If the private investor is free to select both capacity and toll charge, social

welfare gain is not guaranteed as its major concern is its own profit; if both capacity and toll charge is regulated by the government, optimal social welfare can be realized, but the project may become less attractive to private investors due to rigid regulations and may even become infeasible because of a net loss in revenue. To avoid these two extreme situations, two kinds of government policies can be considered: one is that the government sets an (optimal) level of the toll charge but leaves the road capacity to be freely determined by the investor; the other is that the government fixes the (optimal) road capacity while the investor selects the toll charge freely. These two regimes may both lead to market failure, but the intrinsic reasons are different. One failure is associated with the fact that when profit is maximized with respect to capacity, the investment may not be optimally set; while the other is caused by the allowance for the private investor to exploit its market power over toll (Spence 1975).

The changes in the profit and welfare gain generated by the new commercial road can be calculated for various combinations of toll and capacity. Figure 1 shows an example of the profit and welfare gain contours in the two-dimensional capacity-toll space. This graphical representation allows for an intuitive characterization and discussion of the possibilities of profitability and welfare gain open to a road planner.

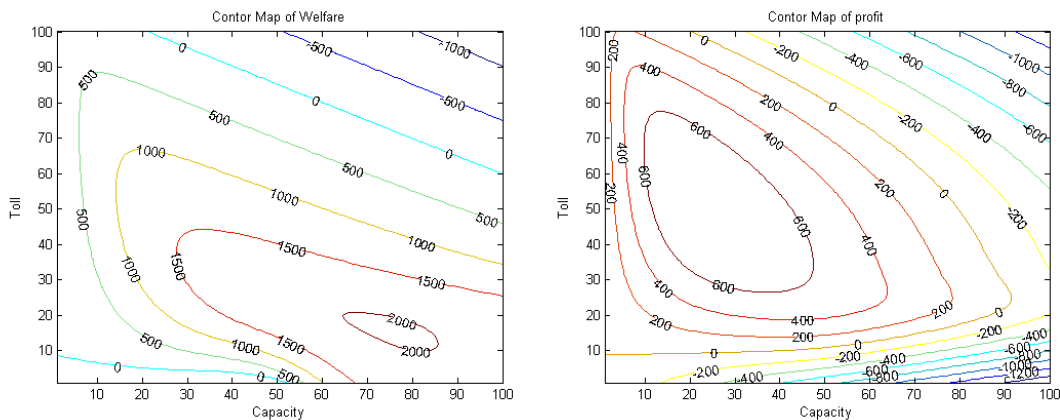


Figure 1 An example of profit and welfare gain contours

Different government policies may lead to different equilibria represented by points in Figure 1. We can see from the comparison of the two contours that actually the profit-oriented behavior will induce a higher toll rate and lower investment (thus lower capacity) than the social optimum, which implies that giving the franchise to private sector to build the transportation facility may not be fully efficient.

The above results were obtained with the assumption of identical travelers, that is, all the travelers have the same value-of-time (VOT). VOT, arguably the most central parameter

in transportation economics, is often calculated as a trade-off ratio between the in-vehicle time coefficient and the cost coefficient. Travel time savings usually constitute a very large share of total benefits in cost benefit analyses of infrastructure projects (Hensher, 2001; Mackie et al., 2001) and cost benefit analyses are in turn a main part of the information provided to decision makers on new projects.

In real life, travelers' value-of-time can differ significantly, and this difference in the valuation of time by commuters can affect investment and regulatory decisions by the firm and government agencies, respectively. When forecasting market share for a tolled road, for example, both the average VOT and its distribution are found to be significant factors (Hensher and Goodwin, 2004). Thus it is important to consider user heterogeneity in analyzing the BOT scheme. The choice of capacity and prices is more complex when two or more competing firms operate multiple toll roads, because their profits are interrelated due to demand inter-dependence in the network. Apart from the consideration of road user responses, each firm must consider what its competitors' choices are likely to be in making its capacity design and pricing decisions.

2. Related Literature

Previous analytical studies on private toll road modeling with homogeneous commuters

Issues have been studied by considering only homogeneous road users with a single VOT. Yang and Meng (2000) investigate the profitability or self-financing and social welfare gain of a single new toll road in a general network through numerical experiments. Later they show that the self-financing theorem holds for each road individually in a full network and consequently for the network in aggregate, provided each link is optimally priced and all capacities are optimized (Yang and Meng, 2002). Verhoef and Rouwendal (2004) address some implications of both the first-best and second-best congestion pricing for the applicability of the self-financing theorem using a numerical approach, and they find that the volume-capacity ratio in the social optimum is identical for all links if they have the same marginal cost of capacity. This observation is derived numerically rather than theoretically on a general network with the assumption of a linear inverse demand function and a traditional BPR (Bureau of Public Road) travel time function. De Borger and Van Dender (2005) analyzed a model with two substitute congestible facilities under three administrative regimes: (a) social optimum, (b) monopoly, and (c) duopoly in a sequential capacity-then-toll game. They showed that equilibrium time

delays are equal in regimes (a) and (b), but higher in regime (c). Namely, pricing and capacity choices under monopoly do result in the socially optimal service quality. This result is derived theoretically but with a linear inverse demand function. Xiao et al. (2007) obtained a more general result when studying the inefficiency of the oligopolistic equilibria of toll road competition. They proved that at both oligopolistic equilibria and social optimum, the volume-capacity ratio of each road remains unchanged and is only determined by the road's own unit construction cost. A one-shot game was considered where the road capacity and level of toll charge are determined simultaneously by each firm subject to the resulting traffic flow being in equilibrium.

Previous analytical studies on private toll road modeling with heterogeneous commuters

A few existing works relax the limitation of a single VOT by considering different VOTs for different users. In Cheung et al. (1999), users are divided into a number of groups or classes according to their VOTs. Each group has a distinct group-specific demand function to characterize its trip rates. Users are assumed to minimize their individual generalized cost in choosing their routes on the basis of travel time and monetary cost. Various possibilities of profitability and welfare gain of a private toll road in a given network under various combinations of road capacity and toll charge are presented. They also compare and contrast the outcomes with the case of a single average VOT, and investigated how the VOT distribution affects the traffic flow and profit forecasts. Yang et al. (2002) further examined the impact of user heterogeneity on the profitability and social welfare gain of new toll roads. Mayer and Hansen (2000) developed a model for steady-state congestion pricing in which the VOT has a continuous distribution. They mainly focused on the difference of optimal tolls when the social welfare function is measured in money and time units respectively. A relevant earlier work by Spence (1975) deals with market problems that arise when a monopoly sets some aspect of product quality as well as price, and makes some interesting findings. For example, if users value product quality equally (corresponding to homogeneous travelers with identical VOT), then profit-maximizing and socially optimal quality levels coincide (in the case of congested highways considered in this paper, product "quality" just corresponds to congestion delay). Spence also discussed about the advantages of the rate-of-return regulation for the supposed market model.

3. One Toll Road Competes with a Free Road Network

In this section we first consider a simplified model to study the steady-state congestion pricing problem. Suppose in a general network in the morning rush period, there is a fixed amount of commuters N traveling from node A to node B , where A is the residential area and B is the CBD area. To shorten the travel time between A and B , the government decides to build a highway directly connecting the two nodes. Because of financing insufficiency, the government has to adopt the BOT scheme by providing a private firm the franchise to build and operate the highway. After the construction of the highway, the population of N commuters have two choices: they may use the new highway, in which case they are charged a toll τ . Their travel time follows a function $T(v, y)$, where v is the flow of commuters on the highway and y is the highway capacity. Without loss of generality, we assume that $\partial T/\partial v > 0$, $\partial^2 T/\partial v^2 > 0$, and $\partial T/\partial y < 0$, which indicates that the highway performance function is strictly increasing and convex with respect to the flow on highway and strictly decreasing with respect to the highway capacity; or they can take the alternative: a network of arterial roads free of charge with a fixed travel time t_0 , since the arterial road network is large enough and can be assumed not congested. And it is reasonable to assume that commuters using the arterial road will spend more time to get to the CBD area, i.e. $t_0 > T(0)$, where $T(0)$ is the free flow travel time of the highway, so that the tolled highway will certainly attract some users with a relative high value-of-time. A schematic representation of the situation modeled is given in Figure 2.

