STATE OF CALIFORNIA DEPARTMENT OF TRANSPORTATION

TECHNICAL REPORT DOCUMENTATION PAGE

TR0003 (REV. 10/98)

1. REPORT NUMBER CA13-2270	2. GOVERNMENT ASSOCIATION NUMBER	3. RECIPIENT'S CATALOG NUMBER
4. TITLE AND SUBTITLE Development of Improved Guidelines for Seismic Analysis and Design of Earth Retaining Structures		5. REPORT DATE July 3, 2013 6. PERFORMING ORGANIZATION CODE UCLA
7. AUTHOR(S) Ertugrul Taciroglu		8. PERFORMING ORGANIZATION REPORT NO. UCLA/CA13-2270
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Civil and Environmental Engineering University of California, Los Angeles UCLA, 5731E Boelter Hall Los Angeles, CA 90095-1593		10. WORK UNIT NUMBER 11. CONTRACT OR GRANT NUMBER 65A0413
12. SPONSORING AGENCY AND ADDRESS California Department of Transportation Engineering Service Center 1801 30th Street, MS 9-2/5i Sacramento, California 95816 California Department of Transportation Division of Research and Innovation, MS-83 1227 O Street Sacramento CA 95814		13. TYPE OF REPORT AND PERIOD COVERED Final Report 6/1/2011 – 5/31/2012 14. SPONSORING AGENCY CODE 913
15. SUPPLEMENTAL NOTES Prepared in cooperation with the State of California Department of Transportation.		

16. ABSTRACT

Earth retaining structures shall be designed to withstand lateral earth and water pressures, the effects of surcharge loads, self-weight of the wall, and earthquake loads. These are the *safety* requirements. In addition, earth retaining systems shall be designed to provide adequate structural capacity with acceptable movements, adequate foundation capacity with acceptable settlements, and acceptable overall stability of slopes adjacent to walls. These are the *serviceability* requirements. The tolerable levels of lateral and vertical deformations are controlled by the type and location of the wall structure and surrounding facilities.

17. KEY WORDS Retaining Walls, Standards	18. DISTRIBUTION STATEMENT No restrictions. This document is available to the public through the National Technical Information Service, Springfield, VA 22161	
19. SECURITY CLASSIFICATION (of this report) Unclassified	20. NUMBER OF PAGES 243	21. PRICE

FUNDING AGENCY ACKNOWLEDGMENT

Support for this research was provided by the California Department of Transportation under Research Contract No. 65A0413 (and amendments thereto), which is gratefully acknowledged. We would like to acknowledge the valuable assistance and technical support of Caltrans staff in this project, particularly Anoosh Shamsabadi. Caltrans Research Project Manager Peter Lee is recognized for his assistance in contract administration.

OTHER ACKNOWLEDGMENTS

The authors would like to acknowledge the following graduate and undergraduate UCLA students who participated in the preparation of this report: Rajaa Alrayyes, Weichun Chen, Samuel Delwiche, Yajuan Duan, Taher Ghaemi, Dorian Krausz, Tuyen Nguyen, Adena Nikarakelyan, Emily Yagi.

DISCLAIMER STATEMENT

UCLA

Structural & Geotechnical Engineering Laboratory

Development of Improved Guidelines for Analysis and Design of Earth Retaining Structures

Anoosh Shamsabadi

Shi-Yu Xu

Ertugrul Taciroglu

University of California, Los Angeles

Department of Civil & Environmental Engineering

UCLA - SGEL

Report 2013/02

Development of Improved Guidelines for Analysis and Design of Earth Retaining Structures

by

Anoosh Shamsabadi, Senior Bridge Engineer
California Department of Transportation

Shi-Yu Xu, Post-doctoral Research Associate
University of California, Los Angeles

Ertugrul Taciroglu, Professor (Principal Investigator)
University of California, Los Angeles

A Final Report submitted on research conducted under Grant No. ***65A0413***
from the California Department of Transportation (Caltrans).
This is also the **Caltrans Final Report No. CA13-2270**

Department of Civil and Environmental Engineering
University of California, Los Angeles

July 2013

EXECUTIVE SUMMARY

This report presents the *Caltrans Reference Manual for Design of Earth Retaining Structures*, preparation of which was commissioned by Caltrans to the UCLA team led by Prof. E. Taciroglu under contract no. 65A0413. This reference manual is extracted and revised from a document originally prepared by Caltrans Senior Bridge Engineer, Dr. Anoosh Shamsabadi, which is henceforth referred to as the *Original Document* (OD). In the following sections of this executive summary, the amendments made to the OD and the related work carried out by the UCLA team are described.

The design examples provided in the manual are prepared in accordance with the features of the computer software, CT-Rigid and CT-FLEX, developed by Caltrans. The solutions of all the examples demonstrated in this manual can be reproduced using these two programs.

I. General Amendments

General revisions to the *OD* include: (1) correcting the typographical errors throughout the manuscript, and various errors in the equations, (2) revising the design guideline such that the LRFD factors to reflect the most up-to-date changes in the *AASHTO LRFD Specification* (2010), (3) completing the missing figures and equations, (4) correcting the erroneous figures and figure annotations, (5) reorganizing the structure of the solutions to the example problems, and providing a step-by-step solution procedure for the design of gravity and non-gravity earth retaining systems, (6) rebuilding the solutions and figures of most of the design examples, (7) completing the model description and theoretical development of the Log-Spiral-Rankine Model, (8) preparing the design charts for the evaluation of passive seismic earth pressures adopting the Log-Spiral-Rankine Model, (9) recreating the design charts for the evaluation of active seismic earth pressures using the Trial-Wedge Method, (10) adding a reference list to the document, (11) re-ordering and re-labeling the equations and figures, (12) proof-reading the equations and modifying the inappropriate grammar and narrative, and (13) removing the sections that were irrelevant to the Caltrans CT-Rigid and CT-FLEX programs.

The major amendments made to each chapter are summarized in the subsequent sections.

II. Major Amendments to Chapter 1

An introduction section is added to the beginning of Chapter 1 to provide the readers with an overall picture of the design considerations for earth retaining systems, and the scope and structure of the manual.

III. Major Amendments to Chapter 2

Chapter 2 reviews various prevailing analytical models available for evaluating the static and/or seismic earth pressures. A large number of the cited equations and figures, however, included typos and errors. These have been fixed.

The references to the analytical models, cited equations, and cited tables are now provided.

The Log-Spiral-Rankine Model, previously available only for the passive case, has been enhanced and extended to accommodate the active case. The theoretical background of the model is delineated in more details in the manual, and the reference to the model is now available.

The original solution of the example problem in this section was incorrect. It has been corrected.

IV. Major Amendments to Chapter 3

In Chapter 3, the design procedure and considerations of gravity and semi-gravity earth retaining systems sitting on the spread footings and on the pile foundations are introduced. For the reason that the considered limit states and LRFD factors in *AASHTO LRFD Specification* being revised, the context of this chapter is significantly modified to reflect the latest development, including the equations, figures, tables, and symbols. Some important concepts substantial to the structural design of RC retaining walls are also added to enrich and complete the design examples.

All the design examples in this chapter have been redone. The solutions are reorganized and figures redrawn. Specifically, the references to the equations cited from *AASHTO LRFD Specification* are now explicitly listed for every single equation. All the solutions now follow an identical solution procedure, which can be served as the template for the design of gravity and semi-gravity retaining wall. The solutions of the example problems have been double checked by the UCLA graduate students and by the postdoctoral researcher, using (1) the Caltrans CT-Rigid program and (2) the equivalent MATLAB code developed at UCLA.

V. Major Amendments to Chapter 4

Chapter 4 introduces the design considerations and procedure for the design of non-gravity earth retaining systems. Like in the Chapter 3, the context of Chapter 4 was considerably revised due to the changes of limit states and LRFD factors adopted in the latest version of *AASHTO LRFD Specification*. Many equations, figures, and tables are revised accordingly.

A few missing figures are prepared and inserted to the document.

All the examples in this chapter have been redone and the associated figures redrawn. The solutions of the example problems are again double checked by the UCLA graduate students and by the postdoctoral researcher. The computer programs used to check the solutions include (1) the Caltrans CT-FLEX program and (2) the equivalent MATLAB code developed at UCLA.

Some sections that are irrelevant to the Caltrans CT-FLEX program were removed (e.g., the p-y curves and the computation of deflection along the walls).

VI. Major Amendments to Appendices A and B

Appendix A provides the design charts for assessing the seismic active earth pressures using the Trial Wedge Method; and Appendix B that for passive case adopting the Composite Log-Spiral Method. Originally they were presented in terms of “horizontal” earth pressure coefficient vs. horizontal seismic coefficient.

The Composite Log-Spiral Method constitutes the basis and foundation of the lately developed Log-Spiral-Rankine Model. The Composite Log-Spiral Method, however, possesses some theoretical defects, which introduced some unnecessary errors into the predicted results. Also, many of the design charts in Appendix B were duplicate charts (i.e., same charts used in different combinations of input parameters) and thus were incorrect. Therefore, the design charts are all redone using the Log-Spiral-Rankine Model.

Furthermore, since the similar relationships of most of the other prevailing models are provided in the form of "total" earth pressure coefficient versus the horizontal seismic coefficient, all the design charts in Appendices A and B are redone and now are presented in this fashion.

VII. Broader Outcomes

With the funding provided by Caltrans to support the research, the UCLA team examined the formulation and the derivation of the Composite Log-Spiral Method (Shamsabadi et al., 2005, 2007) and was able to identify several necessary improvements to that model. Modifications and enhancements to the model were proposed and presented in two archival journal articles[1,2]. The improved model—*viz.,* the Log-Spiral-Rankine Model—is based on a limit-equilibrium approach, and utilizes a composite logarithmic spiral failure surface along which the Mohr Coulomb failure criterion is enforced. The model explicitly accounts for the magnitude of earthquake acceleration, the structure's height, the backfill soil properties (e.g., internal friction angle, and cohesion), and the mobilized interface friction angle between the backfill and the earth-retaining structure. The Log-Spiral-Rankine Model is physically sound, mathematically rigorous, intuitive, and offers a more complete picture of the problem. It is the most generalized and robust earth pressure model to date. The model has been implemented as a stand-alone executable computer code with a graphical user interface by the UCLA team, and will be made available for distribution to the general public.

The UCLA team has also developed two other computer programs with MATLAB. Many of the figures in the design examples are produced using these two programs. They are respectively equivalent to the Caltrans CT-Rigid and CT-FLEX programs. The UCLA team has leveraged these programs to identify any "bugs" in CT-Rigid and CT-FLEX. Several have been identified and reported to Dr. Shamsabadi who used this information to update the two CT- codes.

---

[1] Shamsabadi A, Xu SY, Taciroglu E (2013). A generalized log-spiral-Rankine limit equilibrium model for seismic earth pressure analysis, *Soil Dynamics & Earthquake Engineering*, 49, 197-209.

[2] Xu S-Y, Shamsabadi A, Taciroglu E (2013). Evaluation of active and passive seismic earth pressures considering internal friction and cohesion, *Soil Dynamics & Earthquake Engineering* (submitted for publication).

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1 **INTRODUCTION**

Earth retaining structures shall be designed to withstand lateral earth and water pressures, the effects of surcharge loads, self-weight of the wall, and earthquake loads. These are the *safety* requirements. In addition, earth retaining systems shall be designed to provide adequate structural capacity with acceptable movements, adequate foundation capacity with acceptable settlements, and acceptable overall stability of slopes adjacent to walls. These are the *serviceability* requirements. The tolerable levels of lateral and vertical deformations are controlled by the type and location of the wall structure and surrounding facilities.

This reference manual describes the geotechnical and structural design procedures for earth retaining systems in accordance with the general principles and recommendations stipulated in AASHTO LRFD Specifications (2010). Per AASHTO LRFD Specifications, there are three distinct limit states that must be examined for the design of earth retaining systems: (1) Service Limit State, (2) Strength Limit State, and (3) Extreme Event Limit State. Earth retaining systems shall be designed to satisfy the strength requirements of *all three* states. First, “unfactored” loads shall be determined, which are the estimated forces acting on the retaining structures imposed by the soil medium, pore water pressure, any live load surcharges, and seismic forces before the safety factors are considered. Depending on the type of earth retaining systems, the calculated “unfactored” loads shall be multiplied by appropriate load factors associated with the aforementioned limits states, and applied in combinations that represent the possible worst-case scenarios, which earth retaining structures may face (LRFD load combinations).

What follows in Chapter 1 of this document is a brief introduction to various types of widely used earth retaining systems, including the rigid gravity and semi-gravity walls, the non-gravity cantilever and anchored walls, the mechanically stabilized earth walls, the soil nail walls, and the prefabricated modular walls.

Selection of an appropriate method to determine the magnitudes and locations of the unfactored lateral loads is essential for the safe design of earth retaining systems. Chapter 2 provides a review on the classical analytical procedures, which adopt the limit-equilibrium concept. The models introduced in Chapter 2 include the Rankine Theory, the Coulomb Theory, the Mononobe-Okabe Model, the Trial Wedge Method, and the recently developed Log-Spiral-Rankine Model, which, incidentally, is the most general limit equilibrium model to date.

Chapter 3 discusses the structural analysis and design of semi-gravity cantilever retaining walls. Step-by-step design procedures are provided, and two examples are presented respectively for the design of cantilever retaining walls supported by a spread footing and by a pile foundation.

In Chapter 4, the structural behaviors of non-gravity cantilever and anchored retaining walls are investigated. Analysis methods and design requirements for this type of earth retaining systems are introduced in this section. Three examples are provided, which demonstrate the analysis and design procedures for cantilever and anchored sheet pile walls subject to a single layer or multiple layers of backfill materials.

1.1. TYPES OF EARTH RETAINING STRUCTURES

Retaining walls are generally classified as gravity, semi-gravity, non-gravity cantilevered, non-gravity anchored, and soil nail. Gravity walls derive their capacity to resist lateral loads through the dead weight of the wall. The gravity wall type includes rigid gravity walls, mechanically stabilized earth (MSE) walls, and prefabricated modular gravity walls. Although semi-gravity walls are similar to gravity walls, semi-gravity walls rely on their structural components to mobilize the dead weight of backfill to derive their capacity to resist lateral loads.

Non-gravity cantilevered walls rely on structural components of the wall partially embedded in foundation material to mobilize passive resistance to resist lateral loads. Anchored walls derive their capacity to resist lateral loads by restraining their structural components with tension elements connected to anchors, and possibly additionally by partially embedding their structural components into the foundation materials.

Soil nailing is an economical technique for stabilizing slopes and constructing retaining walls from the top down. This ground reinforcement process uses steel tendons, which are drilled and grouted into the soil to create a composite mass similar to a gravity wall. A shotcrete facing is typically applied, though many architectural options such as precast panels or "green" vegetated cells are available for permanent wall facings.

1.2. RIGID GRAVITY AND SEMI-GRAVITY WALLS

Rigid gravity walls may be constructed of stone masonry, un-reinforced concrete, or reinforced concrete as shown in Figure 1-1. These walls can be used in both cut and fill applications. They have relatively narrow base widths, are generally not used when deep foundations are required, and are most economical at low wall heights.

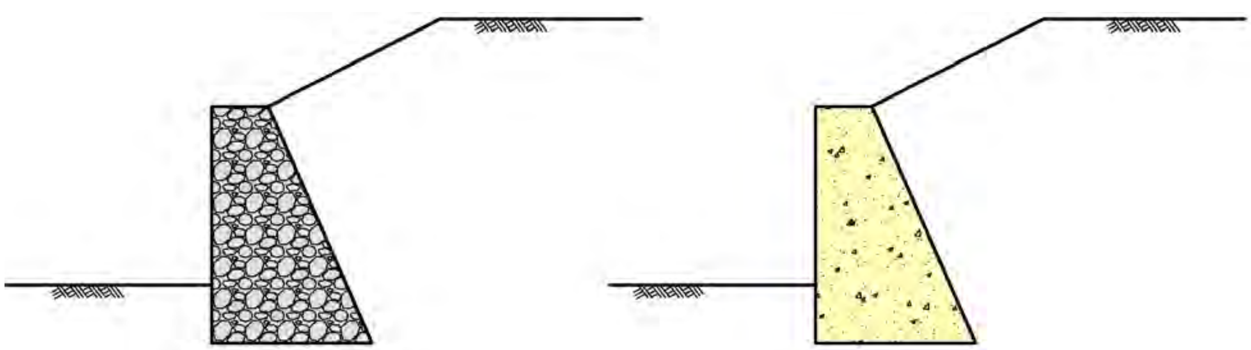


Figure 1-1: Gravity Walls

Semi-gravity cantilever, counterfort and buttress walls are constructed using reinforced concrete. They can also be used in both cut and fill applications, and have relatively narrow base widths. They can be supported by both shallow and deep foundations as shown in Figure 1-2 to Figure 1-4. The position of the wall stem relative to the footing can be varied to accommodate right-of-way constraints. These walls can support soundwalls, sign structures, and other highway features, and are most economical at low to medium wall heights. The analysis and design procedures for semi-gravity walls are demonstrated in Chapter 3.