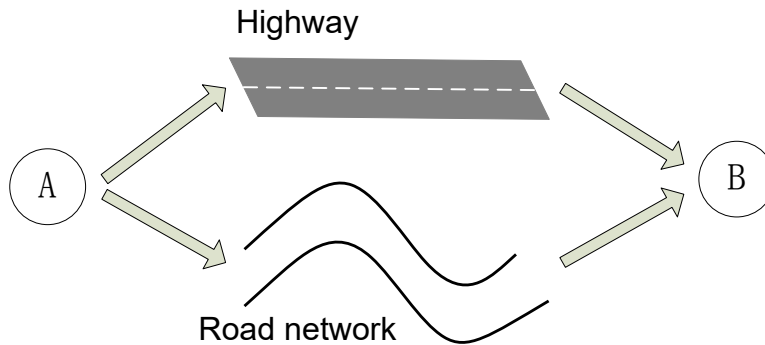


Figure 2 Study network

The volume/capacity ratio, $\gamma = V/y$, is a representation of the level of service on the highway. To simplify the discussion, we bring forward the following two assumptions

Assumption 1. *The link travel time function $T(V, y)$ is homogeneous of degree zero in both link flow V and link capacity y .*

Assumption 2. (Constant return to scale in road construction) There are constant returns to scale in road construction, namely, $E_I^y = 1$ or $I(y) = ky$, where E_I^y is the elasticity of investment cost, I , with respect to output capacity, y , and $I(y)$ is the link construction cost function of the highway; k denotes the unit capital cost.

Assumption 1 is equivalent to assuming that the speed of traffic on the road is dependent only on the volume-capacity ratio γ of the road, that is, $T(V, y) = T(\gamma)$. A good example is the widely used BPR (Bureau of Public Roads) type of function. Except where we specifically mention, these two assumptions are used throughout the study.

Instead of just a single VOT for the whole population N , the more realistic situation that each commuter has a unique VOT is considered here. The distribution of VOT across the population is characterized by a continuous function $\beta(v)$. If the population is ordered in decreasing order of VOT, $\beta(v)$ gives the VOT of the v -th commuter. Assume that $\beta(v)$ is continuous and differentiable in its domain of definition, then from our definition, $\beta'(v) < 0$. A representative VOT curve is presented in Figure 3.

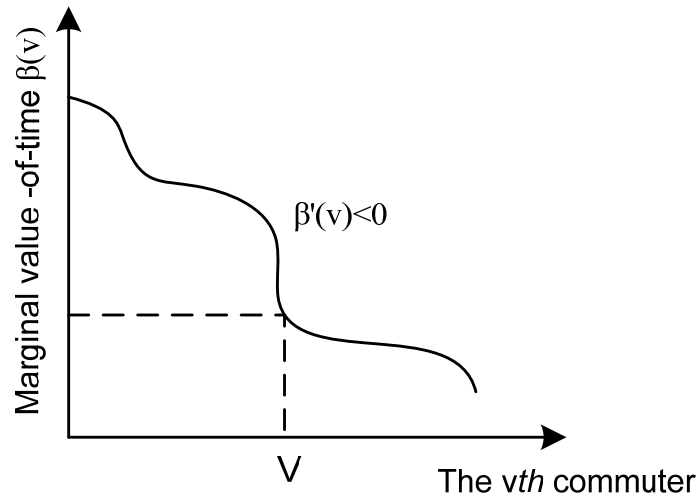


Figure 3 Continuously distributed VOT

Based on this definition, we are ready to obtain the relationship between $\beta(v)$ and F (the cumulative function of VOT)

$$\beta(v) = F^{-1}\left(1 - \frac{v}{N}\right) \quad (1)$$

and the following relationship between $\beta'(v)$ and f (the probability density function of VOT)

$$\beta'(v) = -\frac{1}{Nf(\beta(v))} < 0 \quad (2)$$

Eqns.(1) and (2) provide a convenient way to derive $\beta(v)$ from the density or cumulative function of value-of-time established by the survey data.

There are trade-offs between traveling by highway and arterial road network. For those who choose to drive on highway, they spend more money for the exchange of a shorter travel time. It's not hard to observe that commuter v will choose the highway if $\tau + \beta(v) \cdot T(\gamma) \leq \beta(v) \cdot t_0$, where V is the number of commuters using the highway. Thus the full price that the v -th commuter is willing to pay to use the highway is

$$D(v) = \beta(v)t_0 \quad (3)$$

For any equilibrium where $V < N$, the benefit of the V -th commuter, who is the highway user with the lowest VOT and therefore the lowest willingness to pay for using the highway, is zero. Thus we have the following equilibrium condition

$$\tau = \beta(V) \cdot (t_0 - T(\gamma)) \quad (4)$$

V also represents the total amount of highway users. The equilibrium condition (4) implicitly defines V as a function of τ and γ . Utilizing the implicit function theorem, we can obtain the two derivatives V_τ and V_γ

$$V_\tau = \frac{1}{(t_0 - T)\beta' - \beta T_V} \quad (5)$$

$$V_\gamma = \frac{\beta T_\gamma}{(t_0 - T)\beta' - \beta T_V} \quad (6)$$

Because $\beta' < 0$, $T_V > 0$ and $T_\gamma < 0$, we have $V_\tau < 0$ and $V_\gamma > 0$, which implies that the highway usage will increase if we lower the toll or expand the capacity of the highway. IF the private firm is in charge of the construction and operation of the highway, it cannot set the toll rate and the highway capacity too low or too high because of the existence of the competing arterial road: If it sets the toll rate too high or the capacity too low, then the highway becomes less attractive and the firm will lost its customers and the revenue is reduced; However, if the toll rate is too low or the capacity is too high, then the revenue still cannot be guaranteed and the construction cost is higher. Thus it's necessary to

investigate the behavior of the private firm under such a competition circumstance and the resulting outcomes in terms of the profit, social welfare gain and project efficiency. Because of the profit-oriented behavior of the private firm, the market outcome may not be efficient in terms of social welfare. Thus difference policies might be carried out by the government to be imposed on the firm to prevent tremendous welfare loss. In the following sub-sections, two commonly used regulations (the price-ceiling regulation and the investment-floor regulation) are modeled as two types of sequential games B and C and compared with the do-nothing alternative, modeled by game A, in which the private firm and the DOT, are assumed to choose both of the toll and capacity levels simultaneously

- Game A. Simultaneous game without limitation on either toll or capacity;
- Game B. Monopoly with upper-bound of toll;
- Game C. Monopoly with lower-bound of capacity.

And then the rate-of-return regulation is recommended in response to the disadvantage of the price-ceiling and investment-floor regulations. At the end of this section our model will be implemented to evaluate the performances of different policies on the highway franchising with HOV lanes. SR91 express lane is chosen as the experiment site for the data collection.

3.1 Simultaneous game with both toll and capacity as decision variables

If the government does nothing to restrict the private firm's behavior, the private firm will have the entire power of setting both the toll and the investment of the highway, although a free arterial network exists to compete with the private company. We model the monopoly behavior of the private firm as a simultaneous game, i.e. the private firm is assumed to choose toll and capacity simultaneously. Profit maximization gives the following two first-order optimality conditions

$$\pi_y = V_y \cdot \tau - I' = \frac{-\beta^2 T' \gamma(t_0 - T)}{(t_0 - T)y\beta' - \beta T'} - I' = 0 \quad (7)$$

$$\pi_\tau = V + V_\tau \cdot \tau = V + \frac{y\beta(t_0 - T)}{(t_0 - T)y\beta' - \beta T'} = 0 \quad (8)$$

From eqns.(7) and (8) the following results are obtained

$$T'(\gamma)\gamma^2 = \frac{I'}{\beta(V)} \quad (9)$$

$$\tau = T'(\gamma)\gamma \frac{\beta^2(V)}{\beta(V) + V\beta'(V)} \quad (10)$$

$$\tau = \frac{I'}{\gamma} \frac{1}{1 + \frac{V\beta'(V)}{\beta(V)}} \quad (11)$$

To measure the efficiency loss of the monopoly behavior of the private firm, we should also examine the social welfare maximizing result, assuming that the government has all the information of the market, where the social welfare is measured in units of money

$$\begin{aligned} W(\tau, y) &= \int_0^V D(v) dv - \int_0^V \beta(v) \cdot T(\gamma) dv - ky \\ &= (t_0 - T(\gamma)) \int_0^V \beta(v) dv - I(y) \end{aligned} \quad (12)$$

Taking the first-order derivatives yields

$$W_y = \frac{1}{y} T' \gamma \int_0^V \beta(v) dv - I' = 0 \quad (13)$$

$$W_V = \tau - \frac{1}{y} T' \int_0^V \beta(v) dv = 0 \quad (14)$$

From eqns.(13) and (14) the following two results are obtained (where ‘-’ represents the social optimal solution)

$$T'(\bar{\gamma})\bar{\gamma}^2 = \frac{I'}{\frac{1}{\bar{V}} \int_0^{\bar{V}} \beta(v) dv} \quad (15)$$

$$\bar{\tau} = T'(\bar{\gamma})\bar{\gamma} \left(\frac{1}{\bar{V}} \int_0^{\bar{V}} \beta(v) dv \right) = T'(\bar{\gamma})\bar{\gamma}\bar{\beta} \quad (16)$$

$$\bar{\tau} = \frac{I'}{\bar{\gamma}} \quad (17)$$

By comparing eqns.(15) and (9), the following proposition is obtained

Proposition 1. *With Assumptions 1 and 2, there is a difference in toll charge level between the monopoly market and the social optimum if the commuters are heterogeneous, which means that the monopoly market is inefficient; and the monopoly market is fully efficient when the commuters are homogeneous.*

Proof. Suppose monopoly market has the same toll and capacity levels with socially optimal solution, then the traffic flow must also be the optimal flow, thus we have $\bar{V} = V$ and $\bar{\gamma} = \gamma$. However, with Assumption 1 and from eqns. (15) and (9) it follows that

$$T'(\gamma)\gamma^2 = \frac{k}{\beta(V)} > \frac{k}{\frac{1}{V} \int_0^V \beta(v) dv} = T'(\bar{\gamma})\bar{\gamma}^2 \quad (18)$$

Which implies $\bar{\gamma} < \gamma$. This conflicts with the assumption that $\bar{\gamma} = \gamma$.