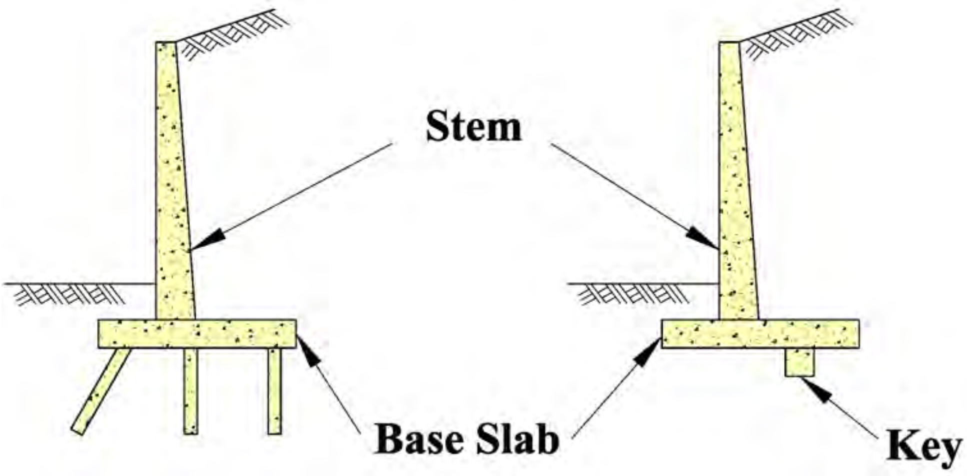


Figure 1-2: Semi-Gravity Cantilever Retaining Walls

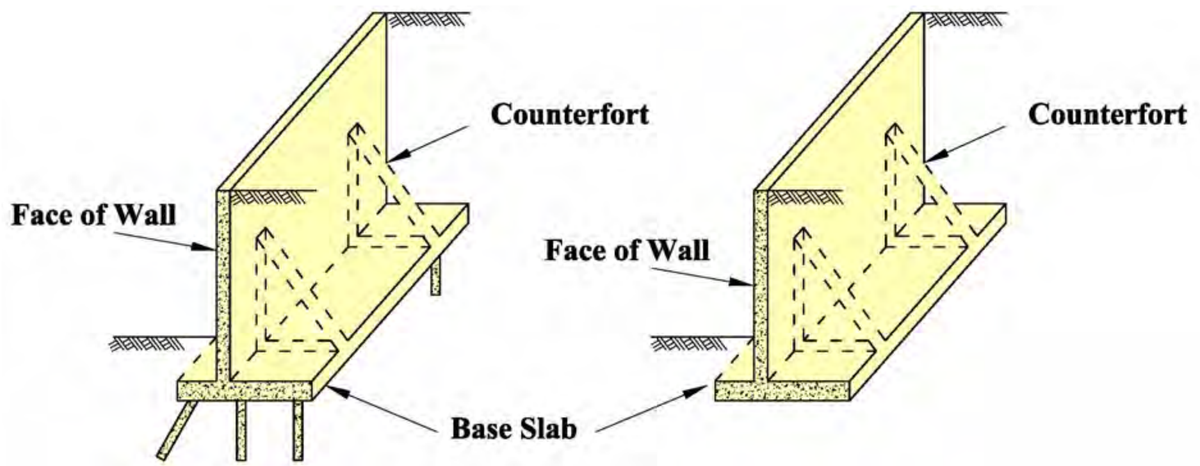


Figure 1-3: Counterfort Retaining Walls

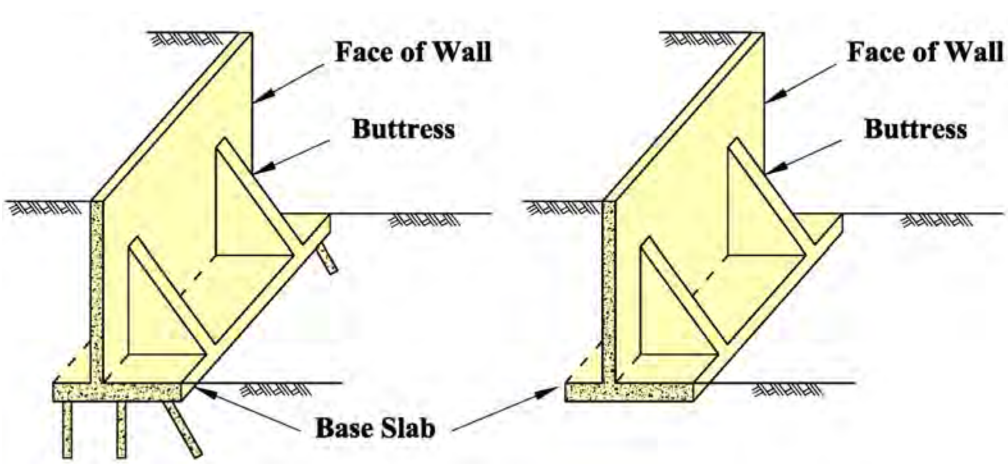


Figure 1-4: Buttressed Retaining Walls

1.3. **NON-GRAVITY CANTILEVER WALLS**

Non-gravity cantilever walls are constructed of vertical structural members consisting of partially embedded soldier piles or continuous sheet piles as shown in Figure 1-5. Soldier piles may be constructed with driven steel piles, treated timber, precast concrete or steel piles placed in drilled holes and backfilled with concrete or cast-in-place reinforced concrete. Continuous sheet piles may be constructed with driven precast pre-stressed concrete sheet piles or steel sheet piles. Soldier piles are faced with either treated timber, reinforced shotcrete, reinforced cast-in-place concrete, precast concrete, or metal elements. This type of wall relies on the passive resistance of the foundation material and the moment resisting capacity of the vertical structural members for stability. Therefore, its maximum height is limited by the competence of the foundation material and the moment resisting capacity of the vertical structural members. The economical height of this type of wall is generally limited to a maximum of 18 feet.

1.4. **NON-GRAVITY ANCHORED WALLS**

Anchored walls are typically composed of the same elements as non-gravity cantilevered walls, but derive additional lateral resistance from one or more levels of anchors as shown in Figure 1-6. The anchors may be ground anchors (tiebacks) consisting of drilled holes with grouted in pre-stressing steel tendons extending from the wall face to an anchor zone located behind potential failure planes in the retained soil or rock mass. Anchored walls are typically constructed in cut situations in which construction proceeds from the top to the base of the wall. The vertical wall elements should extend below potential failure planes associated with the retained soil or rock mass. Anchored walls may be used to stabilize unstable sites. Provided that adequate foundation material exists at the site for the anchors, economical wall heights up to 80 feet are feasible. The analysis and design procedures for non-gravity cantilever and anchored walls are demonstrated in Chapter 4.

1.5. **MECHANICALLY STABILIZED EARTH WALLS**

Mechanically stabilized earth (MSE) walls use either metallic (inextensible) or geosynthetic (extensible) soil reinforcement in the soil mass, and vertical or near-vertical facing elements as shown in Figure 1-7. MSE walls behave as gravity walls, deriving their lateral resistance through the dead weight of the reinforced soil mass the facing the structure. MSE walls are typically used where conventional reinforced concrete retaining walls are considered, and are particularly well suited for sites where substantial total and differential settlements are anticipated. The allowable differential settlement is limited by the deformability of the wall-facing elements within the plane of the wall. The practical height of an MSE wall is limited by the competence of the foundation material at a given site.

1.6. **SOIL NAIL WALL**

A soil nail wall consists of steel bars grouted into a drilled hole inclined back into the retained mass of soil as shown in Figure 1-8. Soil nails are typically spaced about 4 to 6 feet apart in both the horizontal and vertical directions, and usually vary in length from 0.7 to 1.2 times the wall height. In permanent soil nail walls, the soil nail bars have an additional layer of corrosion protection (usually epoxy coating). After the soil nails are installed, prefabricated drainage panels are placed against the cut slope, and the slope is then covered with reinforced shotcrete connected to the nail "heads." The shotcrete can be left with a rough "nozzle finish" or a smoother "cut finish." In the case of visible permanent walls, the shotcrete can be carved and stained to resemble the surrounding soil or rock, or finished to a smooth surface.

1.7. PREFABRICATED MODULAR WALLS

Prefabricated modular walls use stacked or interconnected structural elements, some of which utilize soil or rock fill, to resist earth pressures by acting as gravity retaining walls as shown in Figure 1-9. Structural elements consisting of treated timber or precast reinforced concrete are used to from a cellular system, which is filled with soil to construct a crib wall. Additionally, steel modules can be bolted together to form a similar system to construct a bin wall. Rock filled wire gabion baskets are used to construct a gabion wall, while solid precast concrete units or segmental concrete masonry units are stacked to form a gravity block wall. The aesthetic aspects of some of these types of walls are governed by the nature of the structural elements used. Those elements consisting of precast concrete may incorporate various aesthetic treatments. This type of wall is most economical for low to medium height walls.

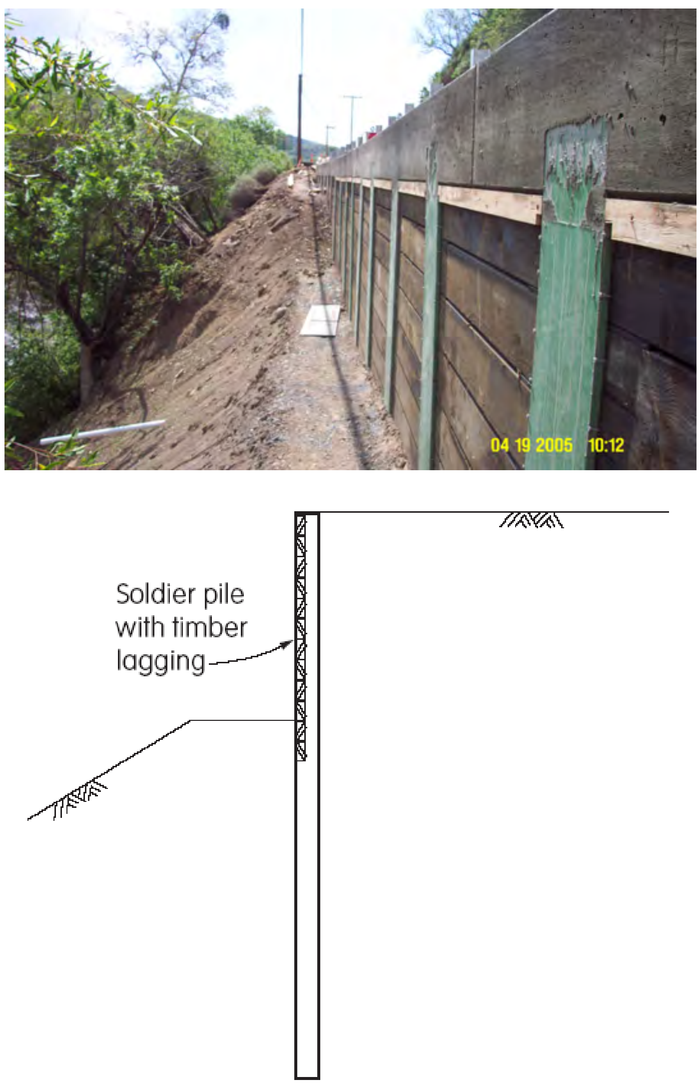


Figure 1-5: Non-Gravity Cantilever Walls

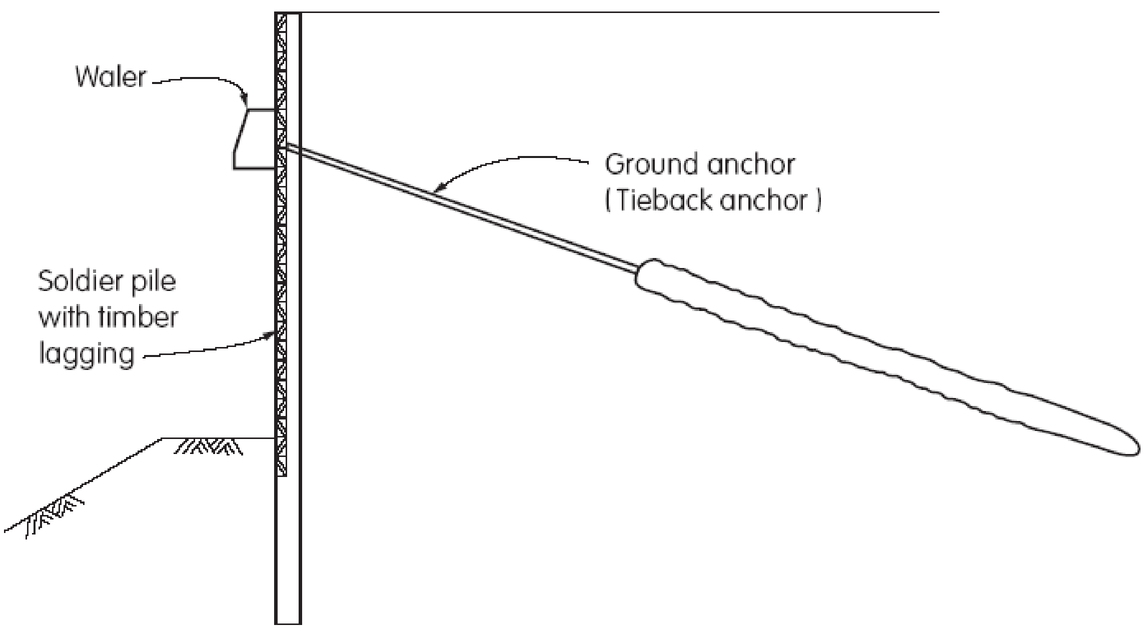


Figure 1-6: Single Tieback System

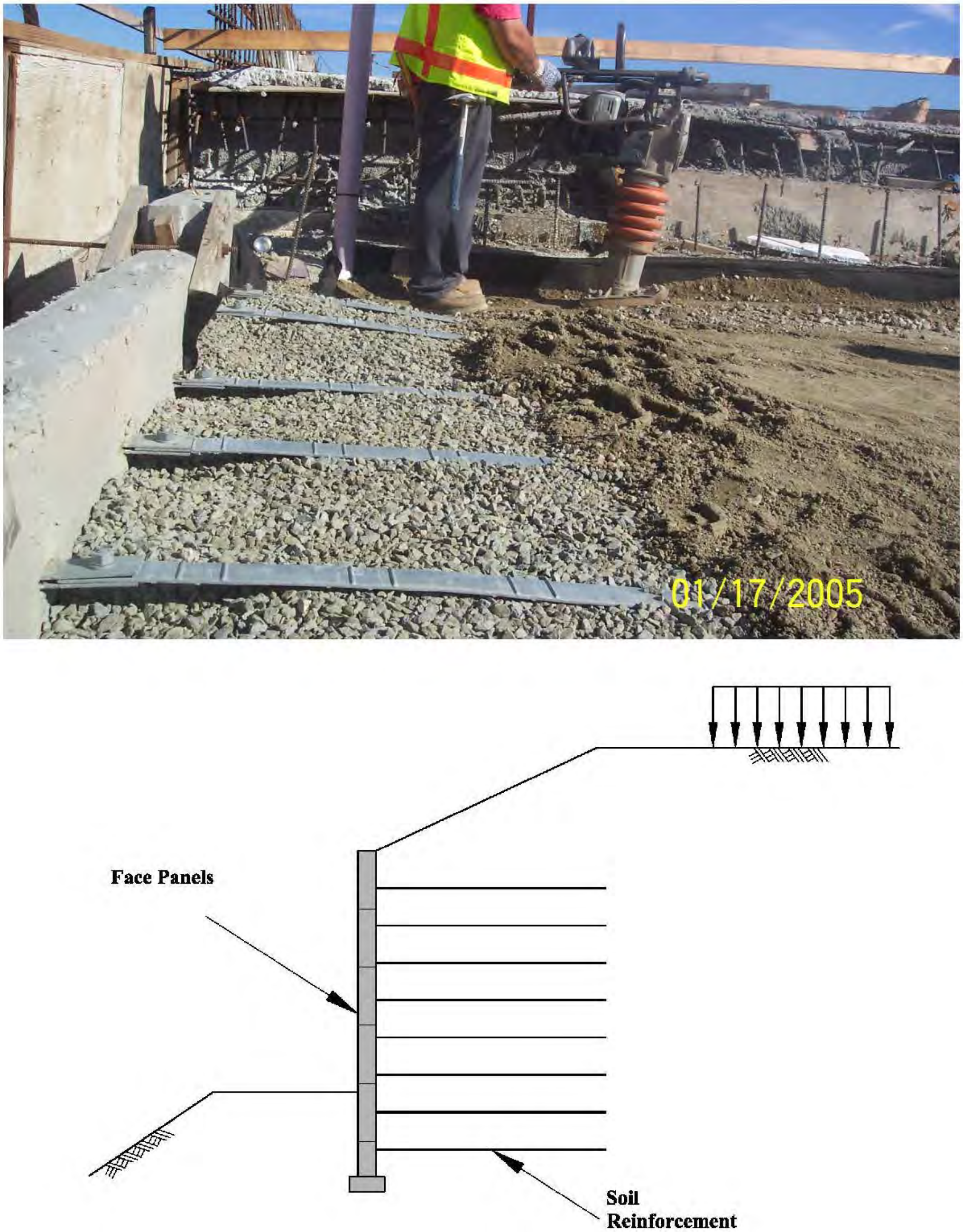
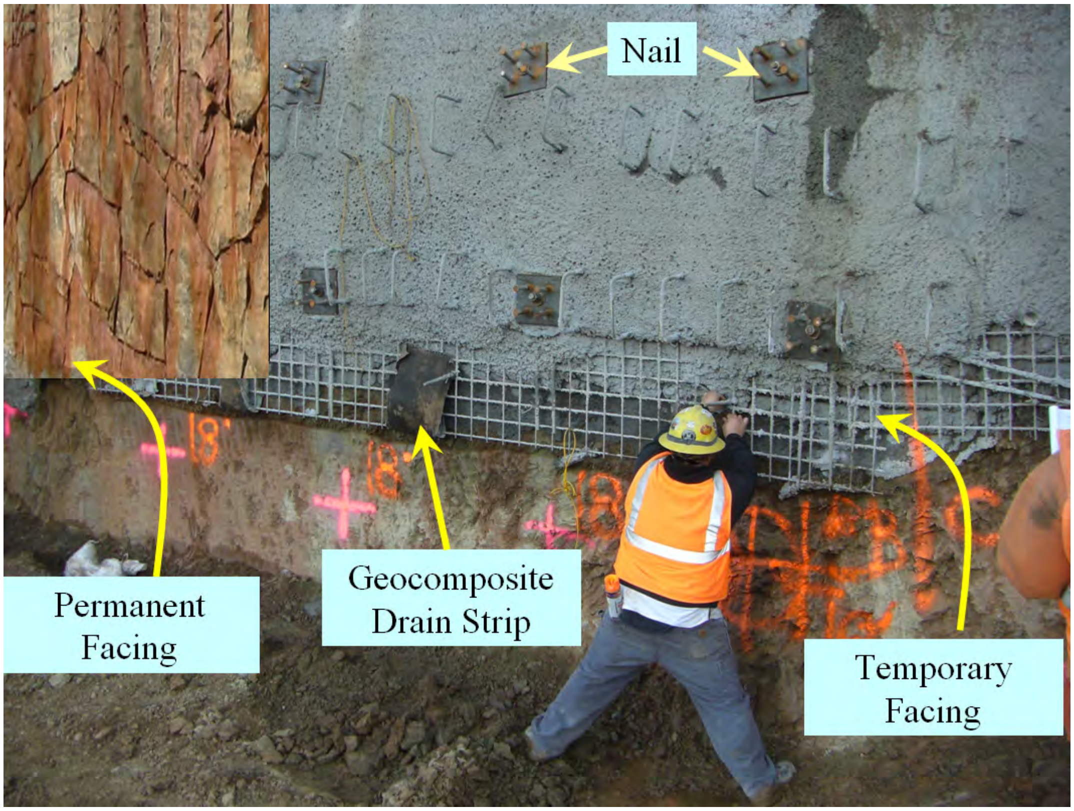


Figure 1-7: MSE Wall with Precast Concrete Face Panels



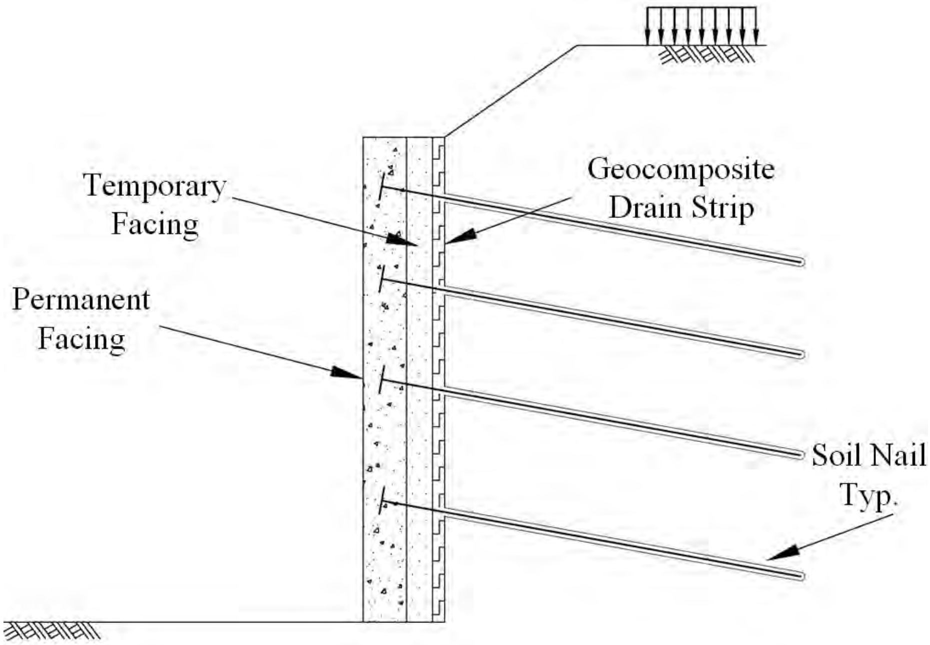


Figure 1-8: Soil Nail Wall

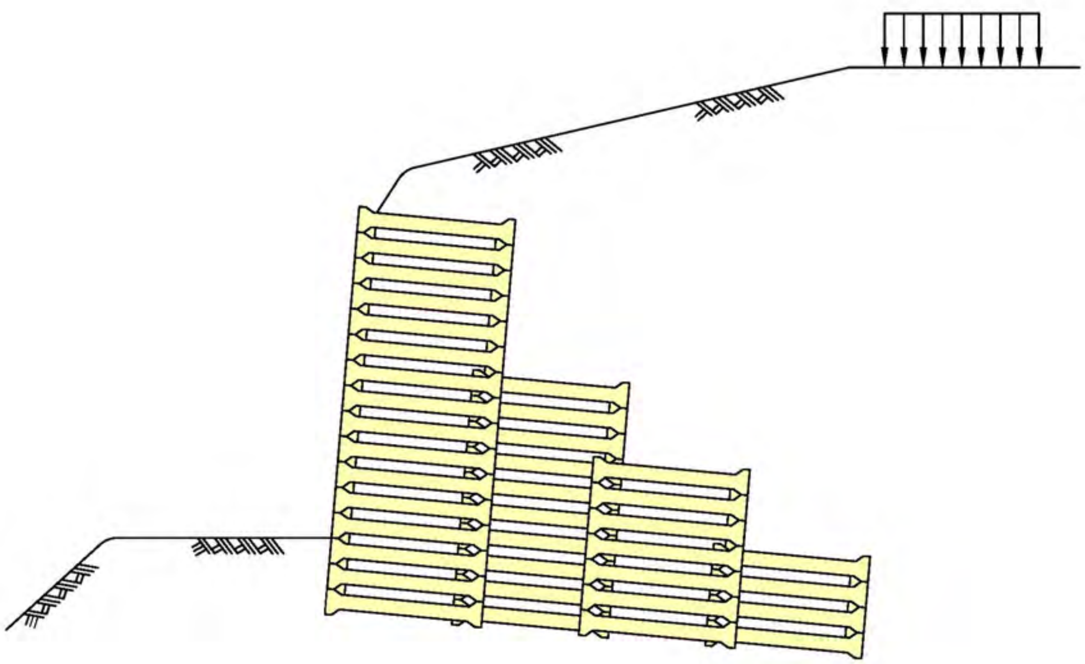


Figure 1-9: Precast Concrete Crib Walls

CHAPTER 2 **LATERAL EARTH PRESSURES AND THE LIMIT EQUILIBRIUM APPROACH**

A major issue in providing a safe earth retaining system design is determining the loading diagram, which shall be calculated through appropriate earth pressure theories. The magnitude of the earth pressure load depends on the following:

- Physical properties of the soil backfill
- Geometry of the backfill
- Nature of the soil-structure interface
- Surcharge loads
- Seepage force
- Seismic loads
- Location of the resultant load
- Possible modes of deformation and structural stiffness of the earth retaining system

Depending on the modes of deformation, the lateral earth pressure can be classified into three categories:

1. At-Rest Earth Pressure
2. Active Earth Pressure
3. Passive Earth Pressure

The at-rest earth pressure develops when the wall experiences no lateral movement to mobilize the shear strength of the backfill. Examples of such structures are integral bridge abutments and retaining structures, which are restrained at the top by roof framing systems and at the bottom by slab foundations. Earth retaining systems of this type must be designed to withstand the full hydrostatic earth pressure.

The active earth pressure develops when the earth retaining system is free to move away from the backfill as shown in Figure 2-1. Earth retaining systems, which are allowed to move away from the backfill must be designed for full active earth pressure.

The passive earth pressure develops when the earth retaining system moves toward the soil mass as shown in Figure 2-2 An example of such an earth retaining system is a seat-type bridge abutment backwall, which moves toward the backfill in the longitudinal direction of the bridge during a seismic event.

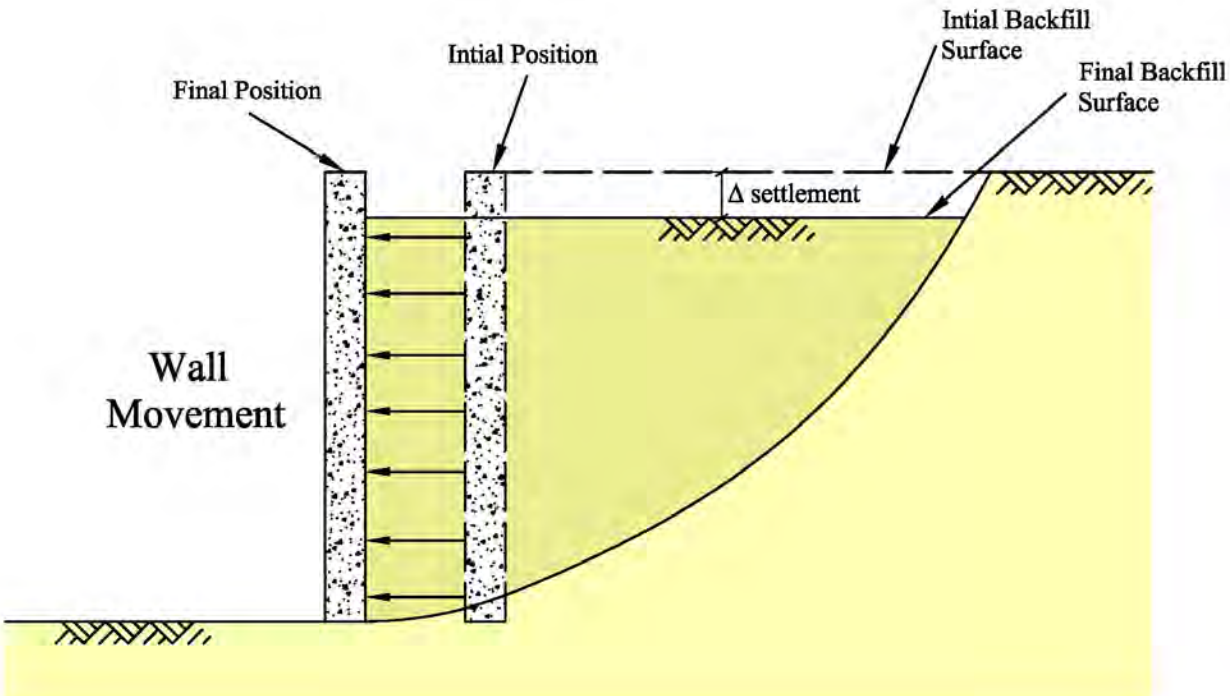


Figure 2-1: Active Earth Pressure

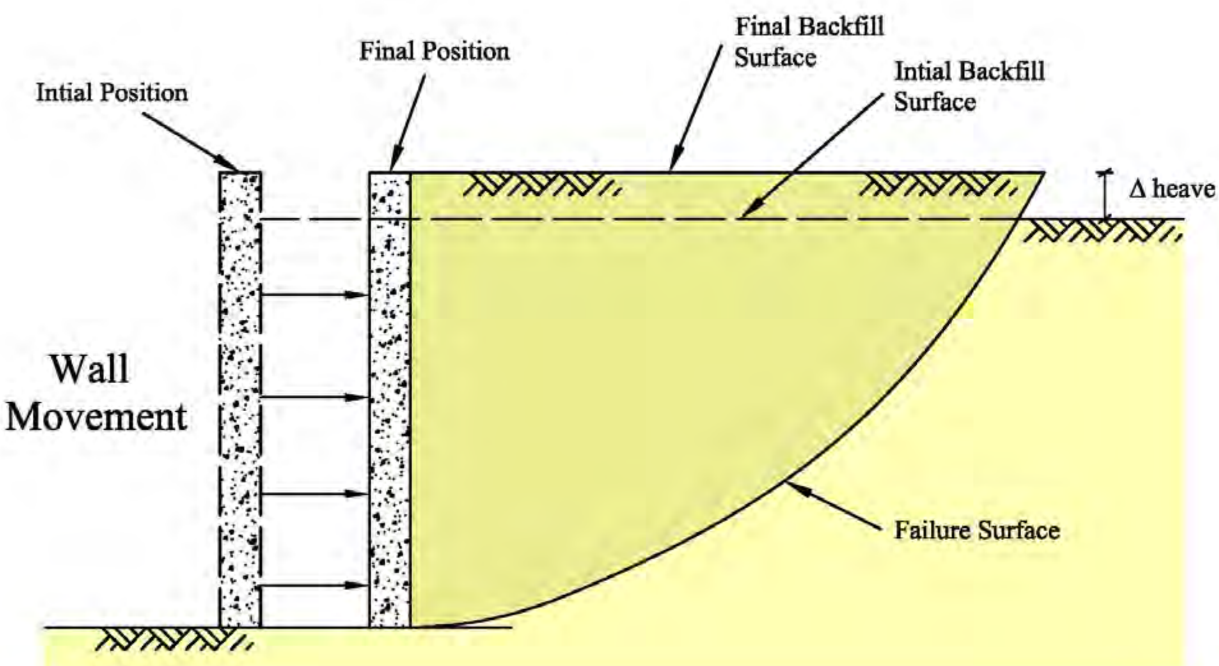
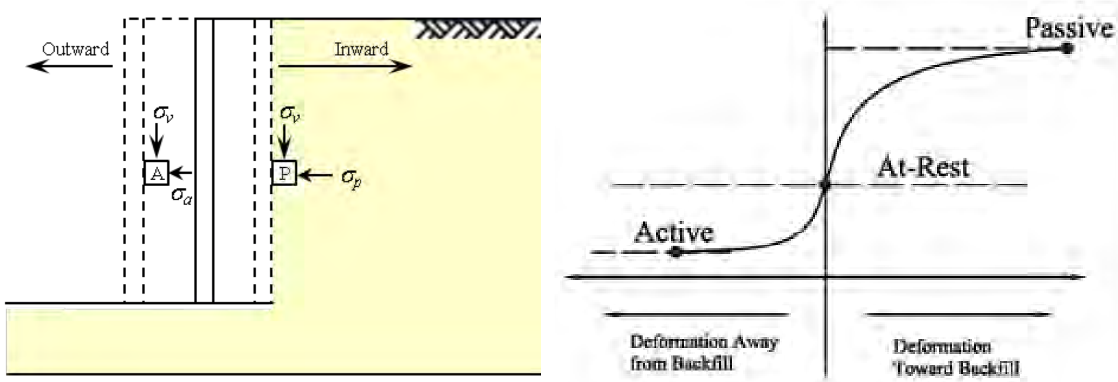


Figure 2-2: Passive Earth Pressure

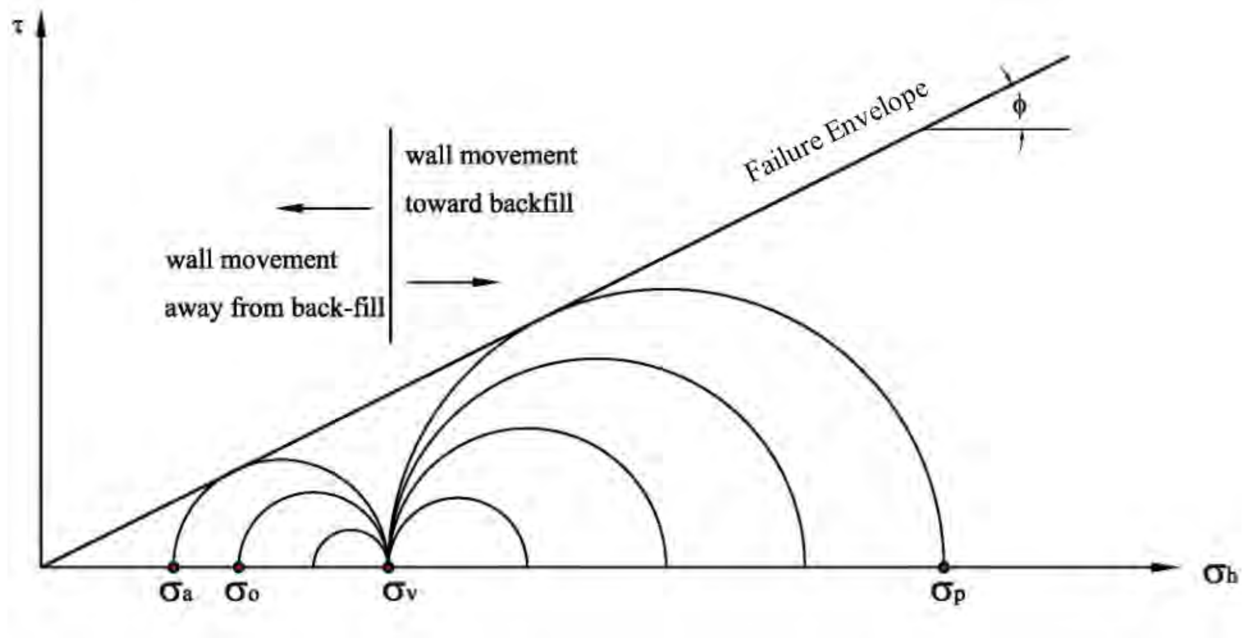
The variation of lateral stress between the active and passive earth pressure values can be brought on only by lateral movements within the soil mass (i.e., the backfill). Consider an element of the granular soil below the surface as shown in Figure 2-3. It is assumed that the shear stress within the backfill is governed by the Mohr-Coulomb shear strength criterion. It is also assumed that, at a particular point, such as point A, the vertical stress (σ_v) remains constant. If wall moves away from the backfill, the lateral stress (σ_h) gradually decreases, until the limiting value of active earth pressure (σ_a) is reached. If the wall moves toward the backfill, the lateral stress (σ_h) gradually increases, until the limiting value of passive earth pressure (σ_p) is reached. Various typical values of these mobilizing movements relative to the wall height are given in Table 2-1 (Clough, 1991).

Table 2-1: Normalized movements of active and passive pressures for various types of backfills

Type of Backfill	**Value of Δ/H**	
	Active	Passive
Dense Sand	0.001	0.01
Medium Dense Sand	0.002	0.02
Loose Sand	0.004	0.04
Compacted Silt	0.002	0.02
Compacted Lean Clay	0.01	0.05
Compacted Fat Clay	0.01	0.05
Note: Δ denotes the movement of the wall that is required to reach the minimum active or maximum passive pressure by tilting or by lateral translation. H is the wall height.		



(a) Wall movement



(b) Mohr circle representation of stress states at elements A and P

Figure 2-3: Mobilizing Earth Pressure Coefficient Relative to Wall Deformation

Selection of the methods for evaluating the static and seismic earth pressures is a crucial step in the design of earth retaining systems. The following section describes analytical procedures for computing static and dynamic lateral loads for various earth retaining systems. Depending on the backfill properties and wall geometry, the following methods may be used to compute active and passive earth pressures:

- Rankine theory
- Coulomb theory
- Log-Spiral-Rankine method
- Trial Wedge method

For the purpose of the initial discussion, it is assumed that the backfills are level, homogeneous, and isotropic; and the distribution of vertical stress (σ_v) with depth is hydrostatic, as shown in Figure 2-4. The horizontal stress (σ_h) is linearly proportional to depth, and is a multiple of vertical stress (σ_v), as shown in Eqn. (1).

$$\sigma_h = \sigma_v K = \gamma\, h\, K, \tag{1}$$

$$P = \frac{1}{2}\sigma_h\, h. \tag{2}$$

Depending on the wall movement, the coefficient K in Eq. (1) represents the active (K_a), the passive (K_p), or the at-rest (K_o) earth pressure coefficient.

The resultant lateral earth force (P), which is equal to the area of the load diagram, is assumed to act at the point located at $h/3$ above the base of the wall, where h is the height of the pressure surface measured from the surface of the ground to the base of the wall. This resultant force P is the force that causes bending, sliding and overturning in the wall.

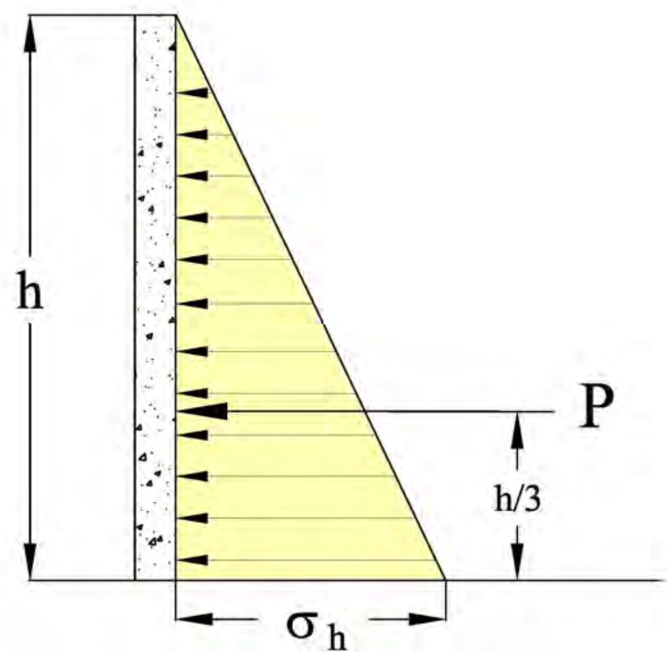


Figure 2-4: Lateral Earth Pressure Variation with Depth

2.1. AT-REST EARTH PRESSURE

For a zero shear strain condition, the horizontal and vertical stresses are related to each other by the Poisson's ratio (μ) as follows:

$$K_0 = \frac{\mu}{1-\mu}. \tag{3}$$

For normally consolidated soils and vertical walls, the coefficient of at-rest lateral earth pressure may be taken as,

$$K_0 = (1 - \sin\phi)(1 + \sin\beta) \tag{4}$$

For over-consolidated soils comprising a level backfill behind a vertical wall, the coefficient of at-rest lateral earth pressure may be assumed to vary as a function of the over-consolidation ratio (*OCR*) or stress history and may be taken as

$$K_o = (1 - \sin\phi)(OCR)^{\sin\phi} \tag{5}$$

Variables β and ϕ in Eqn. (4) are the slope angle of the ground surface behind the wall, and the internal friction angle of the soil, respectively.

2.2. ACTIVE AND PASSIVE EARTH PRESSURE THEORIES

Depending on the earth retaining system, the value of the active and/or passive pressure can be determined using the Rankine theory, the Coulomb theory, the Log-Spiral-Rankine method, or the Trial Wedge method.

The state of the active or passive earth pressure depends on the transformation (via expansion or compression) of the backfill from the elastic state to the state of plastic equilibrium. The concept of active and passive earth pressure theories can be explained using a continuous deadman near the ground surface for the stability of a sheet pile wall, as shown in Figure 2-5. As a result of wall deflection, Δ, the tierod is pulled until the active and passive wedges are formed behind, and in front of, the deadman. The elements *P*, in the front of the deadman, and the elements A, at behind the deadman, are acted on by two principal stresses—namely, a vertical stress (σ_v), and a horizontal stress (σ_h). In the active case, the horizontal stress (σ_a) is the minor principal stress and the vertical stress (σ_v) is the major principal stress. In the passive case, the horizontal stress (σ_p) is the major principal stress and the vertical stress (σ_v) is the minor principal stress. The resulting failure surfaces within the soil mass corresponding to active and passive earth pressures for a cohesionless soil are shown in Figure 2-5.