The proof of the second part is straightforward. When $\beta = \text{constant}$, since $T(\gamma)$ is convex, from eqns. and it follows that

$$\beta(V) = \frac{1}{V} \int_0^V \beta(v) dv \Rightarrow \bar{\gamma} = \gamma \quad (19)$$

Also, from eqns.(11) and (17), $\tau = \bar{\tau}$. Thus the two situations coincide. ■

It is hard to compare the toll and capacity levels under monopoly and at social optimum, since the overall performance of the monopolist is jointly affected by the two partial effects in Sections 3.1 and 3.2. The firm with market power deviates from the optimum in either one or both of the two aspects: on the one hand, the firm intends to give a higher toll than the optimum, which causes a lower traffic on the highway. On the other hand, the firm sets γ too low in comparison with optimal capacity.

Corollary 1. *With Assumptions 1 and 2, the monopoly market cannot have both a lower toll and a higher capacity compared with the social optimum.*

The potential market failure can be addressed by the different VOT of the commuters. Consider a firm which is contemplating a small increase in the investment for the construction of capacity. The increase in capacity will increase costs, say by ΔI . It will also increase revenues. The increase in capacity reduces the highway travel time and thus increases the unit monetary benefit of the highway to the marginal commuter (the one who is just willing to pay for the toll) by $\Delta\tau(v)$, where v is the number of commuters. The firm will increase revenues by $v\Delta\tau(v)$. This increase is desirable for the firm if

$v\Delta\tau(v) > \Delta I$. On the other hand, the capacity increase is desirable for the society if the total benefits $-\Delta T(v) \int_0^V \beta(v) dv$ exceeds the cost ΔI . Comparing the two situation, it is found that the social benefits correspond to the increase in the revenues of the firm only if the marginal benefit of the commuter is equal to the average, that is, when $-\frac{1}{V}\Delta T(V) \int_0^V \beta(v) dv = \Delta\tau(v)$, or $-\Delta T(V)\bar{\beta} = \Delta\tau(v)$. Here, for the demand to remain unchanged, $\Delta\tau(v)$ and $\Delta T(V)$ must have the relationship: $\Delta\tau(v) = \beta(v)\Delta T(V)$. Thus, only when $\beta = \bar{\beta}$, the social benefits correspond to the increase in the revenues of the firm. But according to the definition of $\beta(v)$, $\bar{\beta}$ cannot be less than β and they are equal only if all the commuters have the same VOT.

Now we examine the level of service, which is measured by the v/c ratio on the highway. From eqns.(4), (9) and (11) it follows that

$$\frac{t_0 - T(\gamma)}{T'(\gamma)\gamma} = \frac{1}{1 - |E_{\beta}^V|} \quad (20)$$

Similarly, from eqns. (4), (15) and (16) we have

$$\frac{t_0 - T(\bar{\gamma})}{T'(\bar{\gamma})\bar{\gamma}} = \frac{\bar{\beta}}{\beta} \quad (21)$$

Let $g(\gamma) = (t_0 - T(\gamma))/T'(\gamma)\gamma$. Obviously $g(\gamma)$ is decreasing with respect to γ . And from eqns.(20) and (21) we have

$$\frac{g(\gamma)}{g(\bar{\gamma})} = \frac{\beta(\bar{v})}{\bar{\beta}(\bar{v})} \frac{1}{1 - |E_{\beta}^V|} \quad (22)$$

Following the similar discussion, it is not hard to find that whether the v/c ratio of the highway controlled by the firm is higher or lower relative to the optimum depends on the absolute value of the elasticity of VOT at V .

It's also worth noting that when government decides both toll rate and capacity supply, the profit realized for the firm is

$$\pi = \bar{\tau} \cdot V(\bar{\tau}, \bar{\gamma}) - I(\bar{\gamma}) = \frac{k}{\bar{\gamma}} \bar{V} - I(\bar{\gamma}) = 0 \quad (23)$$

3.2 The toll-ceiling regulation

Based the discussion in Section 4.1, we know that the private firm tends to set a higher toll rate or lower capacity of the highway which causes the efficiency loss. Thus it's natural to think that a ceiling of the toll rate may be imposed on the private firm to improve the social welfare gain. In this section, we assume that a ceiling of the toll rate is set by the government first with the aim of maximizing the social welfare and then the capacity is selected by the private firm that wishes to maximize its profit. The game follows a two-stage process: at the first stage, the government sets a toll level, based on all the market information; at the second stage, the firm decides the amount of capacity to provide under the toll given to try to maximize its own profit.

Suppose the toll given by the government is $\bar{\tau}$, the firm tries to maximize its profit given by the following function with respect to y

$$\max \pi(y) = \bar{\tau} \cdot V(\bar{\tau}, y) - I(y) \quad (24)$$

Taking the first-order derivative we have

$$\pi'(y) = V_y \bar{\tau} - I' = 0 \quad (25)$$

$$\bar{\tau} = \frac{I'}{V_y} \quad (26)$$

On the other hand, the social welfare maximizing solution under $\bar{\tau}$ is given by the following function

$$W(V, y) = (t_0 - T(V, y)) \int_0^V \beta(v) dv - I(y) = \bar{\tau} \frac{1}{\beta} \int_0^V \beta(v) dv - I_y \quad (27)$$

The first-order optimality condition of (27) is

$$\frac{\partial W}{\partial V} = \bar{\tau} \left(1 - \frac{\beta'}{\beta^2} \int_0^V \beta(v) dv \right) V_y - I' = 0 \quad (28)$$

From (28) we have

$$\bar{\tau} = \frac{I'}{V_y} \frac{1}{1 - \frac{\beta'}{\beta^2} \int_0^V \beta(v) dv} \quad (29)$$

If the private firm chooses the optimal capacity level which is preferred by the government, then the traffic flows must also be the optimal flow on the highway. From eqns.(26) and (29) we have

$$\frac{1}{1 - \frac{\beta'}{\beta^2} \int_0^v \beta(v)dv} = 1 \quad (30)$$

Since $\beta' < 0$, eqn.(30) cannot hold. Thus from this contradiction we conclude that the capacity will not be optimally set under monopoly and the traffic flows differ. With assumption 2, from eqns.(26) and (29) we have

$$0 < V_y(\tau, \bar{y}) < V_y(\tau, y) \quad (31)$$

Inequality (31) states that whether the firm selects a lower or higher capacity than the government is actually determined by the marginal effect of capacity on traffic flow. If the marginal influence of capacity on the traffic flow is diminishing, the company will set a capacity lower than the socially optimal one, and conversely. If all the commuters have exactly the same value-of-time, i.e. $\beta' = 0, \forall v \in (0, N)$, the profit-oriented behavior exactly obtains the same result with the optimum.

Here we summarize those partial effects into the following Proposition 2.

Proposition 2. *With Assumption 2, for any given toll charge, there is a difference in capacity level between the monopoly market and the social optimum if the commuters are heterogeneous, which means that the monopoly market is inefficient; the firm undersupplies capacity relative to the optimum when the marginal influence of capacity on the traffic flow is diminishing, and conversely, capacity is oversupplied.*

3.3 The investment-floor regulation

Most studies of capacity-and-price competition adopt a two-stage game framework, in which capacities are chosen in the first stage and prices in the second. The two-stage sequential game is natural in the context of toll roads since adjusting road capacity is more costly and time-consuming than adjusting tolls. Indeed, unless tolls are heavily regulated it is easy to adjust tolls using electronic toll collection technology. Thus it could be more reasonable if we consider the sequential game, in which the level of construction

capacity is given by the government first and the toll charge level is selected by firm for profit maximization. Similarly, the game also follows a two-stage process.