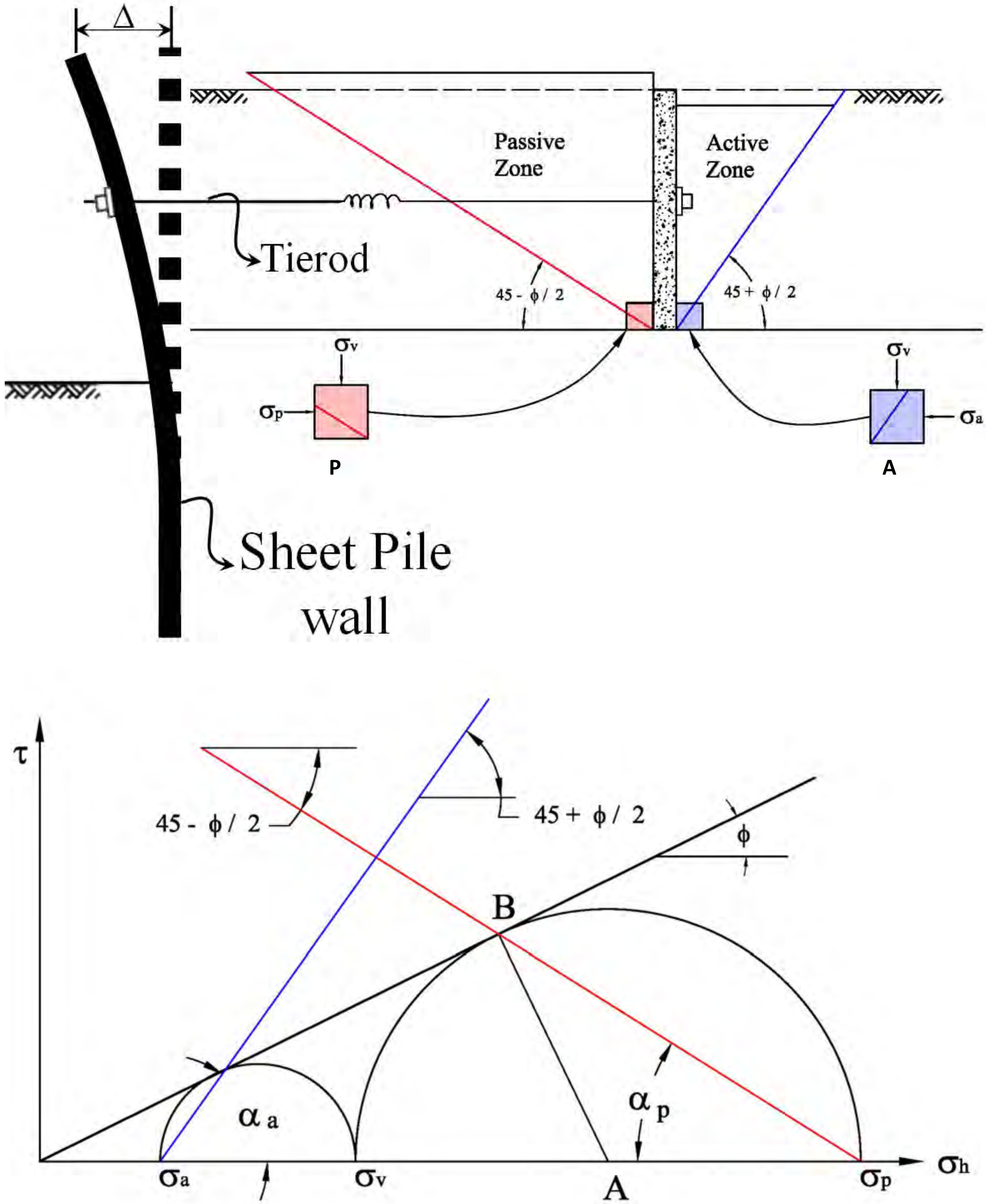


Figure 2-5: Mohr Circle Representation of the Stress State for a Cohesionless Backfill

For a cohesionless soil, Figure 2-5 can be used to derive the relationship for the active and passive earth pressures.

$$\sin\phi = \frac{AB}{OA} = \frac{\frac{\sigma_v - \sigma_a}{2}}{\frac{\sigma_v + \sigma_a}{2}} \quad (6)$$

where AB is the radius of the circle, and OA is the distance from center of circle to the origin. It follows that

$$\sin\phi = \frac{AB}{OA} = \frac{\sigma_v - \sigma_a}{\sigma_v + \sigma_a}. \quad (7)$$

$$\sigma_v \sin\phi + \sigma_a \sin\phi = \sigma_v - \sigma_a. \quad (8)$$

Collecting the terms yields

$$\sigma_a + \sigma_a \sin\phi = \sigma_v - \sigma_v \sin\phi, \quad (9)$$

$$\sigma_a(1+\sin\phi) = \sigma_v(1-\sin\phi), \quad (10)$$

$$\frac{\sigma_a}{\sigma_v} = \frac{(1-\sin\phi)}{(1+\sin\phi)}. \quad (11)$$

Using the trigonometric identities,

$$\frac{(1-\sin\phi)}{(1+\sin\phi)} = \tan^2\left(45^\circ - \frac{\phi}{2}\right) \quad (12)$$

$$\frac{(1+\sin\phi)}{(1-\sin\phi)} = \tan^2\left(45^\circ + \frac{\phi}{2}\right) \quad (13)$$

We have, for the active case,

$$K_a = \tan^2\left(45^\circ - \frac{\phi}{2}\right), \text{ where } K_a = \frac{\sigma_a}{\sigma_v} \quad (14)$$

and for the passive case,

$$K_p = \frac{\sigma_p}{\sigma_v} = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45^\circ + \frac{\phi}{2}\right). \quad (15)$$

For a cohesive soil, Figure 2-6 can be used to derive the relationships for the active and passive earth pressures.

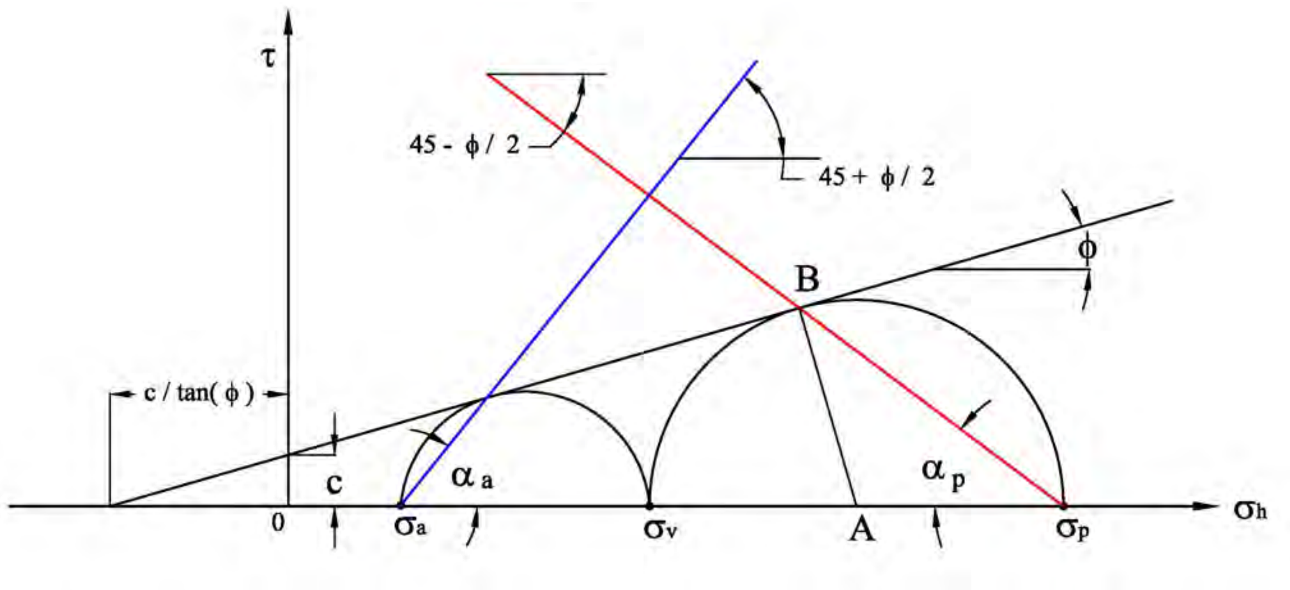


Figure 2-6: Mohr Circle Representation of State of Stress for Cohesive Backfill

For the active case,

$$\sin\phi = \frac{\frac{\sigma_v - \sigma_a}{2}}{\frac{\sigma_v + \sigma_a}{2} + \frac{c}{\tan\phi}}. \tag{16}$$

Then,

$$\sigma_v \sin\phi + \sigma_a \sin\phi + 2c\cos\phi = \sigma_v - \sigma_a \tag{17}$$

Collecting the terms yields

$$\sigma_v(1-\sin\phi) = \sigma_a(1+\sin\phi) + 2c\cos\phi. \tag{18}$$

This can be solved for σ_a to obtain,

$$\sigma_a = \frac{(1-\sin\phi)}{(1+\sin\phi)}\sigma_v - 2c\frac{\cos\phi}{(1+\sin\phi)}. \tag{19}$$

Using the trigonometric identities,

$$\frac{\cos\phi}{1+\sin\phi} = \tan\left(45^{\circ} - \frac{\phi}{2}\right) \tag{20}$$

$$\frac{\cos\phi}{1-\sin\phi} = \tan\left(45^{\circ} + \frac{\phi}{2}\right) \tag{21}$$

We have, for the active case,

$$\sigma_a = \sigma_v \tan^2\left(45^o - \frac{\phi}{2}\right) - 2c\tan\left(45^o - \frac{\phi}{2}\right), \tag{22}$$

$$\sigma_a = \sigma_v K_a - 2c\sqrt{K_a}, \text{ where } \sigma_v = \gamma\ z. \tag{23}$$

For the passive case, solving for σ_p we get

$$\sigma_p = \frac{(1+\sin\phi)}{(1-\sin\phi)}\sigma_v + 2c\frac{\cos\phi}{(1-\sin\phi)}, \tag{24}$$

$$\sigma_p = \sigma_v \tan^2\left(45^o + \frac{\phi}{2}\right) + 2c\tan\left(45^o + \frac{\phi}{2}\right), \tag{25}$$

$$\sigma_p = \sigma_v K_p + 2c\sqrt{K_p}, \text{ where } \sigma_v = \gamma\ z. \tag{26}$$

2.2.1 Rankine's Earth Pressure Theory

Rankine's theory is the simplest formulation proposed for earth pressure calculations. The geometry of the Rankine's wedge is shown in Figure 2-7 and Figure 2-8, and the assumptions implicit Rankine's earth pressure theory are as follows:

- The wall is smooth and vertical.
- There is no friction or adhesion between the wall and the soil.
- The failure wedge is a plane surface and is a function of soil's friction ϕ and the backfill slope β, as shown in Eqns. 29 and 32.
- Lateral earth pressure varies linearly with depth.
- The direction of the lateral earth pressure acts parallel to the slope of the backfill, as shown in Figure 2-7 and Figure 2-8.
- The resultant earth force acts at a distance equal to one-third of the wall height from the base.
- The backfill slope must be less than the backfill friction angle.

The values for the coefficient of active lateral earth pressure using the Rankine's theory are given by

$$K_a = \cos\beta\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \xrightarrow{\text{if } \beta=0} \frac{1-\sin\phi}{1+\sin\phi}. \tag{27}$$

The magnitude of active earth pressure can be determined using

$$P_a = \frac{1}{2}\gamma\ h^2\ K_a. \tag{28}$$

The failure plane angle α_a is given by

$$\alpha_a = \left(45^o + \frac{\phi}{2}\right) - \frac{1}{2}\left[\sin^{-1}\left(\frac{\sin\beta}{\sin\phi}\right) - \beta\right]. \tag{29}$$

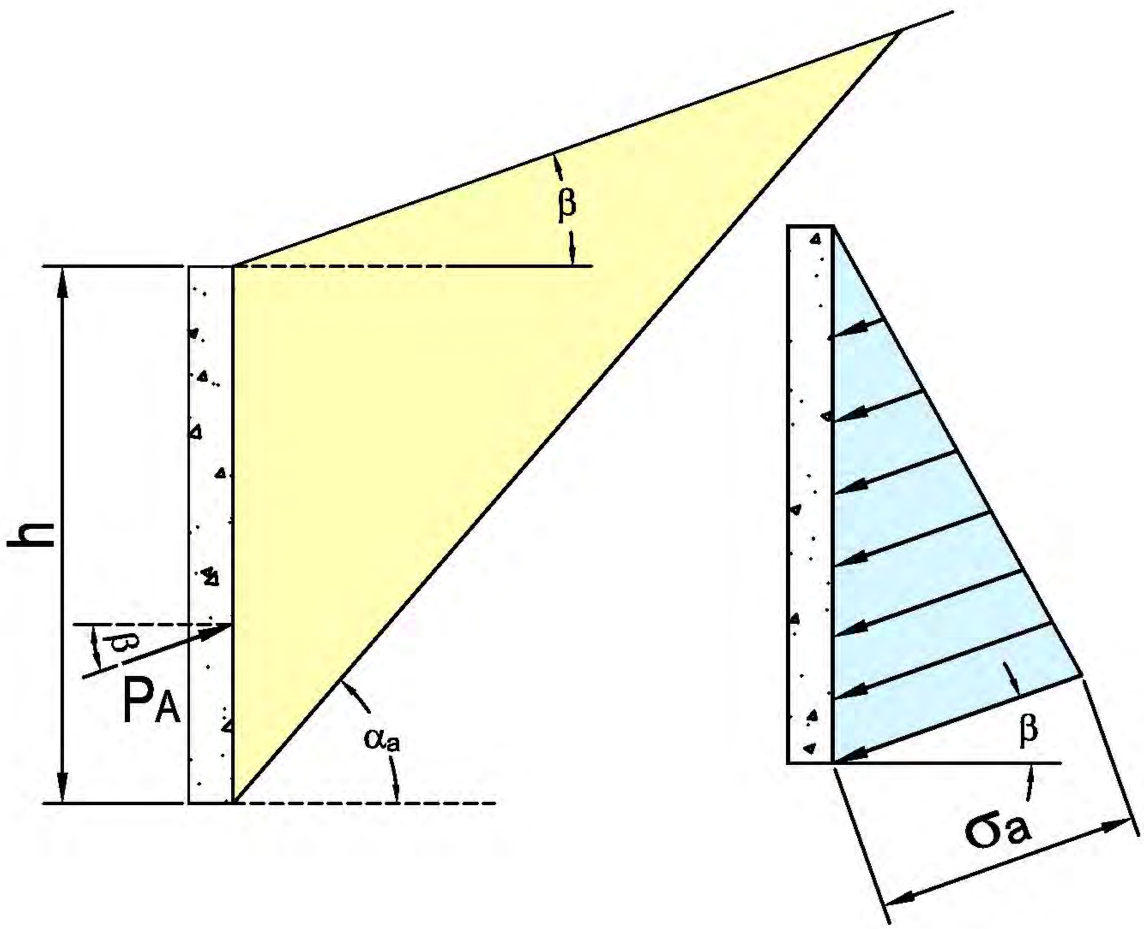


Figure 2-7: Rankine's Active Wedge

Rankine made similar assumptions to calculate the passive earth pressure. The values for the coefficient of active lateral earth pressure using the Rankine's theory are given by

$$K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \xrightarrow{\text{if } \beta=0} \frac{1+\sin\phi}{1-\sin\phi} \tag{30}$$

The magnitude of passive earth pressure can be determined using

$$P_p = \frac{1}{2}\gamma\, h^2\, K_p. \tag{31}$$

The passive failure plane angle α_p is given by

$$\alpha_p = \left(45^o - \frac{\phi}{2}\right) - \frac{1}{2}\left(\sin^{-1}\left(\frac{\sin\beta}{\sin\phi}\right) + \beta\right). \tag{32}$$

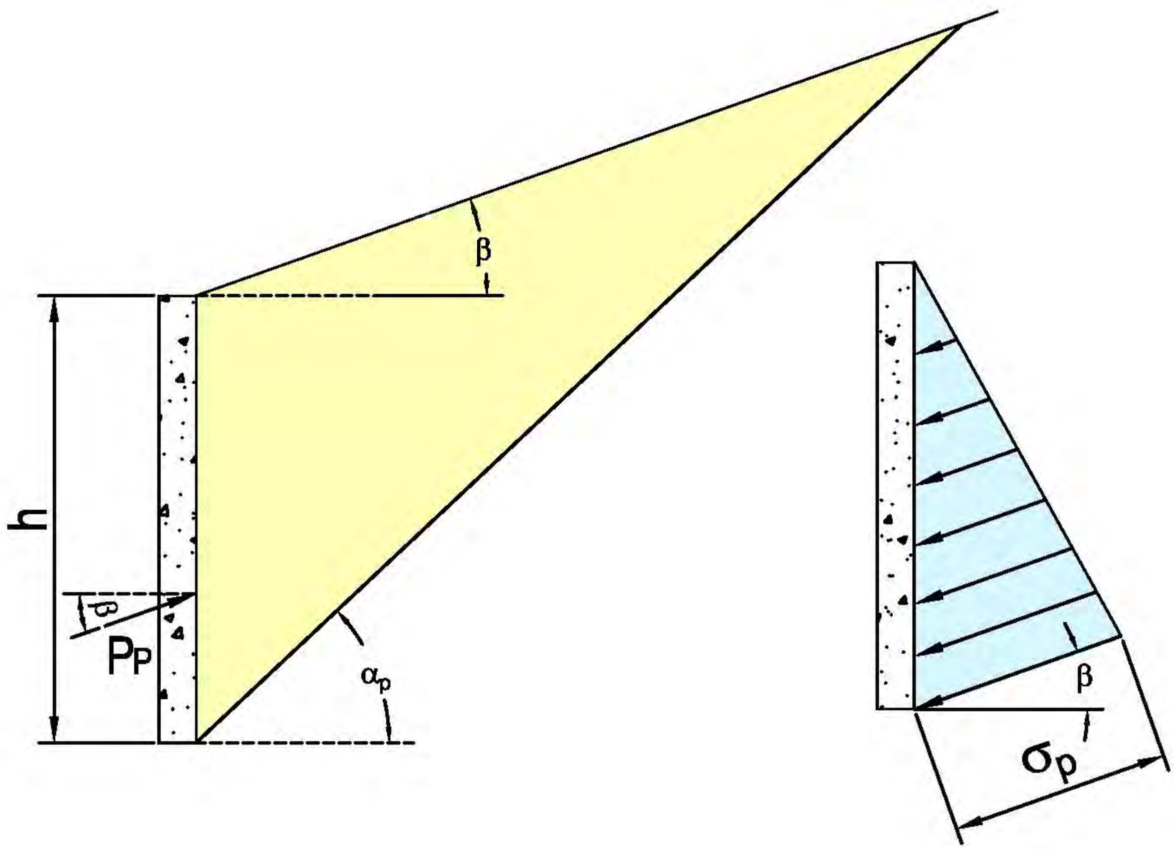


Figure 2-8: Rankine' s Passive Wedge

While Rankine's equation for the passive earth pressure is provided above, it should not be used to calculate the passive earth pressure if the backfill angle is greater than zero ($\beta > 0$). In fact, the K_p values for positive ($\beta > 0$) and negative ($\beta < 0$) backfill slopes are identical; and therefore, the Rankine equation to calculate the passive earth pressure coefficient for sloping ground should be avoided.

2.2.2 Earth Pressure For Cohesive Backfill

Neither Coulomb's nor Rankine's theories explicitly incorporate the effect of cohesion into the lateral earth pressure computations. Bell (1952) modified Rankine's solution to include this effect. Bell's derivation and equations for active and passive pressures follow the same steps that are provided in section §2.2. Caution is advised when evaluating soil stresses in cohesive soils. The evaluation of the stress induced by cohesive soils is highly uncertain due to their sensitivity to shrinkage-swell, wetnessdryness, and the degree of saturation. Tension cracks (gaps) can form, which may considerably affect the nature the assumptions employed for estimating the stresses. The development of tension cracks from the surface to depth h_{cr} is illustrated in Figure 2-9.

The active earth pressure (σ_a) normal to the back of the wall at depth h is equal to

$$\sigma_a = \gamma \, h \, K_a - 2C\sqrt{K_a}, \tag{33}$$

$$P_a = \frac{1}{2}\gamma \, h^2 \, K_a - 2C\sqrt{K_a}h. \tag{34}$$

According to Eqn. 33, the lateral stress (σ_a) at some point along the wall is equal to zero. Therefore,

$$\gamma \, h \, K_a - 2C\sqrt{K_a} = 0. \tag{35}$$

The depth of the tension cracks can then be obtained from Eqn. 35, as in

$$h = h_{cr} = \frac{2C\sqrt{K_a}}{\gamma \, \mathrm{K_a}}. \tag{36}$$

The passive earth pressure (σ_p) normal to the back of the wall at depth h is equal to

$$\sigma_p = \gamma \, \mathrm{h} \, \mathrm{K_p} + 2C\sqrt{K_p}. \tag{37}$$

Thus,

$$P_p = \frac{1}{2}\gamma \, h^2 \, K_p + 2C\sqrt{K_p}h. \tag{38}$$

The forces and stresses corresponding to these limiting states are shown in Figure 2-10. The effect of the surcharges and ground water are not included in this figure. In the presence of water, the hydrostatic pressure in the tension crack needs to be considered.

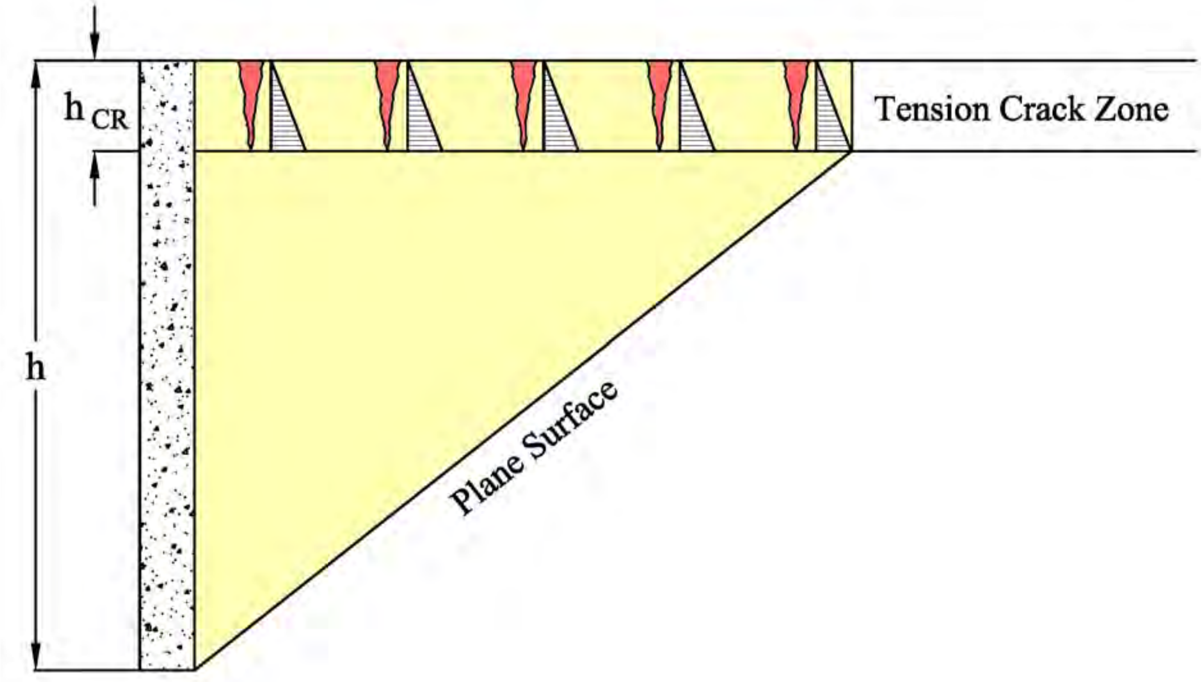


Figure 2-9: Tension Crack Zone

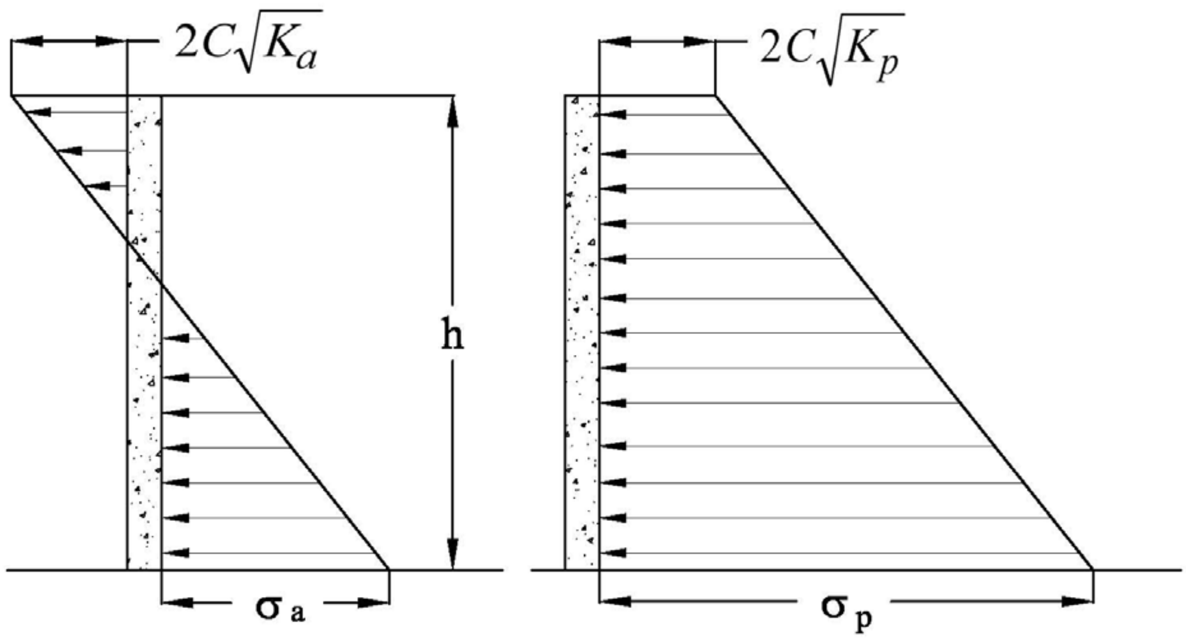
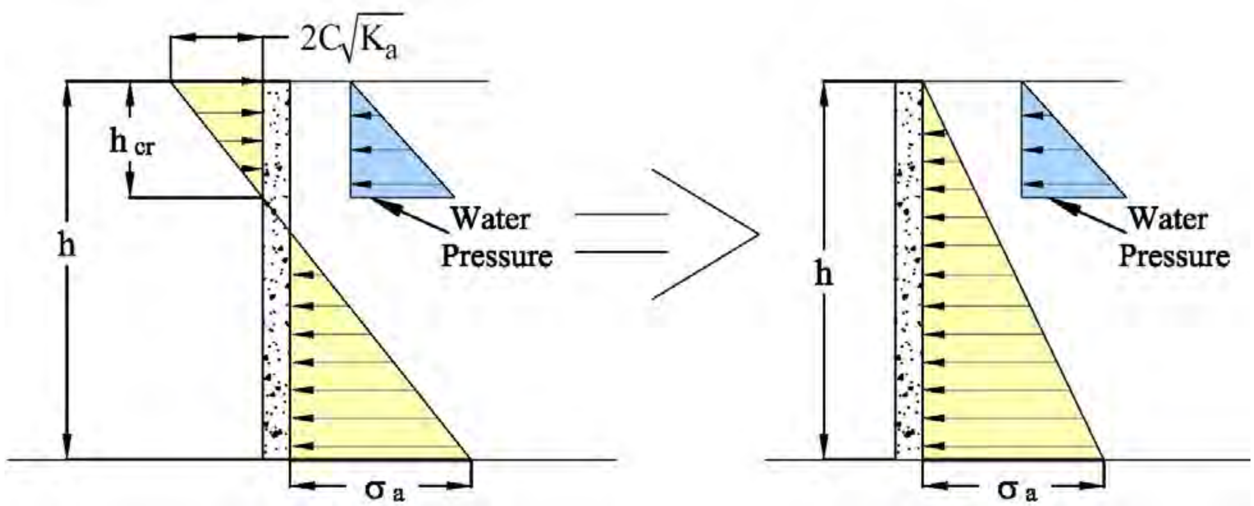


Figure 2-10: Tension Crack with Hydrostatic Water Pressure

When designing earth retaining systems that support cohesive backfills, the tensile stress distributed over the tension crack zone should be ignored, and the simplified lateral earth pressure distribution acting along the entire wall height h—including the pore water pressure—should be used, as shown in Figure 2-11.



(a) Tension Crack with Water (b) Recommended Pressure Diagram for Design

Figure 2-11: Stress Distribution for Cohesive Backfill Considered in Design

The apparent active earth pressure coefficient, K_{ap}, is defined as

$$K_{ap} = \frac{\sigma_a}{\gamma\ h} \geq 0.25. \tag{39}$$

Eqn. 39 indicates that the active lateral earth pressure (σ_a) acting over the wall height (h) in a cohesive soil should be taken no less than 0.25 times the effective overburden pressure at any depth. It is also required in design practice that in the case of lightweight backfill soils, the active earth pressure coefficient should not be taken less than 36 pcf divided by the specific weight of soil (i.e., $\gamma\ K_a \geqq$ 36 pcf).

2.2.3 Coulomb's Earth Pressure Theory

Unlike Rankine's earth pressure theory, Coulomb's (1776) earth pressure theory assumes that the wall is not frictionless. The geometries of Coulomb's wedges are shown in Figure 2-12 and Figure 2-13. The effect of the wall-backfill interface friction is essentially the introduction of shear stresses between the back of the wall and the backfill, which changes the direction of the principal planes. The following assumptions are implicit in Coulomb's theory:

- The wall is rough.
- There is friction or adhesion between the wall and the soil.
- The failure wedge is a plane surface and is a function of the soil friction (ϕ), wall friction (δ), the backfill slope (β), and the slope of the wall (ω).

- Lateral earth pressure varies linearly with depth.
- The direction of the lateral earth pressure is at an angle δ with the surface normal of the wall.
- The resultant earth force acts at a distance equal to one-third of the wall height from the base.
- The backfill slope must be less than the backfill friction angle.

The values for the coefficient of active lateral earth pressure may be are given by

$$K_a = \frac{\cos^2(\phi - \omega)}{\cos^2 \omega \cos(\omega + \delta)\left(1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\cos(\delta + \omega)\cos(\omega - \beta)}}\right)^2}, \tag{40}$$

and the magnitude of active earth force can be determined using

$$P_a = \frac{1}{2}\gamma\, h^2\, K_a. \tag{41}$$

The active failure angle can be calculated using

$$\alpha_A = \varphi + \tan^{-1}\left[\frac{-\tan(\varphi - \beta) + c_{1A}}{c_{2A}}\right], \tag{42}$$

where

$$c_{1A} = \sqrt{\tan(\phi - \beta)\left[\tan(\phi - \beta) + \cot(\phi - \omega)\right]\left[1 + \tan(\delta + \omega)\cot(\phi - \omega)\right]}, \tag{43}$$

$$c_{2A} = 1 + \tan(\delta + \omega)\left[\tan(\phi - \beta) + \cot(\phi - \omega)\right]. \tag{44}$$

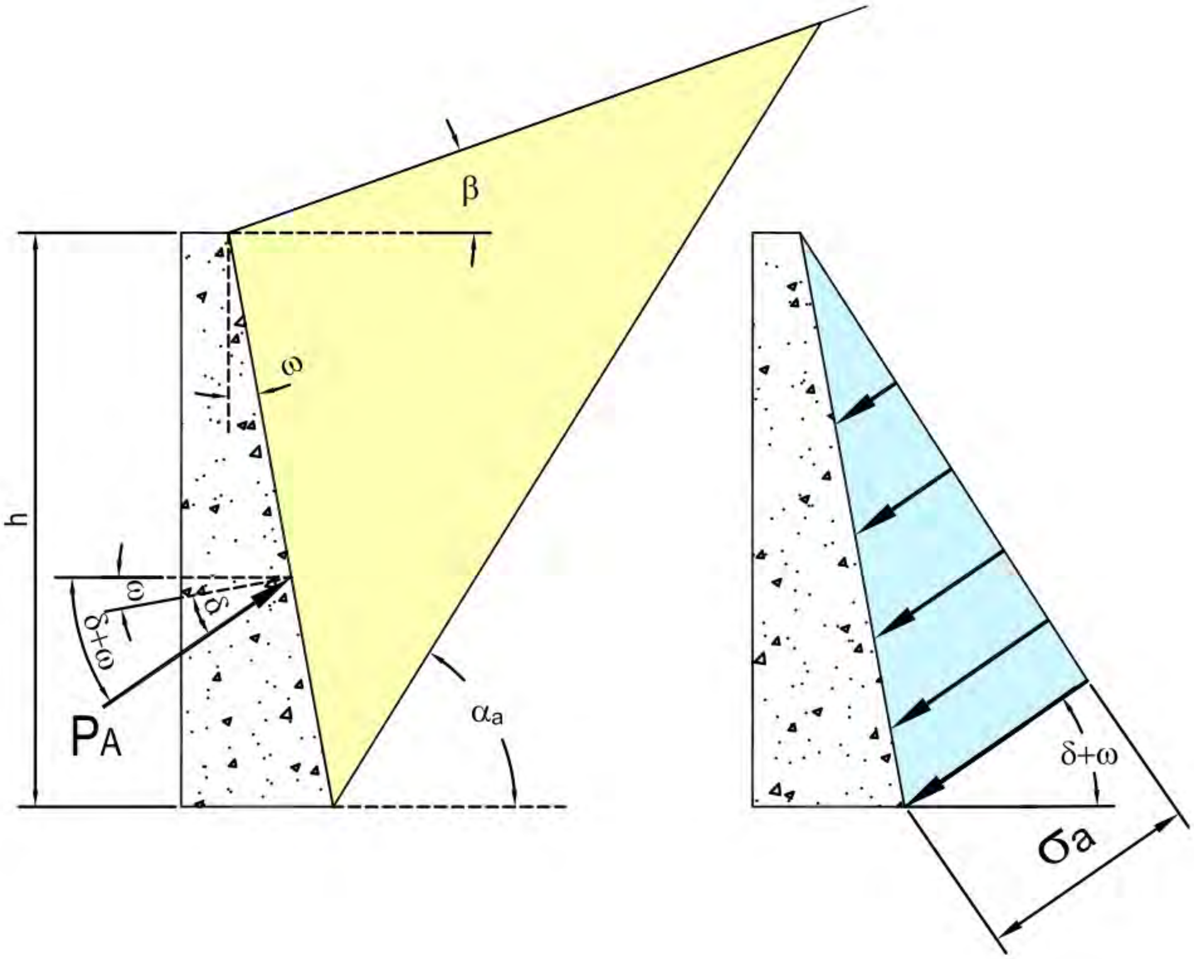


Figure 2-12: Coulomb's Active Wedge

Coulomb's passive earth pressure formulas are derived in similarly fashion to the active earth pressure formulas; however, the inclination of the force is different, as shown in Figure 2-13. The values for the coefficient of passive lateral earth pressure may be evaluated using

$$K_p = \frac{\cos^2(\phi+\omega)}{\cos^2\omega\cos(\delta-\omega)\left(1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi+\beta)}{\cos(\omega-\delta)\cos(\omega-\beta)}}\right)^2}. \tag{45}$$

The magnitude of the passive earth pressure can be determined using

$$P_p = \frac{1}{2}\gamma\, h^2\, K_p. \tag{46}$$

The passive failure angle can be calculated using

$$\alpha_P = -\varphi + \tan^{-1}\left[\frac{\tan(\varphi+\beta)+c_{3P}}{c_{4P}}\right], \tag{47}$$

where

$$c_{3P} = \left(\sqrt{\tan(\phi+\beta)\left[\tan(\phi+\beta)+\cot(\phi+\omega)\right]\left[1+\tan(\delta-\omega)\cot(\phi+\omega)\right]}\right), \tag{48}$$

$$c_{4P} = 1+\left[\tan(\delta-\omega)\right]\left[\tan(\phi+\beta)+\cot(\phi+\omega)\right]. \tag{49}$$

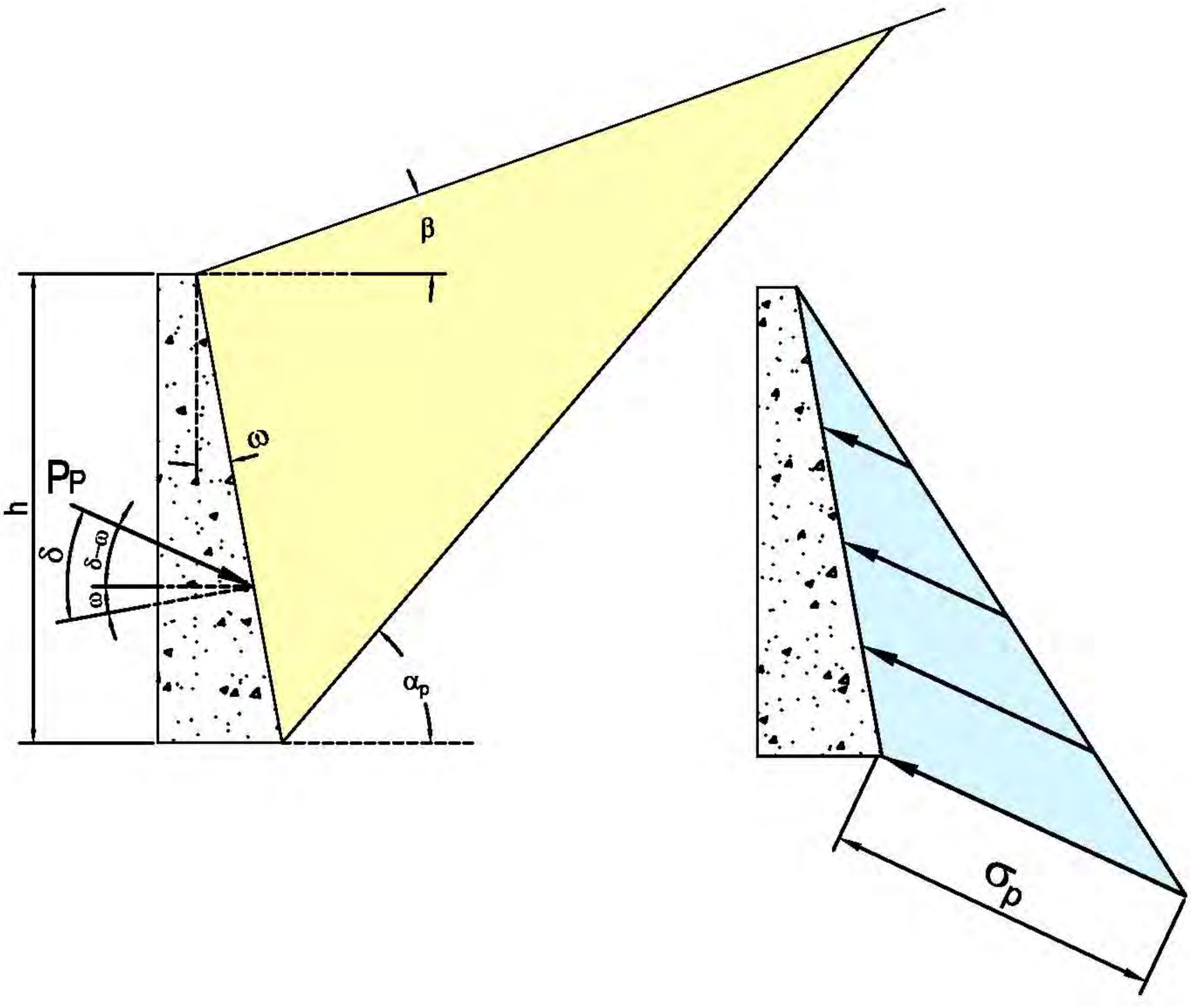


Figure 2-13: Coulomb's Passive Wedge

2.2.4 The Log-Spiral Method

In Rankine's and Coulomb's earth pressure theories, the failure surface is assumed to be planar. It has been long recognized that when there is a significant friction at the wall-soil interface, the assumption of a planar failure surface becomes unrealistic. Instead, a logarithmic failure surface develops, as illustrated in Figure 2-14