Because the lower-bound of capacity of the highway is confined by the government, the private firm can maximize its profit given by the following function only with respect to τ

$$\max \pi(\tau) = \tau \cdot V(\tau, \bar{y}) - I(\bar{y}) \quad (32)$$

Taking the first-order derivative we have

$$\pi'(\tau) = V + V_{\tau}\tau = V + \tau \frac{1}{\frac{\beta'}{\beta}\tau - \beta T_V} = 0 \quad (33)$$

Thus

$$\tau = \frac{\beta T_V V}{1 + \frac{\beta'}{\beta} V} \quad (34)$$

In contrast, the social welfare is maximized with respect to V by the following function

$$W(V, y) = (t_0 - T(V, y)) \int_0^V \beta(v) dv - I(y) \quad (35)$$

The first-order optimality condition is

$$\frac{\partial W}{\partial V} = \tau - T_V \int_0^V \beta(v) dv = 0 \quad (36)$$

Thus we have

$$\bar{\tau} = T_V \int_0^V \beta(v) dv = VT_V \bar{\beta} \quad (37)$$

If we assume the toll on the highway remains unchanged under both monopoly and at social optimum, then the traffic flows will also be the same, from eqns.(37) and (34) we have

$$\frac{\tau}{\bar{\tau}} = \frac{\beta}{\bar{\beta} \left(1 + \frac{\beta'}{\beta} V\right)} = 1 \quad (38)$$

Here the term $\beta' / \beta V$ is actually the elasticity of VOT with respect to the ordered commuter at V , which can be expressed by the elasticity E_{β}^V . Thus eqn.(38) can be written as

$$\frac{\tau}{\bar{\tau}} = \frac{\beta}{\bar{\beta}(1 - |E_{\beta}^V|)} = 1 \quad (39)$$

Generally, eqn.(39) cannot hold, which implies that the traffic flow patterns of the monopoly and social optimum are different. When $|E_{\beta}^V|$ is small enough, $\tau/\bar{\tau} < 1$, the monopoly firm tends to set a relatively lower toll on the highway than the government, and conversely. Again, if all the commuters have exactly the same VOT, i.e. $\beta' = 0, \forall v \in (0, N)$, then it can be easily found that the profit-maximizing toll is exactly the same as the social optimum. We summarize the partial effects into the following Proposition 3.

Proposition 3. *For any given capacity, there is a difference in toll charge level between the monopoly market and the social optimum if the commuters are heterogeneous, which means that the monopoly market is inefficient; the firm undercharges the toll relative to the optimum when the absolute value of the elasticity of VOT is very small, and conversely toll is overcharged.*

We provide three examples here with three different kinds of VOT distributions.

In the first example, we assume VOT follows a uniform distribution $\beta(v) = b - av, a > 0, b > 0$; then it follows that

$$\frac{\tau}{\bar{\tau}} = \frac{(b - aV)^2}{(b - aV)^2 - abV} > 1 \quad (40)$$

Second, if VOT follows an exponential distribution, $\beta(v) = b - a \ln v, a < 0, b > 0$, then we have

$$\frac{\tau}{\bar{\tau}} = \frac{(b - a \ln V)^2}{(b - a \ln V)^2 - a^2} > 1 \quad (41)$$

Thus for uniform or exponential distributed VOT, the monopoly firm will always set a toll higher than the optimum.

However, one can easily find a counterexample to demonstrate the existence of the opposite situation. For example, if $\beta(v) = a + b/v, a > 0, b > 0$, then

$$\frac{\tau}{\bar{\tau}} = \frac{a^2V + 2ab + \frac{b^2}{V}}{a^2V + ab \ln V} \quad (42)$$

Clearly, when V is large enough, $\tau/\bar{\tau} < 1$.

3.4 Rate-of-return regulation

From the previous discussions, it's not hard to observe that, neither the toll-ceiling nor the investment-floor regulation can achieve the social optimum. Moreover, in reality, it is very difficult for the government to know the true construction costs and the real distribution of VOT, so that it is almost impossible to pre-determine the optimal toll and capacity levels. Facing such informational difficulty, it is shown in this section that a rate-of-return regulation could be a desirable regulation for the government, even under the situation that the VOT distribution is unknown.

The rate-of-return constraint takes the form

$$\frac{\tau V}{I} \leq s \quad (43)$$

Or

$$\pi \leq (s - 1)I(y) \quad (44)$$

which defines an upper-bound of the revenue of the private firm per unit construction cost, s . Since the toll rate on the highway τ , the traffic flow on the highway V and the total construction cost I can all be observed, the only question for the government is to decide the upper-bound of the rate-of-return s . In the following we show that setting an upper-bound of the rate-of-return is equivalent to imposing a lower-bound on constructed capacity

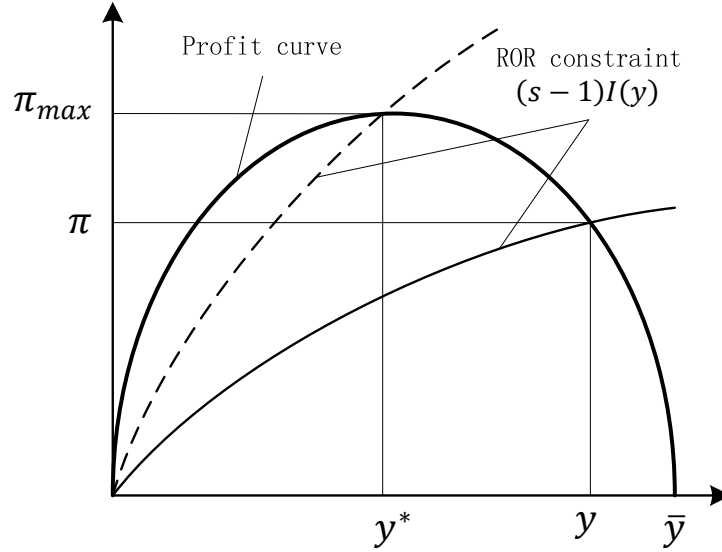


Figure 4 Profit maximization with ROR constraint

Assuming that the profit of the private firm is concave with respect to capacity and has a maximum at point A, as shown in Figure 4, from eqn.(23) the profit curve intersects x-axis at point C(\bar{y} , 0). The curve, $f(y) = (s - 1)I(y)$, is drawn in Figure 4. Obviously, $f'(y) = (s - 1)I' \geq 0$, the rate-of-return constraint is a positively sloped function in profit-capacity space, which means the more the private company invests, the higher profit the company could obtain. In general, the position of the rate-of-return constraint depends on the allowed rate-of-return, s . If we define y^* as the capacity level that maximizes the firm's profit and s^* as the corresponding rate-of-return level, we find that when $1 \leq s \leq s^*$, as s increases profit increases fast and less investment on capacity is induced. When $s \geq s^*$, the capacity will remain at y^* . From eqns.(20), (4) and (44), we have

$$\pi_{max} = \frac{I_y y^*}{1 - |E_\beta^V|} - I = (s^* - 1)I \quad (45)$$

Thus s^* is obtained from eqn.(45) as

$$s^* = \frac{I_y y^*}{I(1 - |E_\beta^V|)} \quad (46)$$

With Assumption 2, $I_y y^* = I$, s^* can be further simplified into

$$s^* = \frac{1}{1 - |E_{\beta}^V|} \quad (47)$$

That is to say, any upper-bound of rate-of-return, $s < (1 - |E_{\beta}^V|)^{-1}$, can drive the private firm to increase its investment on capacity construction when maximizing its own profit and thus to reduce the inefficiency loss under monopoly. The extreme situation is that $s = 1$, the private firm will set a social optimal capacity level and the profit of the private firm is 0. The advantage of the ROR regulation is that the profit of the private firm can be easily controlled by the government and by lowering the rate-of return, the social welfare can be increased without knowing any details about the construction cost and VOT distribution, which, in the reality, are very difficult to obtain.

3.5 An application: the highway franchising with HOV lanes

The competition between an HOV lane and GP (general purpose) lanes can be regarded as a good real life example for our model. California's existing HOV system comprises totally 1,268 lane-miles with 102 lane-miles are under construction and 963 lane-miles are proposed to be constructed (*The statewide HOV lane inventory report, June 2005*). The HOV system in California is initially considered as an innovative traffic management strategy to give time advantage to multiple-occupant vehicles so that people will be motivated to carpool and in return, the overall highway performance will be improved and emissions will be reduced. As traffic demand in California continued to exceed the capacity of metropolitan freeways, the California Department of Transportation has taken HOV lanes to be "an essential alternative for evaluation in the project development process when considering an additional lane by re-striping and/or reconstruction or widening on freeways with three or more lanes in one direction." (*FHWA, California Division Office, Procedure Memorandum D 6103*). The initial motivation of developing HOV lanes is to encourage carpool so that higher person throughput can be realized with fewer vehicles, and as a result, the congestion on the highways will be alleviated, and the total emission reduced. Recent studies indicate that carpool lanes may not reduce as much vehicle trips as were expected from them because most of the carpoolers are from the same household and would not drive a separate vehicle for the trip anyway. For example, McGuckin and Srinivasan (2001) reported that the number of family member carpools is much greater than the casual carpools (83% of carpools for home-based work trips had people from the same household, 97% of whom had only household members). Moreover, some of the HOV facilities are often underutilized, while its neighboring GP

lanes bear the brunt of congestion. This leads to a waste of highway capacity as well as public resistance to HOVninzation in urban highways.