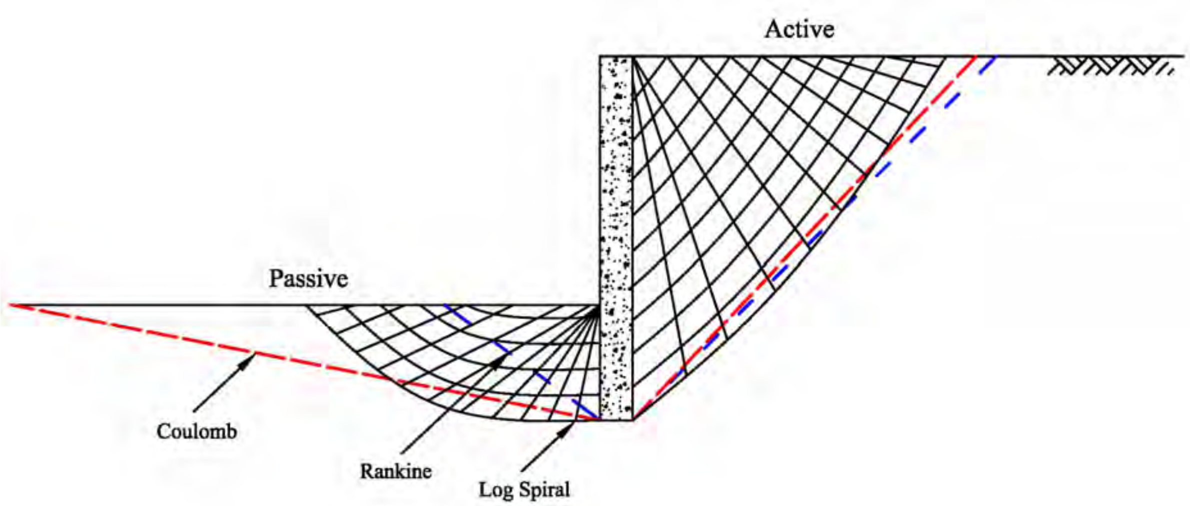
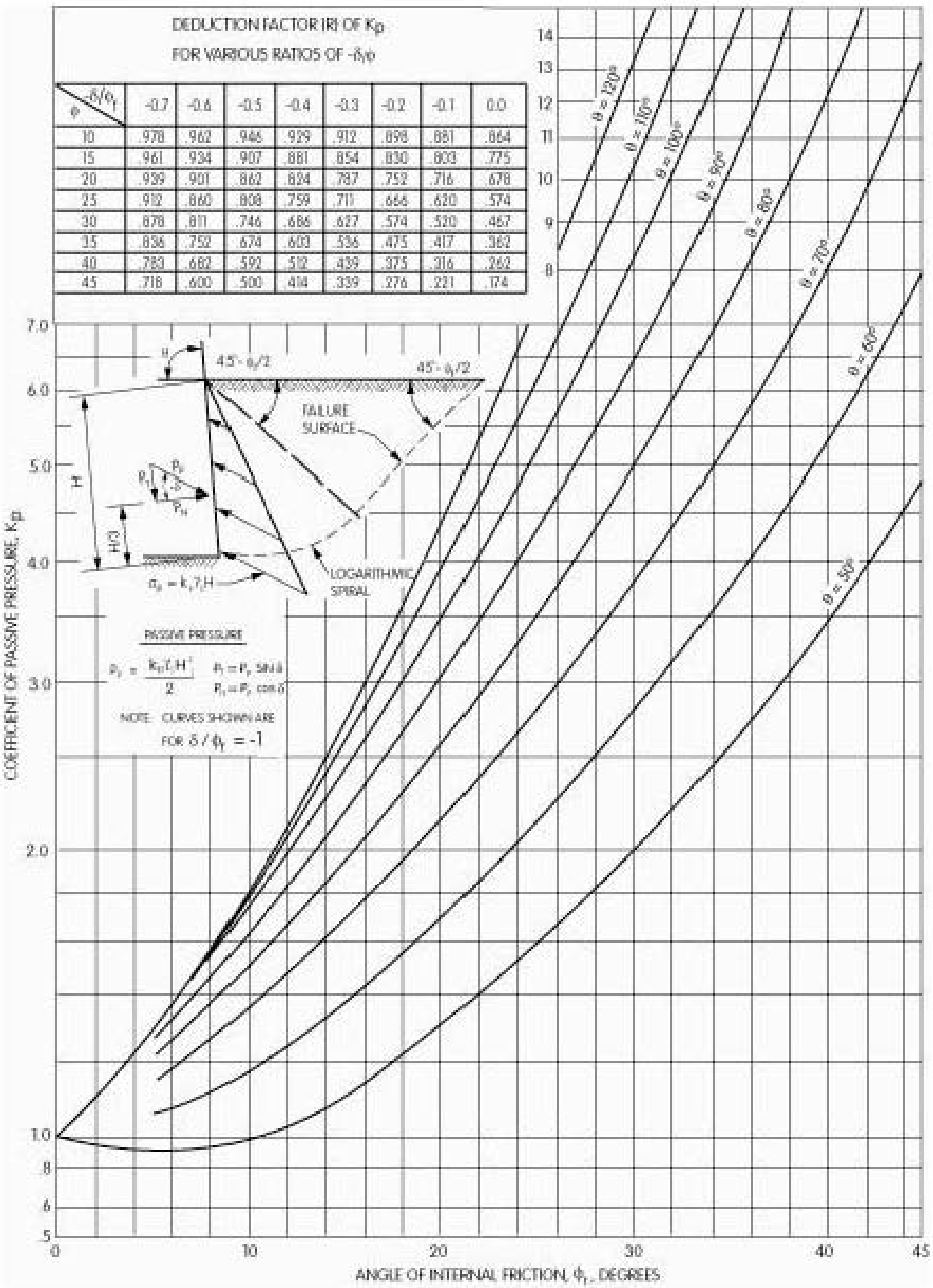


Figure 2-14: Illustration of the Logarithmic Spiral Failure Surface

Figure 2-14 provides a comparison between the potential failure surfaces using Rankine or Coulomb methods versus the log-spiral method for both the active and the passive conditions. For the active case, the failure surfaces determined via the Rankine and Coulomb methods appear to be reasonably close to the log-spiral failure surface. However, for the passive case, the planar failure surfaces determined using the Rankine and Coulomb methods are very different than that determined using the log-spiral method, if the wall-interface friction angle δ is larger than $^1/_3$ of the backfill friction angle, $\boldsymbol{\phi}$. The active and passive earth pressures are functions of the soil mass within the failure surface. The mobilized soil mass within the Coulomb and Rankine active zone is about the same as that of a log-spiral active zone. In contrast, the mobilized soil mass within the Coulomb passive zone is much higher than the log-spiral passive zone, and the mobilized soil mass within Rankine passive zone is much lower than log-spiral passive zone. Thus, it is reasonable state that the Coulomb theory overestimates the magnitude of the passive earth pressure, and the Rankine theory underestimates the magnitude of the passive earth pressure. Therefore, Rankine's earth pressure theory is conservative, Coulomb's theory is non-conservative, and the log-spiral result is the most realistic estimate of the passive earth pressure.

For non-cohesive soils, values of the static passive lateral earth pressure coefficient may be obtained from Figure 2-15 and Figure 2-16. For the seismic case—and for conditions that deviate from those described in Figure 2-15 and Figure 2-16—the active pressure may be calculated by using the Trial Wedge method, and the passive earth pressure may be calculated using the Log-Spiral-Rankine model (Shamsabadi, 2012). Details of the Trial Wedge method and the Log-Spiral-Rankine model are presented in §2.3.2 and §2.3.3, respectively.



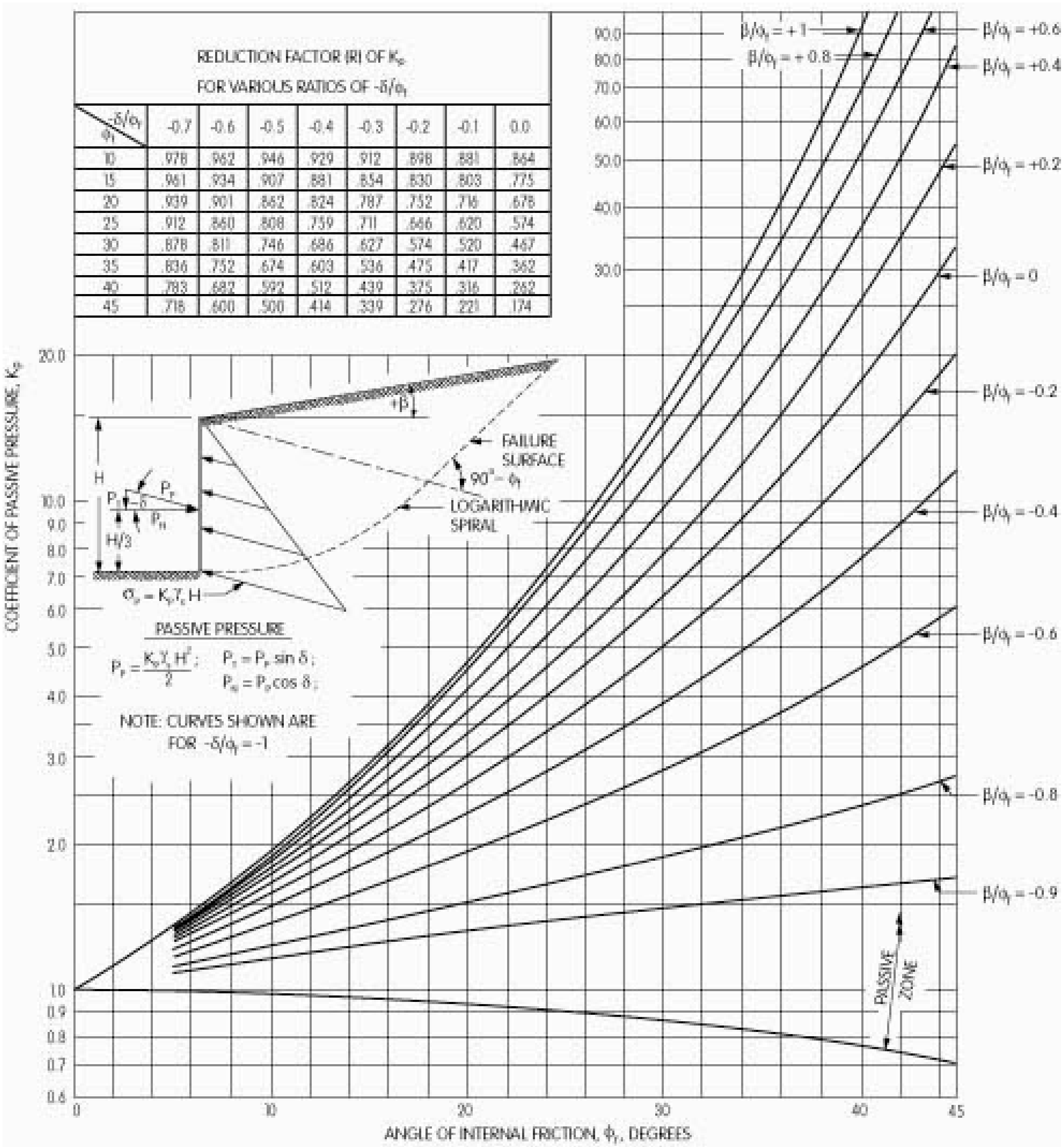
DEDUCTION FACTOR (R) OF K_p FOR VARIOUS RATIOS OF $-\delta/\phi$

ϕ \ $-\delta/\phi_f$	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
15	.961	.934	.907	.881	.854	.830	.803	.775
20	.939	.901	.862	.824	.787	.752	.716	.678
25	.912	.860	.808	.759	.711	.666	.620	.574
30	.878	.811	.746	.686	.627	.574	.520	.467
35	.836	.752	.674	.603	.536	.475	.417	.362
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

$$P_p = \frac{K_p \gamma_t H^2}{2} \qquad P_v = P_p \sin\delta \qquad P_H = P_p \cos\delta$$

$\sigma_p = K_p \gamma_t H$

Figure 2-15: Coefficient of Passive Earth Pressure for Sloping Wall and Horizontal Backfill


REDUCTION FACTOR (R) OF K_p FOR VARIOUS RATIOS OF $-\delta/\phi_f$

ϕ_f \ $-\delta/\phi_f$	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
15	.961	.934	.907	.881	.854	.830	.803	.775
20	.939	.901	.862	.824	.787	.752	.716	.678
25	.912	.860	.808	.759	.711	.666	.620	.574
30	.878	.811	.746	.686	.627	.574	.520	.467
35	.836	.752	.674	.603	.536	.475	.417	.362
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

Figure 2-16: Coefficient of Passive Earth Pressure for Vertical Wall and Sloping Backfill

2.2.5 Trial Wedge Method

The Trial Wedge method of analysis uses the general limit equilibrium approach to calculate forces acting on the earth retaining systems. Trial Wedge method solutions can be used for any wall adhesion and interface friction angle regardless of irregularity of the backfill and surcharges. The sliding wedge is bounded by the ground surface on the top, the rupture surface on one side and the back of the wall on the other side, as shown in Figure 2-17. For a derivation and more detail on the Trial Wedge Method, refer to §2.3.2.

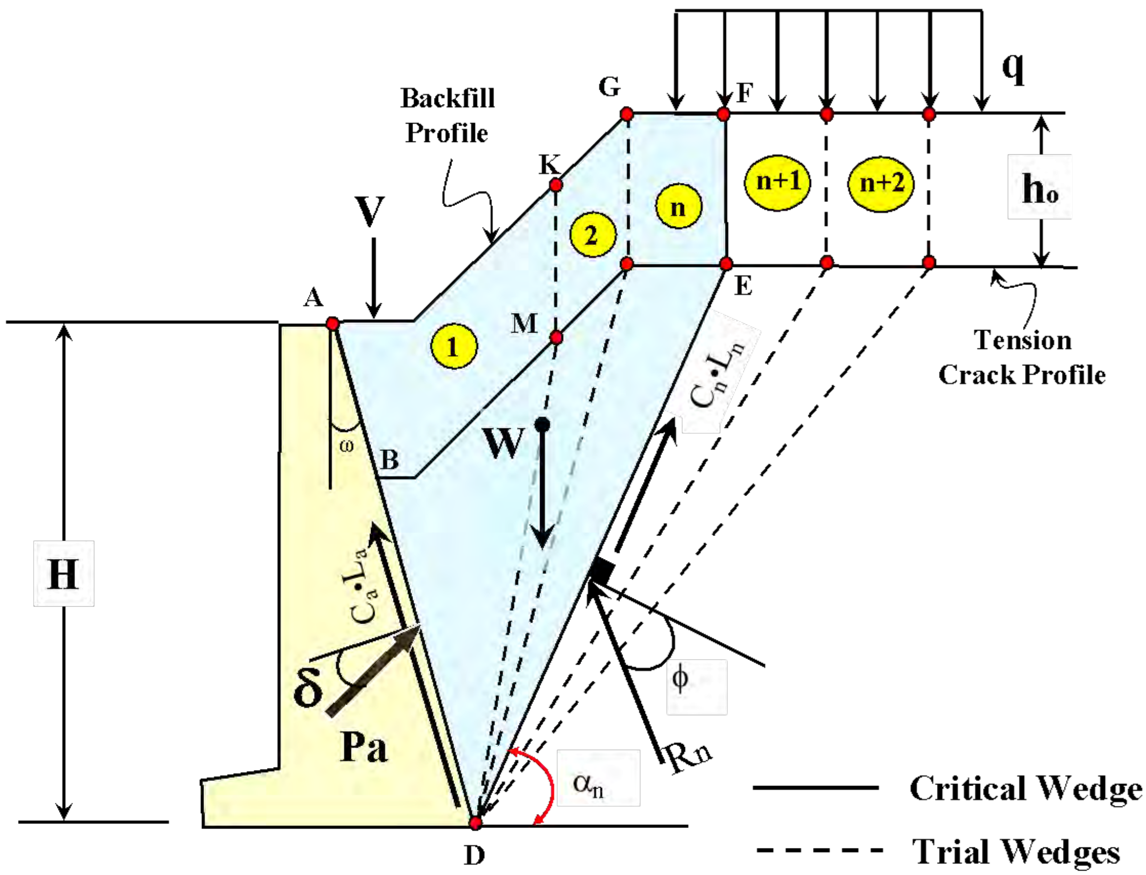


Figure 2-17: Active Trial Wedge

2.3. SEISMIC EARTH PRESSURE THEORY

During a seismic event, energy is released in the form of seismic waves through the soil supporting the earth retaining system foundation. This instantaneously increases the shear stresses and decreases the volume of voids within the backfill. Okabe (1926) and Mononobe and Matsuo (1929) extended Coulomb's (1776) earth pressure theory to include the effects of dynamic earth pressures through the use of a constant horizontal (k_h) and vertical (k_v) earthquake acceleration coefficient.

2.3.1 Mononobe-Okabe Earth Pressure Theories

2.3.1.1 *Seismic Active Earth Pressure*

Okabe (1926) and Mononobe and Matsuo (1929) extended Coulomb's (1776) earth pressure theory, by representing the dynamic inertial forces as pseudo-static forces acting on the Coulomb's wedge. The forces acting on the wedge due to horizontal and vertical ground acceleration are shown in Figure 2-18.