To address both underutilization and fairness, a new concept called HOV&T lanes is suggested here. Different from HOT lanes, where SOV vehicles are charged to use the HOV lane, the HOV&T scheme tolls general purpose lanes while restrict the usage of HOV lanes to HOV vehicles only. This may be more effective in inducing solo drivers to carpool since now they can trade between toll, travel time, and inconveniences of carpool. The idea behind this scheme is to fully utilize the waste capacity on a HOV lane, and from an economic perspective it is more conducive to achieving welfare maximization. At present such as scheme may not be politically feasible, but as we move from a fuel-tax based highway financing system into a VMT-based one, HOV&T lanes could be the wave of future.

In this section we apply our model to evaluate the HOV&T franchise project. The SR91 express lanes are chosen here as a case study site. It is a highway with multiple general purpose and HOV lanes. Since we have the hourly traffic information for all the lanes and the toll schedule on the HOV lane, we can easily establish the relationship between the traffic split (between the tolled lane and free lanes) and the toll rate, so that the VOT distribution of the commuter population can be calibrated. If a private company is franchised to build and operate the GP lanes, the possible outcomes of the toll and capacity levels for the GP lanes have to be carefully examined, not only because the existing HOV lanes will compete with the tolled GP lane for users, but also because inappropriate toll and capacity levels may incur unacceptable loss of social welfare. Our study here is suitable to model such situation and compare the different policies carried out by the government to find out which policy is the best for society. Four different cases are considered: 1. Doing nothing; 2. Toll-ceiling regulation; 3. Investment-floor regulation; and 4. The rate-of-return regulation.

3.5.1 Introduction to SR 91 express lanes.

The SR 91 Express Lanes is a ten-mile high-occupancy toll road/full tollway hybrid contained entirely within the median of the Riverside Freeway (State Route 91) in Orange County, California. They run from the Costa Mesa Freeway (State Route 55) interchange in Anaheim to the Riverside County line (Figure 5).



Figure 5 Orange County SR 91 Express Lanes

The 91 Express Lanes consist of two primary lanes in each direction, separated from the regular, main lanes of the Riverside Freeway with reflective yellow, 3' high, plastic lane markers (as opposed to concrete barriers or a similar "solid" barrier). Limited access to the 91 Express Lanes are provided only at its east and west ends.

All tolls are collected using an open road tolling system, with each vehicle required to carry a FasTrak RFID transponder; there are no toll booths to receive cash. The 91 Express Lanes use a variable pricing system based on the time of day. The road is not truly "congestion priced" because toll rates come from a preset schedule instead of being based on actual congestion. Since January 1, 2008, the toll on the busiest hour on the tollway, 3:00 pm to 4:00 pm eastbound on Fridays, could be as high as \$10.00, or \$1.00 per mile, the highest toll for any toll road in the country. The highest possible toll in the morning rush hour, 7:00 am to 8:00 am westbound on weekdays, is \$4.20. Motorcycles and vehicles with three or more passengers who use the "3+" carpool lanes are not charged a toll, except when traveling eastbound from 4:00 pm to 6:00 pm on weekdays. During that period, they are charged 50% of the full posted toll. Even though there may be no toll charge, a FasTrak transponder is still required on all vehicles using the "3+" carpool lanes.

3.5.2 VOT calibration for the SR91 highway users

The VOT calibration method

Data from PeMs and OCTA (Orange County Transportation Authority) are collected to calibrate the empirical VOT distribution of the commuters. A commuter's VOT can be calculated by the following equation:

$$\beta(P_\tau(t)) = \frac{\tau_H(t)}{T_L(t) - T_H(t)} \quad (48)$$

If we arrange the commuters in the decreasing order of VOT, $\beta(P_\tau(t))$ represents the VOT of the commuter who feels no difference to choose the HOT or GP lane. Any commuter whose VOT is greater than $\beta(P_\tau(t))$ will travel through the HOT lanes, for those whose VOT are lower than $\beta(P_\tau(t))$, they will either use the GP lanes or use the HOT lane by carpooling. $P_\tau(t) = v_\tau(t)/(v_L(t) + v_H(t))$, is the percentage of vehicles paying the toll at time t , where $v_\tau(t)$ is the amount of commuters who pay the toll to use the HOT lanes, $v_L(t)$ is the total number of commuters using the HOT lanes and $v_H(t)$ is the total number of commuters using the GP lanes. T_L and T_H are respectively the total travel times on the GP lane and HOT lane, which can be calculated by the detector data available in PeMs database and vary with respect to time in a day. $\tau_H(t)$ is the toll at time t . $\tau_H(t)$ follows the schedule on the official OCTA website. With 24 groups of data (24 hour a day) we have 24 sample points for one day in one direction. Through the curve fitting, we can thus roughly obtain the VOT distribution for commuters using the SR91 express lanes.

Data collection and VOT calibration

From eqn.(48) we know that to obtain the VOT distribution, we need two sets of data: one is the toll rate on the HOT lanes with respect to time and the other is the corresponding traffic information (travel time, vehicle speed and traffic flow pattern, etc.) on both the HOT and GP lanes on the highway. The hourly toll rate of SR91 express lanes on a typical day (We choose Wednesday to exclude the weekend effects) can be obtained from OCTA (as shown in Figure 6).

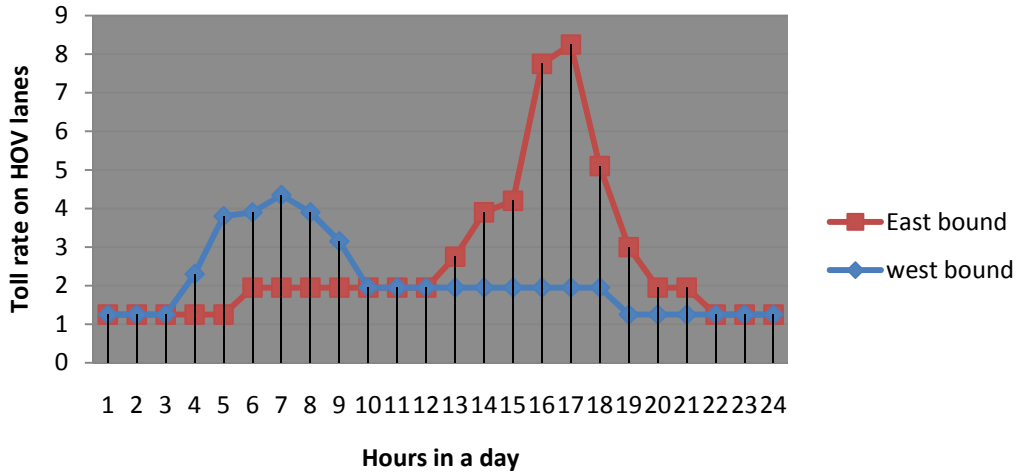


Figure 6 Toll structures of SR91 express lanes

From Figure 6 we can see the toll is high during the morning peak for west bound and evening peak for east bound, which implies that people will commute to work in the morning from the east residential area to the west working area and commute back to home in the evening.

We selected 9 groups of detectors which cover the whole toll road section (The red spots in Figure 7). Each group of detectors comprises two sub-groups: the group of detectors on HOT lanes and the group of detectors on GP lanes. We select the same Wednesday for the data collection. We observe that every detector selected were healthy and received 100% data.

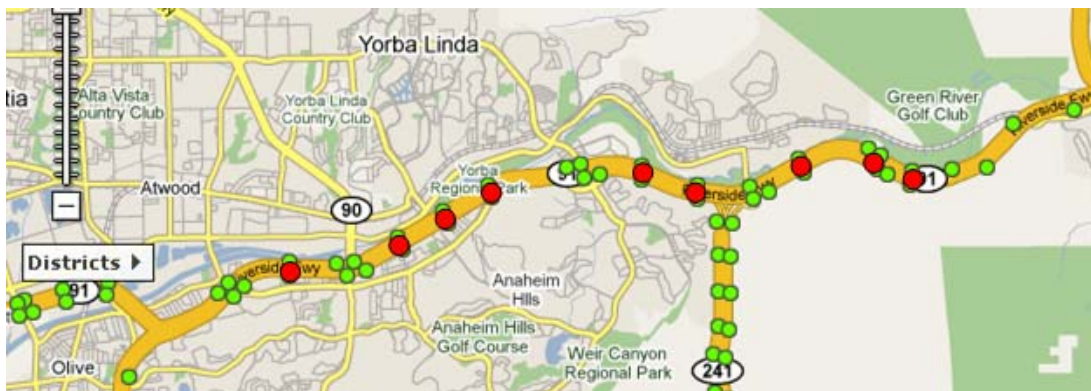


Figure 7 Detectors selected for data collection

Figure 8 shows the change of speed in terms of time for all the nine detectors. Obviously, all the lanes have free flows for most time of the day except that congestion happens

during 15:00-20:00 in the afternoon for the east-bound traffic. Figure 9 shows the total average travel time difference between HOT and GP lanes from 1:00-24:00 in a day for east bound traffic. We can see the average travel time for the whole SR91 express lane section is about 0.16h. The GP lanes are always faster than HOT lanes except during 15:00-20:00. This result shows that the toll charged outside 15:00-20:00 will receive no revenue, since no one will be interested to pay the toll to use the HOT lane because of an even shorter travel time via GP lanes. We can also observe that the HOT lanes are almost always kept uncongested during the whole day.