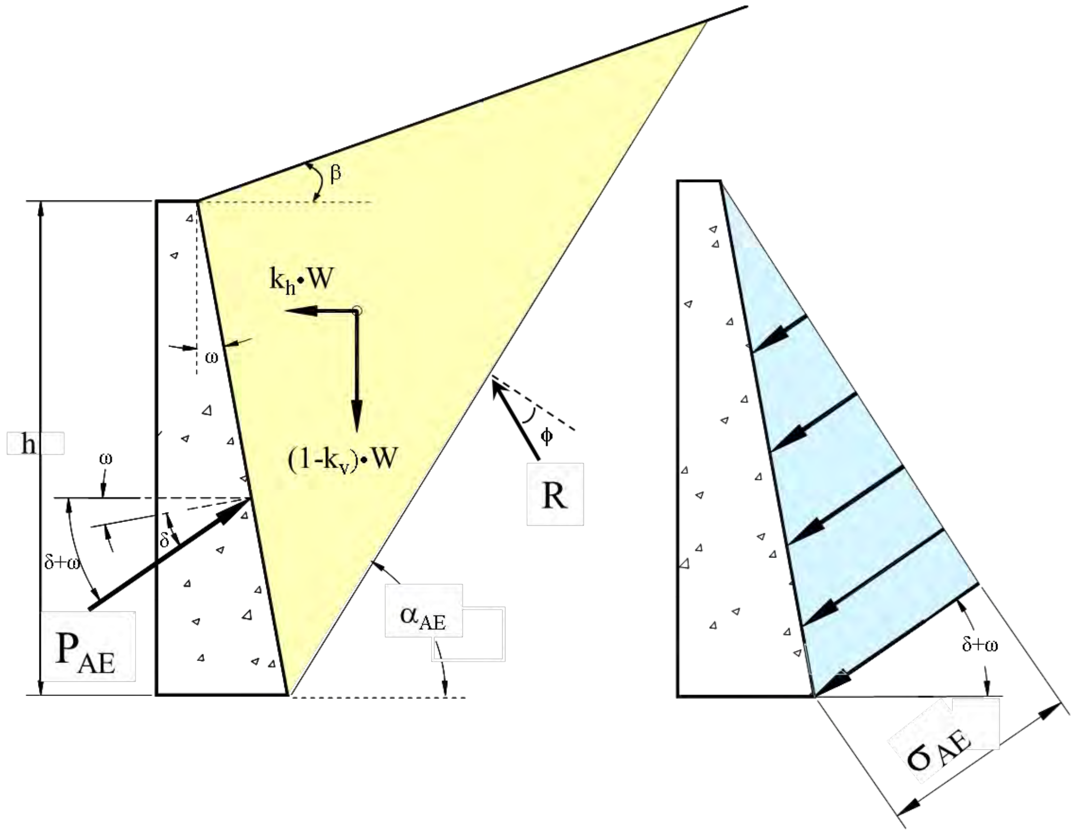


Figure 2-18: Mononobe-Okabe Active Wedge

The active earth force acting on the wall is

$$P_{AE} = \frac{1}{2}\gamma h^2 \left(1-k_v\right) K_{AE}. \tag{50}$$

where K_{AE} is the seismic active earth pressure coefficient expressed as

$$K_{AE} = \frac{\cos^2\left(\phi-\theta-\omega\right)}{\cos\theta\cos^2\omega\cos\left(\theta+\omega+\delta\right)\left(1+\sqrt{\dfrac{\sin\left(\phi+\delta\right)\sin\left(\phi-\theta-\beta\right)}{\cos\left(\delta+\theta+\omega\right)\cos\left(\beta-\omega\right)}}\right)^2}. \tag{51}$$

The seismic inertial angle θ is:

$$\theta = \tan^{-1}\left[\frac{k_h}{1-k_v}\right]. \tag{52}$$

The failure plane angle (α_{AE}) with respect to horizontal is given by (Zarrabi, 1979)

$$\alpha_{AE} = \phi - \theta + \tan^{-1}\left[\frac{-\tan(\phi-\theta-\beta)+c_{1AE}}{c_{2AE}}\right] \quad (53)$$

where

$$c_{1AE} = \sqrt{\tan(\phi-\theta-\beta)\left[\tan(\phi-\theta-\beta)+\cot(\phi-\theta-\omega)\right]\left[1+\tan(\delta+\theta+\omega)\cot(\phi-\theta-\omega)\right]} \quad (54)$$

$$c_{2AE} = 1+\tan(\delta+\theta+\omega)\left[\tan(\phi-\theta-\beta)+\cot(\phi-\theta-\omega)\right] \quad (55)$$

The orientation of the failure surface associated with Eqn. 53 becomes flatter as the level of acceleration increases, and when $\theta + \beta = \phi$, the predicted failure surface is horizontal. In practice cohesionless soil is unlikely to be present for a great distance behind a retaining wall and encompass the entire failure wedge under seismic conditions. In some cases, free draining cohesionless soil may only be placed in the static active wedge with the remainder of the soil being cohesive embankment fill (c-ϕ soil) or even rock. In these instances, earthquake-induced active pressure should be determined using Trial Wedges method described in §2.3.2.1

2.3.1.2 *Seismic Passive Earth Pressure*

The forces acting on the passive wedge due to horizontal and vertical ground acceleration are shown in Figure 2-19. The M-O relationship for the seismic passive earth force, P_{PE}, can be expressed as:

$$P_{PE} = \frac{1}{2}\gamma h^2(1-k_v)K_{PE} \quad (56)$$

where K_{PE} is the seismic passive earth pressure coefficient expressed as

$$K_{PE} = \frac{\cos^2(\phi-\theta+\omega)}{\cos\theta\cos^2\omega\cos(\theta-\omega+\delta)\left[1-\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\theta+\beta)}{\cos(\delta+\theta-\omega)\cos(\beta-\omega)}}\right]^2} \quad (57)$$

The failure plane angle (α_{PE}) with respect to horizontal is given by (Zarrabi 1979)

$$\alpha_{PE} = \theta - \phi + \tan^{-1}\left[\frac{\tan(\phi+\beta-\theta)+c_{3PE}}{c_{4PE}}\right] \quad (58)$$

where

$$c_{3PE} = \left(\sqrt{\tan(\phi+\beta-\theta)\left[\tan(\phi+\beta-\theta)+\cot(\phi+\omega-\theta)\right]\left[1+\tan(\delta-\omega+\theta)\cot(\phi+\omega-\theta)\right]}\right) \quad (59)$$

$$c_{4PE} = 1+\left[\tan(\delta-\omega+\theta)\right]\left[\tan(\phi+\beta-\theta)+\cot(\phi+\omega-\theta)\right] \quad (60)$$

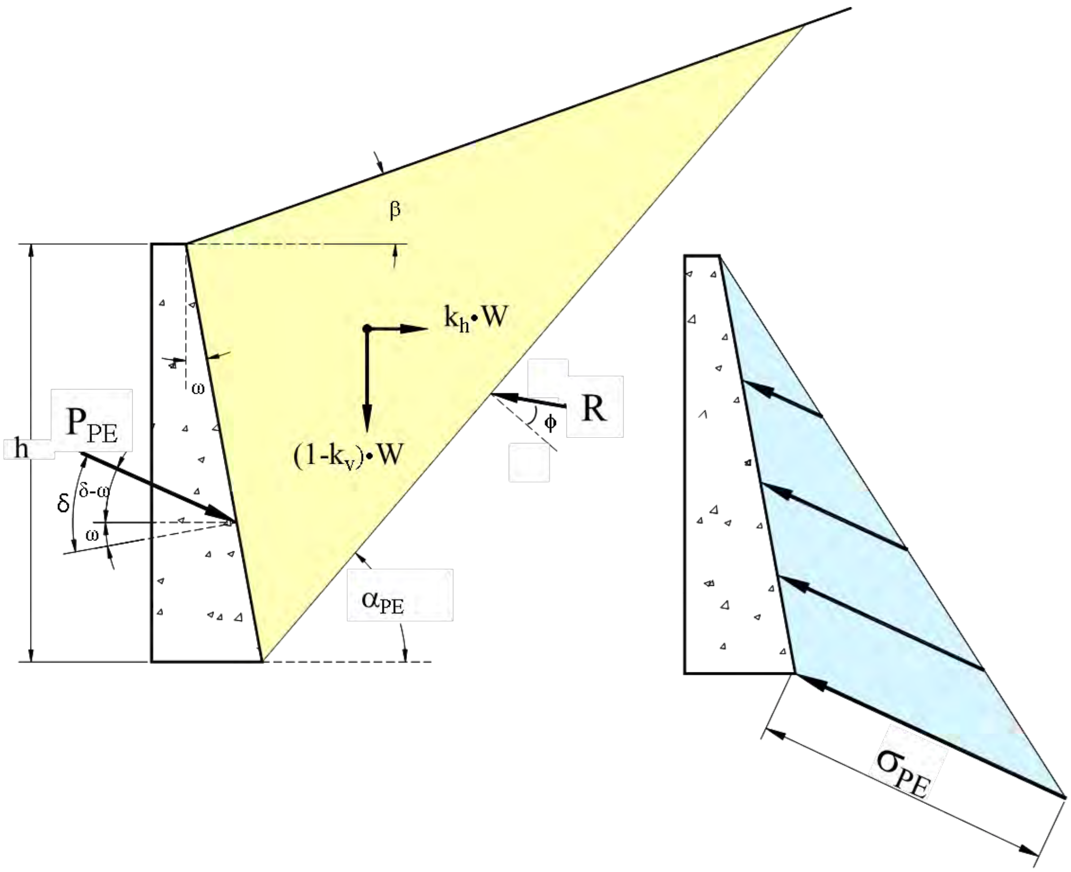


Figure 2-19: Mononobe-Okabe Passive Wedge

2.3.2 Seismic Trial Wedge Method

2.3.2.1 *Seismic Active Earth Pressure*

Figure 2-20 shows the assumptions used to determine the resultant active force for sloping ground with an irregular backfill condition applying the wedge theory. This is an iterative process. The failure plane angle (α_n) for the wedge varies until the maximum value of the horizontal active earth pressure is computed using Eqn. 61. The development of Eqn. 61 is based on the limit equilibrium for a general soil wedge. It is assumed that the soil wedge moves downward along the failure surface and along the wall surface to mobilize the active wedge. This wedge is held in equilibrium by the resultant force equal to the resultant active force (P_{AE}) acting on the face of the wall. Since the wedge moves downward along the face of the wall, this force acts with an assumed wall friction angle (δ) below the normal to the wall in order to oppose this movement.

$$P_{AE} = \frac{WT - COH - ADH - W_a}{\left[1 + \tan(\delta + \omega)\tan(\alpha - \phi)\right]\cos(\delta + \omega)} \quad (61)$$

Seismic earth pressure due to the weight of the wedge is

$$WT = W\left[\left(1-k_v\right)\tan\left(\alpha-\phi\right)+k_h\right]. \quad (62)$$

Seismic earth pressure due to soil cohesion is

$$COH = C_n L_n\left[\sin\alpha\tan\left(\alpha-\phi\right)+\cos\alpha\right]. \quad (63)$$

Seismic earth pressure due to soil-wall adhesion is

$$ADH = C_a L_a\left[\tan\left(\alpha-\phi\right)\cos\left(\omega\right)-\sin\omega\right]. \quad (64)$$

Seismic earth pressure due to water is

$$W_a = \left(U_{st}+U_{sh}\right)\left[\tan\left(\alpha-\phi\right)\cos\alpha-\sin\alpha\right]. \quad (65)$$

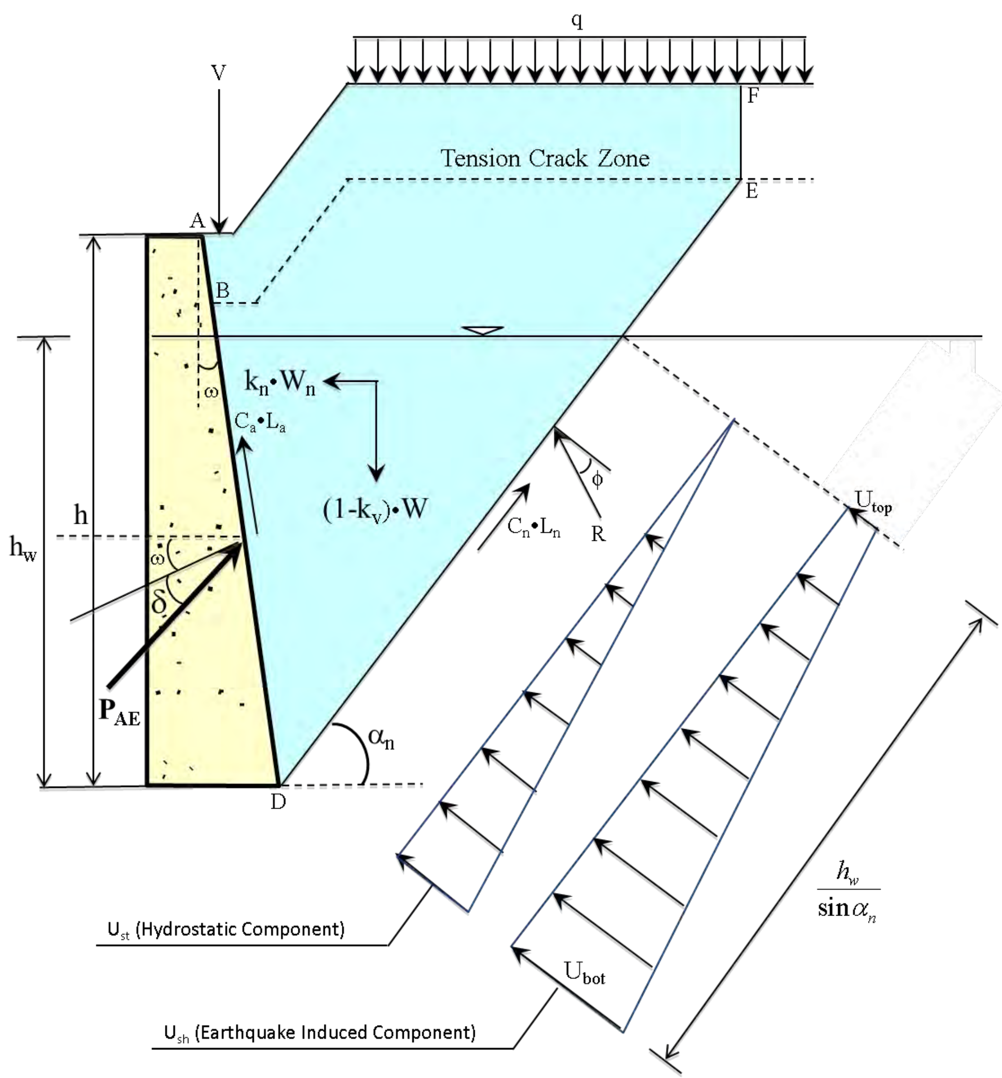


Figure 2-20: Seismic Active Trial Wedge

2.3.2.2 *Seismic Passive Earth Pressure*

Figure 2-21 shows the assumptions used to determine the resultant seismic passive pressure for a broken back slope condition applying the Trial Wedge theory. Using the limit equilibrium for a given wedge, Eqn. 66 calculates the horizontal seismic passive earth pressure on a wall. The iterative procedure that was used for the active case is used here as well. However, the failure surface angle (α_n) is varied until the minimum value of seismic passive force P_{PE} is attained. As mentioned previously, a constant horizontal and vertical acceleration have been added to the equation to take into account the seismic event. The equation can be divided into four components: soil weight, cohesion, adhesion, and water.

$$P_{PEH} = \frac{WT + COH + ADH - W_a}{\left[1 - \tan(\delta - \omega)\tan(\alpha + \phi)\right]\cos(\delta - \omega)} \tag{66}$$

Seismic earth pressure due to the weight of the wedge:

$$WT = W\left[(1 - k_v)\tan(\alpha + \phi) - k_h\right] \tag{67}$$

Seismic earth pressure due to cohesion:

$$COH = C_n L_n\left[\sin\alpha\tan(\alpha + \phi) + \cos\alpha\right] \tag{68}$$

Seismic earth pressure due to adhesion:

$$ADH = C_a L_a\left[\tan(\alpha + \phi)\cos(\omega) - \sin\omega\right] \tag{69}$$

Seismic earth pressure due to water:

$$W_a = (U_{st} + U_{sh})\left[\tan(\alpha + \phi)\cos\alpha - \sin\alpha\right] \tag{70}$$

where

$$U_{st} = \frac{h_w^2 \gamma_w}{2\sin\alpha} \tag{71}$$

$$U_{sh} = (U_{Top} + U_{Bot})\frac{h_w}{2\sin\alpha} \tag{72}$$

$$U_{Top} = \gamma(h - h_w)R_u \tag{73}$$

$$U_{Bot} = \left[\gamma(h - h_w) + (\gamma - \gamma_w)h_w\right]R_u \tag{74}$$

where U_{st} and U_{sh} are the hydrostatic and induced seismic seepage forces acting on the wedge, and R_u is the excess pore water pressure ratio.

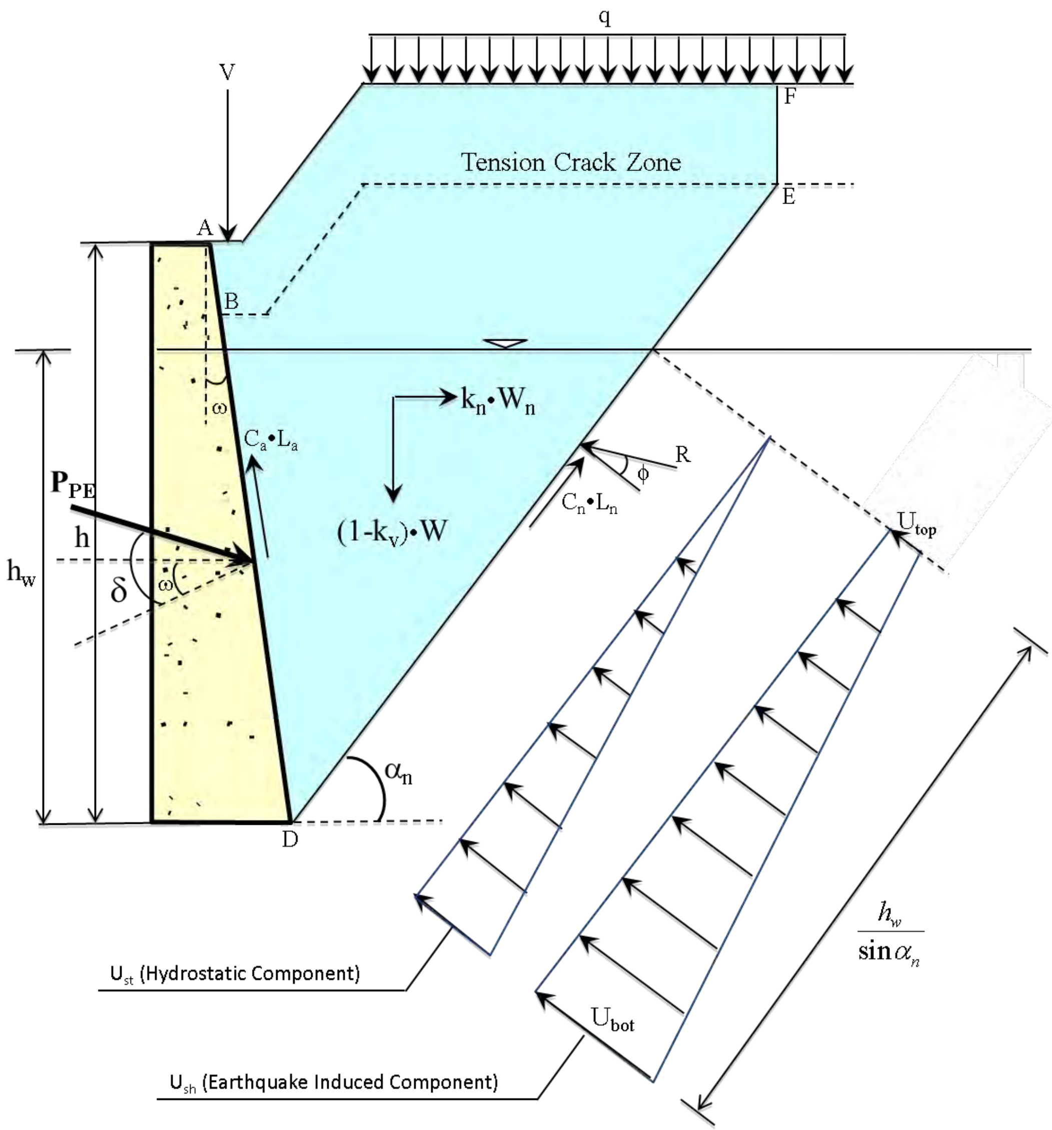


Figure 2-21: Seismic Passive Trial Wedge

2.3.3 Log-Spiral-Rankine Model

Seismic earth pressures should be estimated using procedures that account for the internal friction and cohesion of backfill, as well as the wall-soil interface friction and adhesion. The inertial effects of ground shaking should also be considered on the development of seismic earth pressures. Shamsabadi et al. (2013) have developed a methodology for estimating the seismic earth pressures considering the local and global limit equilibrium of the mobilized soil mass and the states of stress along the nonlinear failure surface within the soil medium. The Log-Spiral-Rankine model (Shamsabadi et al., 2013) is available for both the active and passive earth pressures computation, although it is advantageous over the other models mostly in the passive case.