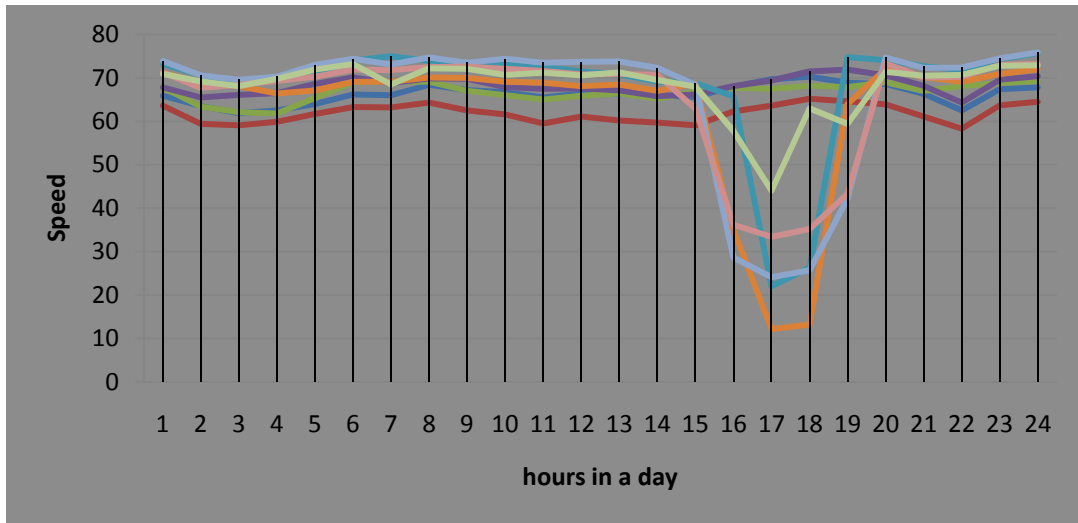


Figure 8 Hourly Speed on SR91 express lanes

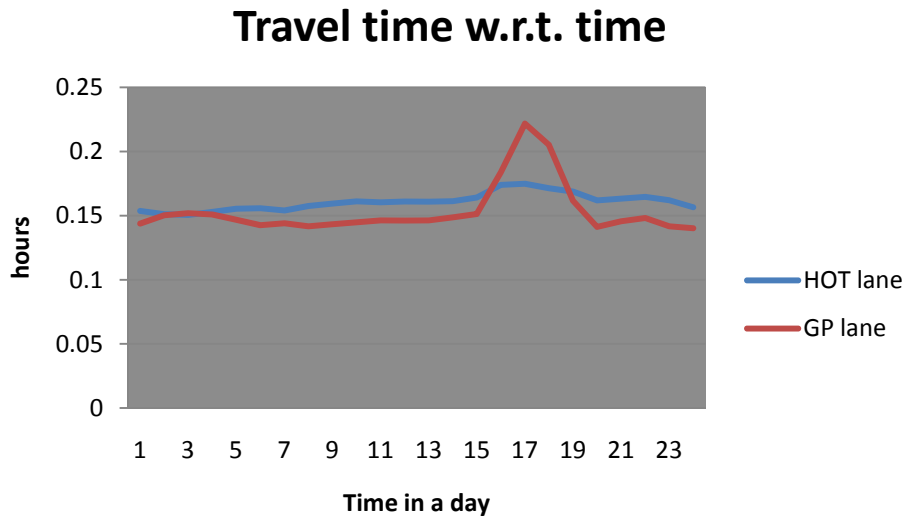


Figure 9 Hourly travel time on HOT and GP lanes

Since there's only 6 hours in a day in this example during which HOT lanes is faster than the GP lanes, we have only 6 sample points to calibrate the VOT distribution. According to the observations made in 1996-1999, the carpool vehicles occupies 7.4% of the total traffic and generally 70% of the HOV traffic was observed to use HOT lanes where available (Sullivan and Burris, 2006). Thus we have

$$P_{\tau}(t) = \frac{v_L(t)}{v_L(t) + v_H(t)} - 7.4\% \times 70\% \quad (49)$$

We find the value ranges from 90\$-245\$/h for the commuters using the HOT lane, which is at 10%-30% of all the users if we order the users in a decreasing order of VOT. This result is much higher than most of the values stated in previous studies.

It is well known that the income distribution of the population in a city is well fitted by a lognormal distribution. And it's reasonable to assume that the VOT follows the same distribution as income. Thus we adopt the lognormal form of distribution of VOT given below:

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} v^{-1} \exp\left(-\frac{(\ln v - \mu)^2}{2\sigma^2}\right) \quad (50)$$

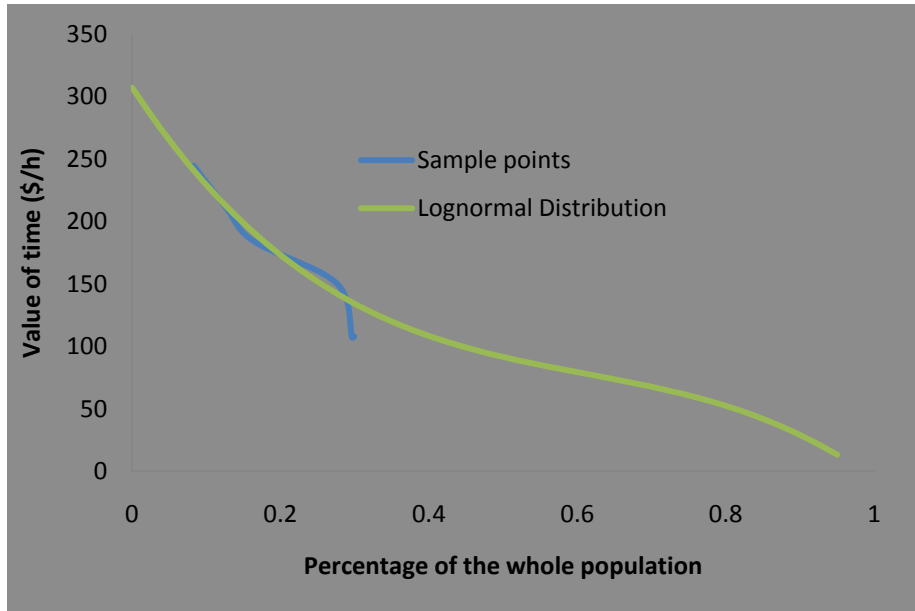


Figure 10 VOT distribution

From the data collected by the detector on the SR91 express lanes, a mean value of $\mu = 4.6041$ and standard deviation $\sigma = 0.6056$ is found by the calibration (See Figure 10).

The average traffic demand during the peak period (15:00-20:00) is 9973 *veh/h*. From Figure 9, we observe that the free flow travel time of the SR91 express lane segment is 0.15*h*. We assume the highway performance function follows the traditional BPR function, which is

$$t(v) = 0.15 \left(1.0 + 0.15 \left(\frac{v}{y} \right)^4 \right) \quad (51)$$

The HOV lanes are never congested. But there is a cost of carpool arrangement, which, if we transfer into time unit, is generally larger than the free flow travel time on the GP lanes. We assume the capital construction cost of the GP lane capacity is $k = 10\$/(\text{veh}/h)$. Table 1 shows the monopoly solution together with the optimum for a comparison.

Table 1. Results for the numerical example

	Monopoly	Toll-ceiling regulation	Investment-floor regulation	Rate-of-return regulation	Social optimum
Toll on highway (\$/h)	39.89	10.20	26.13	10.20	10.20
Highway capacity (veh/h)	3502.96	4743.31	9094.76	9094.76	9094.76
Traffic flow on GP lanes (veh/h)	3442.35	7815.17	6308.38	8914.25	8914.25
Profit of the firm (\$/h)	102289.96	32301.75	73878.20	0.00	0.00
Social welfare (\$/h)	186825.91	145298.36	235363.31	269531.51	269531.51

From the results we have following observations:

1. Due to the profit-driven behavior, the private firm selects a much higher toll and much lower capacity than the social optimal solution. Compared with the optimal solution, the level of service on the GP lanes are lower, which means the GP lanes are more congested. The constructed capacity level is lowest among the five cases and the toll rate is the highest. As a result, the percentage of carpools among the

population is the highest. Compared with the optimal solution, the monopoly behavior experiences 30.7% efficiency loss. It's also worth noting that at optimum solution the profit of the private firm approaches to 0, which is consistent with eqn.(23);

2. When the toll rate is forced to be the optimal level by government regulation, the private firm still intends to undersupply the capacity. As a result, the GP lanes are even more congested. The profit of the firm is greatly reduced. It's surprising to see that the toll-ceiling regulation induces even more efficiency loss than doing nothing, although the resulting capacity is higher than the monopoly case. Since adjusting road capacity is more costly and time-consuming than adjusting tolls, the toll-ceiling regulation may lead to a better result when the franchising to the private firm ends after a certain amount of years, but during the time when the private firm is in charge of the operation of the highway, the efficiency loss will be high;
3. The investment-floor regulation limits the private firm to provide a much higher capacity than it wants. However, under this restriction, the private firm still intends to impose a higher toll rate to the highway users to extract more revenue. The profit of the firm is much higher compared with the toll-ceiling regulation and the social welfare gain is close to the optimal level. Especially, when the highway is returned to the government after the franchising is over, the highway will be operated just to produce the maximum social welfare. Thus we conclude that the investment-floor regulation performs better than the toll-ceiling regulation, no matter before or after the contract ends;
4. The rate-of-return regulation performs the best among the three different regulations. Since it induces exactly the optimal solution, when the rate-of-return is restricted to 1. This result has a significant practical meaning to the regulatory authorities, not only because the regulation provides the optimal results, but because it's very easy and simple to be implemented compared with the other two. The rate-of-return regulation does not need the full information of the market, e.g. the cost structure of the private company and the VOT distribution of the population, which in reality can hardly be known by the government. And without the information, it will be quite difficult for the government to decide a proper toll rate or investment level when utilizing the toll-ceiling or investment-floor contract.

4. Two Toll Roads Competition

Where there are more than one toll roads, the toll operators can form various market strategies and the “games” they place to set the tolls and/or capacity are more complex. Here we consider two typical scenarios, one is oligopoly competition and the other is private company and government competition. Taking both toll and capacity as decision variables makes the problem hard to solve, since now it’s not just an optimization problem but a bi-level Nash equilibrium (the upper level is a Nash equilibrium for two players and the lower level is the user equilibrium). The objective function is no longer convex so that there could be multiple equilibrium points. Moreover, involving VOT distribution makes the problem even more complicated. Actually in practice changing road capacity is more costly and time-consuming than adjusting tolls. Once the road is built, it will be hard to change the capacity in a short period, even under competition. In recognition of this and to simplify the discussion, in this section we consider only price competitions with the capacity of the two toll roads are fixed.

4.1 Oligopoly

This game is defined as a one-shot game where all decisions are made at the same time by both firms in an equal market position. This could be the case where two firms choose their tolls simultaneously when two new private toll roads have already been constructed. The Nash equilibrium in this case of simultaneous moves is found by assuming that each of the operating firms are maximizing profits with regard to its toll charges, given the level of capacity and toll for the other road. In this game we assume that the two toll roads competing with each other are parallel and both subject to congestion. We assume the toll choices of the two firms are respectively τ_1 and τ_2 . And the link performance function of the two road are $t_1(v_1)$ and $t_2(v_2)$, where $v_1 + v_2 = N$. Suppose at the equilibrium, $\tau_1 > \tau_2$ and $t_1 < t_2$ then for those who have relatively high VOT, they will choose to use road 1, otherwise they will use road 2. The equilibrium condition can thus be expressed by

$$\tau_1 - \tau_2 = \beta(v_1)(t_2(v_2) - t_1(v_1)) \quad (52)$$

Eqn.(52) implicitly defines v as a function of τ . Each firm is trying to maximize its own profit, given the choices of the competitor. The profit is equal to the toll revenue subtracted by the construction cost

$$\pi_a = \tau_a v_a(\boldsymbol{\tau}), \quad a = 1, 2 \quad (53)$$

where $\boldsymbol{\tau} = (\tau_1, \tau_2)$ is the toll vector. For simplicity, we assume the unit construction costs for the two firms are identical. According to the definition of equilibrium, at the equilibrium, no firm can improve its profit by changing its toll rate. Thus we have the necessary conditions for the oligopoly equilibrium

$$v_1(\boldsymbol{\tau}) + \tau_1 \frac{\partial v_1(\boldsymbol{\tau})}{\partial \tau_1} = 0 \quad (54)$$

$$v_2(\boldsymbol{\tau}) + \tau_2 \frac{\partial v_2(\boldsymbol{\tau})}{\partial \tau_2} = 0 \quad (55)$$

By solving eqns.(54)-(55), we are able to find the solution of the oligopolistic competition between the two toll roads. It's not easy to obtain analytical results because the solution depends not only on the two road's performance functions, but the VOT distribution of the population. In the end of this session, a numerical example will be provided for the demonstration of the resulting equilibrium and the competition outcomes (toll rate, capacity level, profit of the firm and realized social welfare) will be compared with the socially optimal solution. And the social optimum is given by solving the following objective function

$$\min_{\boldsymbol{\tau}} TC = (v_1 t_1 - v_2 t_2) \int_0^{v_1} \beta(x) dx + v_2 t_2 \int_0^N \beta(x) dx \quad (56)$$

A numerical example is presented here to derive insights on how market structure, regulatory policy and product differentiation affect social welfare, firm profitability, and economic efficiency of the road system. All the values of the parameters in this example are chosen to produce reasonable results. For simplicity, we adopt the uniform VOT distribution, $\beta(v) = 20 - 0.15v$, and link performance function follows the same form in Section 4.5, except that now the two roads are both subject to congestion

$$t_a(v_a) = t_a^0 \left(1.0 + 0.15 \left(\frac{v_a}{y_a} \right)^4 \right), \quad a = 1, 2 \quad (57)$$

where t_a^0 is the free-flow travel time of link a . The basic input data of the link travel time functions are $t_1^0 = 0.15 h$, $t_2^0 = 0.3 h$, and the capacities of the two road $y_1 = y_2 = 50 veh/h$. The total population is 100 veh . Table 2 shows the oligopoly solution together with the social optimum for comparison.

Table 2. Oligopoly vs. SO

	Oligopoly		SO	
	Road 1	Road 2	Road 1	Road 2
Toll (\$)	4.39	3.00	5.09	3.00
Traffic flow (veh/h)	59.29	40.71	50.72	49.28
Profit (\$/h)	210.38	72.12	208.16	97.83
Total travel cost (\$/h)	8833.91		8080.36	

Obviously, the SO solution obtains the lowest total travel cost. Yet we can see the oligopolistic competition does not induce significant efficiency loss (only 9.3%). Because road 1 has shorter free flow travel time, firm 1 can thus take the advantage of firm 2 and obtain much higher revenue. Whereas, the traffic flow on the two roads are almost equal. We can also observe that the toll rates under the competition are lower than the optimal level, which is intuitive because each firm is trying to lower its own toll rate so that more users can be attracted to its own road. Though the toll rates under SO induce less total travel cost, the competition results are also attractive to the planners, since the efficiency loss is acceptable and more importantly, every road user experiences a lower travel cost than the SO solution.

4.2 Private firm vs. the government

This game characterizes the situation of asymmetry regarding the firms' market position. This could be the case where a privately controlled toll road (road 1) is introduced and competes with an existing one (road 2) controlled by the government. We assume that the new road will have a shorter free flow travel time, i.e. $t_1 < t_2$. Both the private firm and the government are unable to simply change their road capacity, whereas the toll rate is free to choose. Since toll adjustment is flexible in the short run, such situation could be described as an outcome following from a dynamic game, where the government and the firm choose their toll levels in turn, giving the competitor's previous choice. Since the government has a different objective (social welfare maximization) from the private firm (profit maximization), the outcome of such kind of competition will be different from the oligopoly market. The government is trying to maximize the social welfare (or in this case equivalent to minimizing the total cost) based on the following equation

$$\min_{\tau_2} TC = (v_1 t_1 - v_2 t_2) \int_0^{v_1} \beta(x) dx + v_2 t_2 \int_0^N \beta(x) dx \quad (58)$$

And the private firm is trying to maximize its profit by choosing the toll rate of the road under its control, given the toll rate on road 2 carried out by the government

$$\max_{\tau_1} \pi_1 = \tau_1 v_1(\tau) \quad (59)$$

From eqns.(58)-(59), the first-order optimality conditions become

$$v_1(\tau) + \tau_1 \frac{\partial v_1(\tau)}{\partial \tau_1} = 0 \quad (60)$$

$$\begin{aligned} & \left(t_1 \frac{\partial v_1}{\partial \tau_2} + v_1 \frac{\partial t_1}{\partial v_1} \frac{\partial v_1}{\partial \tau_2} - t_2 \frac{\partial v_2}{\partial \tau_2} - v_2 \frac{\partial t_2}{\partial v_2} \frac{\partial v_2}{\partial \tau_2} \right) \int_0^{v_1} \beta(x) dx \\ & + \beta(v_1)(v_1 t_1 - v_2 t_2) \frac{\partial v_1}{\partial \tau_2} \\ & + \left(t_2 \frac{\partial v_2}{\partial \tau_2} + v_2 \frac{\partial t_2}{\partial v_2} \frac{\partial v_2}{\partial \tau_2} \right) \int_0^N \beta(x) dx = 0 \end{aligned} \quad (61)$$

By solving eqns.(60)-(61), we are able to find the solution of the competition between the private firm and the government. Similarly, it's still not easy to obtain analytical results because the solution depends not only on the two road's performance functions, but the VOT distribution of the population. The dynamics of the competition is shown in Figure 11 and Figure 12.

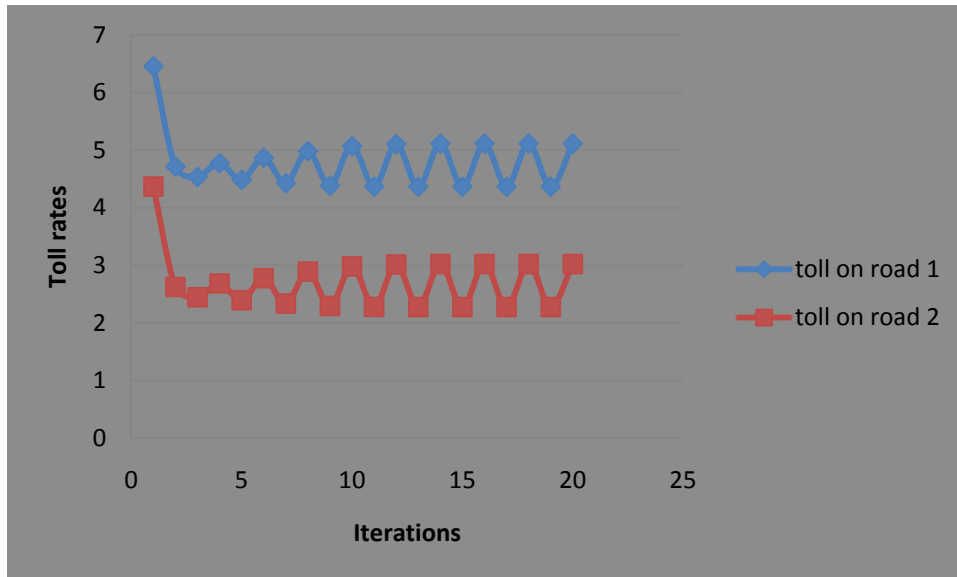


Figure 11 Dynamics of the toll rates

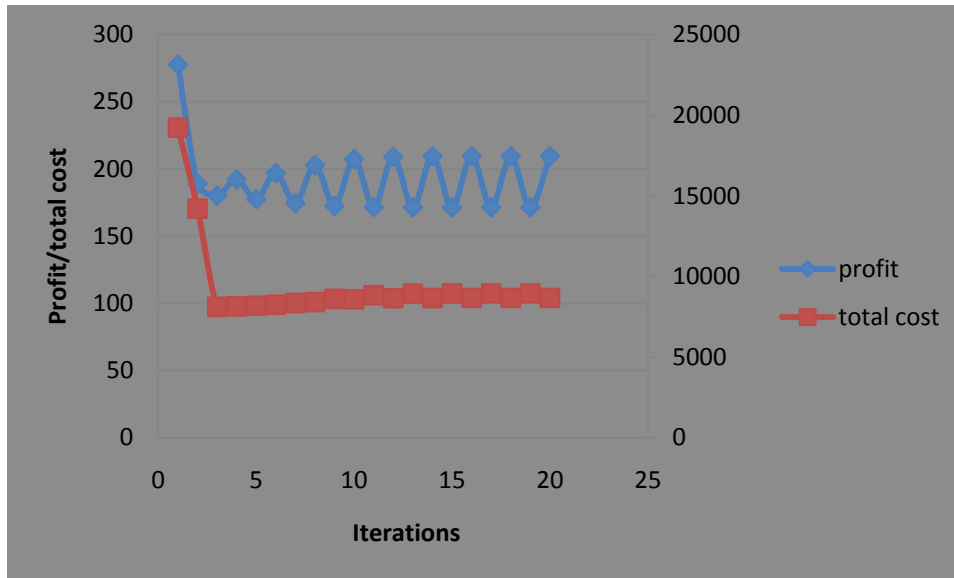


Figure 12 Dynamics of the profit and total travel cost

We can see in this case, the government always tries to enforce the traffic flow split to be optimal so that the total cost can be minimized. However, due to the toll set by the government, the private firm always tries to lower down its own toll rate to attract more users. There is no equilibrium in this case but the toll changing pattern is stable. We list the average toll rates, profit and total travel cost in Table 3. The oligopoly and SO solutions are also listed for comparison.

Table 3. Firm vs. Government

road	Oligopoly		Firm vs. Government		SO	
	1	2	1	2	1	2
Toll (\$)	4.39	3.00	4.74	2.65	5.09	3.00
Profit (\$/h)	210.38	72.12	190.42	99.05	208.16	97.83
Total travel cost (\$/h)	8833.91		8815.14		8080.36	

From Table 3 we observe that when the competitive road is controlled by the government, the profit of the private firm is suppressed. The resulting toll rates are just between the oligopoly outcome and the SO solution. The efficiency is improved but not much difference with the oligopoly competition.

5. Conclusions

In this study, we develop strategic gaming models to study various BOT schemes with non-identical travelers and multiple agents in a one-OD, two-road parallel network and analyze how value-of-time and market structure affect the outcome of social welfare, firm profitability, and efficiency gain/loss. Such study can be applied in predicting the possible outcomes of different government policies on a BOT project. The major conclusions and observations are listed below:

1. In a BOT project, due to the profit-oriented behavior, the private firm tends to impose a much higher toll rate to the users and the capacity will be undersupplied if no intervention is made by the government. The level of service is also lower, which makes the highway more congested. As a result, the monopolistic behavior of the private firm induces as high as 30.7% efficiency loss;
2. When the toll rate is regulated by the government to be at the level that maximizes social welfare, the private firm still intends to undersupply the capacity and the profit of the firm is greatly reduced. The toll-ceiling regulation induces even more efficiency loss than the government doing-nothing, although the resulting capacity is higher than the doing-nothing case. Under such a case, the efficiency loss will be high during the time period the private firm operates the highway,;
3. The investment-floor regulation forces the private firm to provide a much higher capacity than it wants. However, under this regulation, the private firm still intends to charge road users a higher toll rate to extract more revenue. However, the social welfare gain is close to the optimal level. When the highway is returned to the government after the franchising is over, the highway will be operated just to produce the maximum social welfare. Thus the investment-floor regulation performs better than the toll-ceiling regulation, no matter before or after the contract ends;
4. The rate-of-return regulation performs the best among the three regulations, not only because it induces the optimal results, but because it's very easy and simple to be implemented compared with the other two, since no information about the market, e.g. the cost structure of the private company and the VOT distribution of the population, is needed;
5. Road competition improves efficiency. The oligopolistic competition only has an efficiency loss of 9.3%. The traffic flow pattern does not change much under competition compared with the optimal one. And it's worth noting that the toll rates under competition are lower than the optimal level, which implies that the consumer surplus is even higher under road competition;

6. The existence of a publically controlled toll road competing with the BOT project will greatly lower the profit of the private firm. The efficiency is improved only little compared with the oligopoly competition, but the risks of greatly reduced profit makes the BOT project impalpable to the private firms.

As the highway trust fund is expected to have a shortfall in the near future, innovative alternative financing mechanisms play a vital role in maintaining the health of the nation's aging transportation infrastructure. The BOT scheme and congestion pricing schemes, as promising financing instruments, have seen wider adoption in the rest of the world other than the United States. Our study sheds lights to the pros and cons of those finance instruments and show how regulatory policies can improve the social surplus and identify the conditions under which a policy can benefit everyone: the traveling public, the government agencies, and the private toll firm. These results can be used as a reference guide when a state or city decides on a BOT project.

There are several interesting topics arising from this study, which are not addressed here but worthy of future investigations. They are listed below.

The Social Benefit of the HOV/R scheme

We have suggested an innovative pricing strategy called HOV&T that could improve the performance of the existing HOV system in California. Yet this strategy may be ahead of its time: the traveling public is unlikely to approve tolling general purpose lanes as long as they are paying the fuel tax at the pump. However, ramp metering is already a widely accepted form of traffic control, and current practices of metering SOV traffic while letting HOV traffic bypass the meters can be viewed as a form of pricing (we call this the HOV/R scheme): those who are metered paying a price in the form of travel delays. It would be interesting to study this practice under the framework of congestion pricing, and see what type of long term effect this has on travel behavior (mode and departure time choice, for example).

Integrating environmental and energy consumption analysis with road pricing

It is widely recognized that land use and transportation are two sides of the same coin. Road pricing, as an effective way to alleviate urban traffic congestion, is rarely examined in the context of land use, emissions and energy efficiency. By integrating land use models with road pricing models and take into account the environmental constraints

could lead to new solutions and perspectives of our transportation problems

Parking pricing as an alternative of road pricing

Instead of charging people tolls, which could face stronger public resistance for being just another tax or a "perquisite" to the rich, parking fee might be a more acceptable and equitable method to alleviate congestion. Many issues could potentially be addressed utilizing this concept, like the impact of parking fee on the morning commute patterns, private vs. public provision of parking, encouraging mode shifts with parking credits, cordon pricing in downtowns, and so on.

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