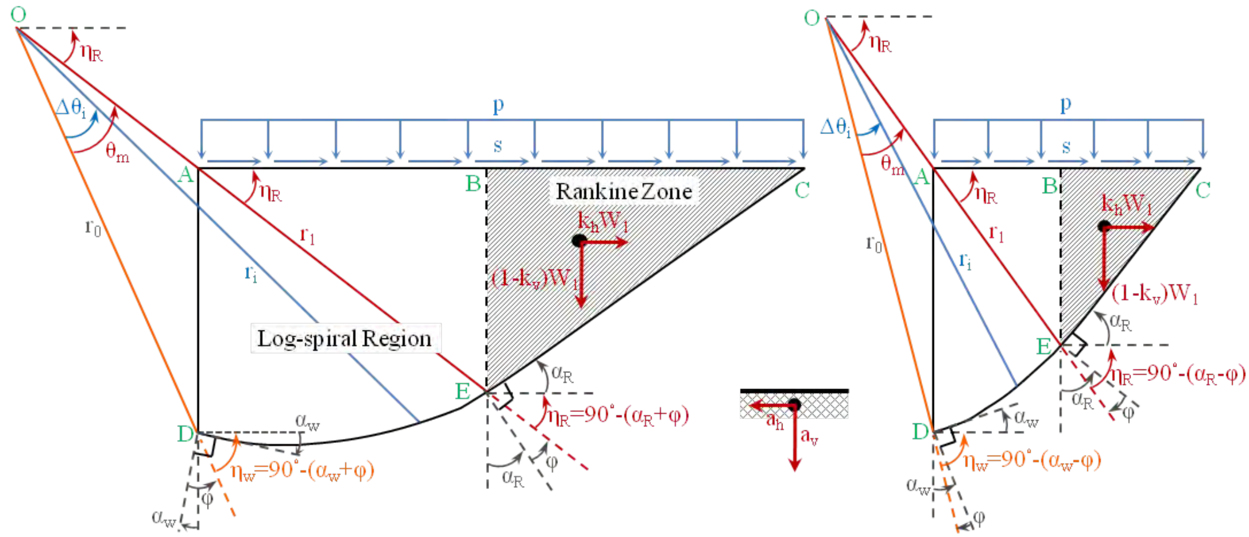
The seismic passive earth pressure becomes important for walls that develop resistance to sliding from the embedded portion of the wall. For these designs it is important to estimate passive pressures that are not overly conservative or non-conservative for the seismic loading condition. This is particularly the case if displacement-based design methods are used, and it can also affect the efficiency of designs based on limit-equilibrium methods.

The M-O equation for seismic passive earth pressure is not recommended for use in determining the seismic passive pressure, despite its apparent simplicity. The M-O equation is based on the Coulomb method to determine the earth pressures, and this method can overestimate the passive earth pressure in some cases. Additionally, the M-O equation does not account for the cohesion of the soil, which can contribute significantly.

A key consideration during the determination of static and seismic passive pressures is the wall friction. Common practice is to assume that some wall friction will be mobilized for static loading. The amount of interface friction for static loading is often assumed to range from 50 to 80% of the soil friction angle. Similar guidance is not available for seismic loading. In the absence of any specific guidance or research results for seismic loading, it is suggested that a wall-soil interface friction angle equal to or greater than $^{2}/_{3}$ of the soil friction angle should be used.

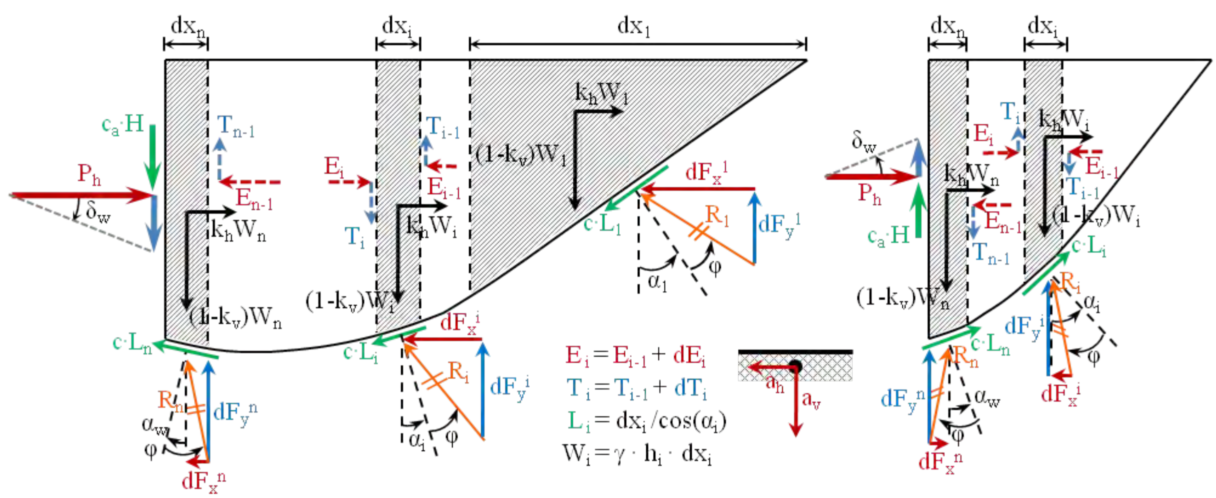
Another important consideration when assessing the seismic passive earth pressure is the amount of deformation required to mobilize this force. The deformation required to mobilize the passive earth pressure during static loading is usually assumed to be large—usually 2% to 6% of the embedded wall height. Similar guidance is not available for seismic loading and therefore the normal approach during design for seismic passive earth pressures is to assume that the displacement to mobilize the seismic passive earth pressure is the same as for static loading.

Realistic seismic earth pressures can be obtained utilizing the Log-Spiral-Rankine model as summarized in this section. In the Log-Spiral-Rankine Model, the (homogeneous) soil body that is mobilized as the retaining system fails under passive or active earth pressures is assumed to be composed of two regions: the log-spiral region and the Rankine zone, as illustrated in Figure 2-22. The triangular region of the mobilized soil body is labeled as the Rankine zone, because the shear stress (τ_{xz}) in this region is induced solely by the horizontal seismic body forces without any contribution from the inter-particle friction or cohesion—i.e., a stress state that is similar to that of the classical Rankine theory.



(a) Passive case (b) Active case

Figure 2-22: Geometry of the Mobilized Failure Surface



(a) Passive case **(b) Active case**

Figure 2-23: Slices of Mobilized Soil Mass

The inclination of the failure surface in Rankine zone (α_R in Figure 2-22) is computed as a function of earthquake acceleration and using Mohr-Coulomb material with cohesion (Richards and Shi, 1994).

$$\alpha_R = (45^{o} - j\phi/2) - \alpha_p^R \tag{75}$$

where

$$\alpha_p^R = \frac{1}{2}\tan^{-1}\left(\frac{2}{(K_R-1)}\frac{k_h}{(1-k_v)}\right) \tag{76}$$

$$K_R \equiv \frac{\sigma_x}{\sigma_z} = \frac{1+\sin^2\phi}{\cos^2\phi} + \frac{2c\tan\phi}{\sigma_z} + \frac{2j}{\cos\phi}\sqrt{\left(\tan\phi+\frac{c}{\sigma_z}\right)^2 - \left(\frac{k_h}{1-k_v}\right)^2} \tag{77}$$

In Eqn. 77, when $j = +1$, σ_x is greater than σ_z and thus the stress state for the passive case is represented; the active case is represented when $j = -1$. The same expression on j is adopted throughout the remainder of this section. To satisfy the ratio of shear stress to horizontal normal stress at the wall-soil interface and to account for the seismic inertia effect, the wall takeoff angle (α_w in Figure 2-22) with respect to horizontal at the base of the wall is given by the following expression:

$$\alpha_w = (45^\circ - j\phi/2) - \alpha_p^w - \alpha_p^R \tag{78}$$

where

$$\alpha_p^w = \frac{1}{2}\tan^{-1}\left(\frac{2K_w\tan\delta}{j(K_w-1)}\right) \tag{79}$$

and

$$K_w \equiv \frac{\sigma_x}{\sigma_z} = \frac{1+\sin^2\phi + \frac{c}{\sigma_z}\sin(2\phi) - 4\frac{c_a}{\sigma_z}\tan\delta + j2\cos\phi\sqrt{\Delta}}{\cos^2\phi + 4\tan^2\delta} \tag{80}$$

where $\Delta = \left(\tan\phi + \frac{c}{\sigma_z}\right)^2 - \left(\tan\delta + \frac{c_a}{\sigma_z}\right)^2 + 4\tan\delta\left(\tan\phi + \frac{c}{\sigma_z}\right)\left[\frac{c}{\sigma_z}\tan\delta - \frac{c_a}{\sigma_z}\tan\phi\right].$

Thus, α_w is positive when it is above the horizontal and negative when it is below the horizontal. The subtended logarithmic arc angle (θ_m in Figure 2-22) can be computed using the following expression:

$$\theta_m = \alpha_R - \alpha_w \tag{81}$$

The geometry of the logarithmic spiral curve, DE, is obtained in Eqn. 82 as follows:

$$r_i = r_0 e^{\Delta\theta_i \cdot \tan(j\cdot\phi)} \tag{82}$$

The log-spiral region can then be discretized into a number of vertical slices as illustrated in Figure 2-23. The increment in the inter-slice shear force from the right face of the i^{th} slice to its left face is denoted as $dT_i \equiv T_i - T_{i-1}$ and given in Eqn. 83. The inter-slice shear angle, δ_i in Eqn. 83, must be solved iteratively adopting either the simplified (cf. Eqn. 83b, Shamsabadi et al., 2013) or the rigorous method (Xu et al., 2013).

$$dT_i = \frac{W_i\left[(1-k_v)\tan(\alpha_i + j\phi) - k_h\right] + jc\,dx_i\left[1+\tan\alpha_i\tan(\alpha_i + j\cdot\phi)\right]}{1 - j\tan\delta_i\tan(\alpha_i + j\phi)}\tan\delta_i \tag{83a}$$

where

$$\delta_i = j\left[(90^{\circ} - j\phi) - 2\alpha_i - \sin^{-1}\left(\frac{\sin\delta_i}{\sin\phi}\right)\right] \quad \text{when } \phi \neq 0 \tag{83b}$$

or

$$\delta_i = \tan^{-1}\left(\frac{c\cos(2\alpha_i)}{\sigma_z + j\,2c\sin(2\alpha_i)}\right) \quad \text{when } \phi = 0 \tag{83c}$$

Finally, the horizontal and total earth-pressure forces, P_h and P_{total}, are obtained as expressed in Eqn. 84.

$$P_h = \frac{C_W + C_a + C_c + C_T}{1 - j\cdot\tan\delta\cdot\tan(\alpha_w + j\cdot\phi)} \;\; ; \;\; P_{total} = \frac{P_h}{\cos\delta} \;\; ; \;\; K_E = \frac{P_{total}}{(1/2\cdot\gamma H^2)} \tag{84a}$$

where

$$C_W = \sum_{i=1}^{n}\left\{(1-k_v)W_i\left[\tan(\alpha_i + j\phi) - \frac{k_h}{1-k_v}\right]\right\} \tag{84b}$$

$$C_a = j\,c_a H\tan(\alpha_w + j\phi) \tag{84c}$$

$$C_c = \sum_{i=1}^{n}\left\{j\,c\,dx_i\left[1 + \tan(\alpha_i)\tan(\alpha_i + j\phi)\right]\right\} \tag{84d}$$

$$C_T = \sum_{i=1}^{n-1}\left\{j\,dT_i\left[\tan(\alpha_i + j\phi) - \tan(\alpha_w + j\phi)\right]\right\} \tag{84e}$$

For the passive case, the critical condition for the retaining structure is when the inertial force is driving the mobilized soil body toward the remaining soil mass, whereas the critical condition for the active case is when the inertial force pushes toward the retaining structure. Therefore, the k_h values in this model are taken as positive for the passive case, and negative for the active case (based on the sign convention adopted in Figure 2-23). The Log-Spiral-Rankine model is used to develop the seismic passive pressure coefficients and the Trial Wedge method is used to develop the seismic active earth pressure coefficients provided in Appendices A and B. Detailed derivation and verification of the Log-Spiral-Rankine model, as well as the predicted point of application of the earth thrust can be found in the literature (Shamsabadi et al., 2013; Xu et al., 2013).

2.4. MAXIMUM SEISMIC COEFFICIENTS FOR DESIGN

The maximum seismic coefficient (k_{max}) for computation of seismic lateral thrust loads shall be determined on the basis of the peak ground acceleration, PGA, at the ground surface as shown in Eqn. 85, where F_{PGA} is the site adjustment factor given in Table 2-2.

$$k_{max} = F_{PGA}\,PGA \tag{85}$$

In the case where the walls are founded on Category A soil (hard rock), the k_{max} shall be estimated based on 1.2 times the site-adjusted peak ground acceleration coefficient as shown bellow.

$$k_{max} = 1.2 F_{PGA} PGA \tag{86}$$

For wall height greater than 20 feet but less than 70 feet, the seismic coefficient used to compute lateral loads can be determined using the following equation:

$$k_{ave} = \alpha k_{max} \tag{87}$$

The α in Eqn. 87 is the fill height-dependent reduction factor and can be determined from Figure 2-24:

Table 2-2: Values of Site Factor (F_{PGA}) at Zero Period on Acceleration Spectrum

Site Class	Peak Ground Acceleration Coefficient (PGA)				
	$PGA \leq 0.10$	$PGA = 0.20$	$PGA = 0.30$	$PGA = 0.40$	$PGA \geq 0.50$
A	0.8	0.8	0.8	0.8	0.8
B	1	1	1	1	1
C	1.2	1.2	1.1	1	1
D	1.6	1.4	1.2	1.1	1
E	2.5	1.7	1.2	0.9	0.9
F*	*	*	*	*	*

* Site-specific geotechnical investigation and dynamic site response analyses should be performed for all sites in Site Class F following the current AASHTO *LRFD Bridge Design Specifications*.

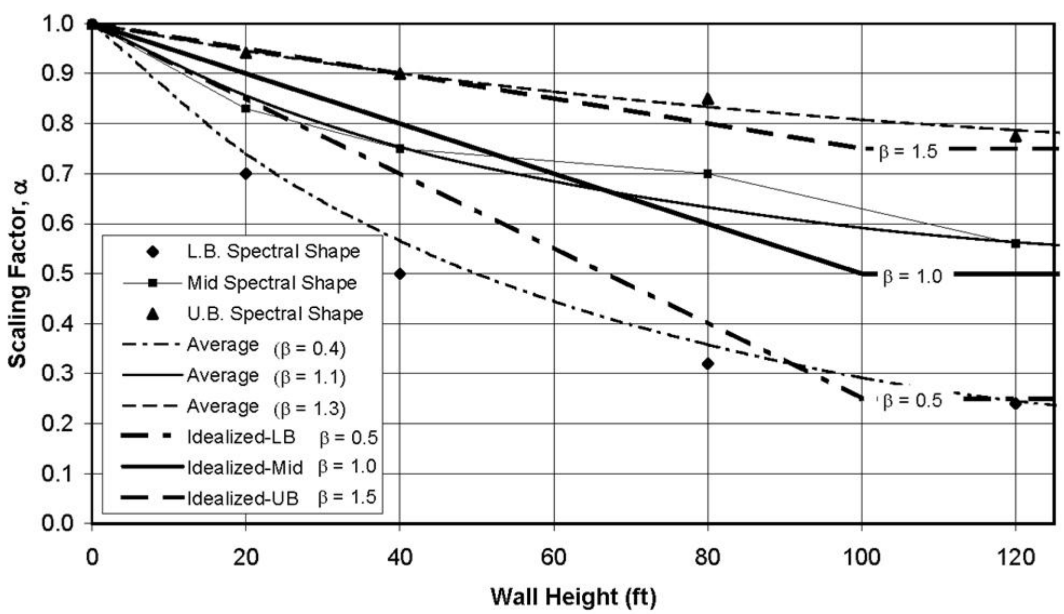


Figure 2-24: Scaling Factor α versus Wall Height H

For wall heights greater than 70 feet, special seismic design studies involving the use of numerical models should be conducted. These special studies are required in view of the potential consequences of failure of these very tall walls, as well as limitations in the simplified wave scattering methodology.

2.5. WALL DISPLACEMENT

Various methods can be used to estimate permanent displacements of earth retaining structures for walls that can move without damaging either adjacent facilities or components of the wall. These methods range from simple Newmark method of analysis to complicated numerical models. For many situations, simple equations or charts will be sufficient; however, as the complexity of the site or the wall-soil system increases, more rigorous numerical modeling methods become advantageous. Per NCHRP 12-70 Project, based on regression analyses, the following simplified relationships may be used to calculate the wall displacement:

- For all sites except Central and East of United States (CEUS) rock sites (Categories A and B):

$$\log(d) = -1.51 - 0.74\log\left(\frac{k_y}{k_{max}}\right) + 3.27\log\left(1-\frac{k_y}{k_{max}}\right) - 0.8\log\left(k_{max}\right) + 1.59\log\left(PGV\right) \quad (88)$$

- For CEUS rock sites (Categories A and B), displacement (in inches) can be estimated by:

$$\log(d) = -1.31 - 0.93\log\left(\frac{k_y}{k_{max}}\right) + 4.52\log\left(1-\frac{k_y}{k_{max}}\right) - 0.46\log\left(k_{max}\right) + 1.12\log\left(PGV\right) \quad (89)$$

Figure 2-25 shows a comparison between the displacements estimated using the old (i.e., AASHTO 2004) and new (i.e., Eqn. 88) equations. Note that the above displacement equations represent mean values, and can be multiplied by 2 to obtain an 84 percent confidence level. When using Eqns. 88 and 89, it is necessary to estimate the peak ground velocity (PGV) and the yield acceleration (k_y). Values of PGV in inch per second may be estimated using the following correlation between the PGV and spectral ordinates at one second (S_1) for Site Class B.

$$PGV = 55F_vS_1 \quad (90)$$

where S_1 is the spectral acceleration at 1 second

F_v is the Site Class adjustment for Site Class B.

Values of the yield acceleration (k_y) can be established by computing the seismic coefficient for global stability that results in a capacity to demand ratio ($^C/_D$) of 1.0.

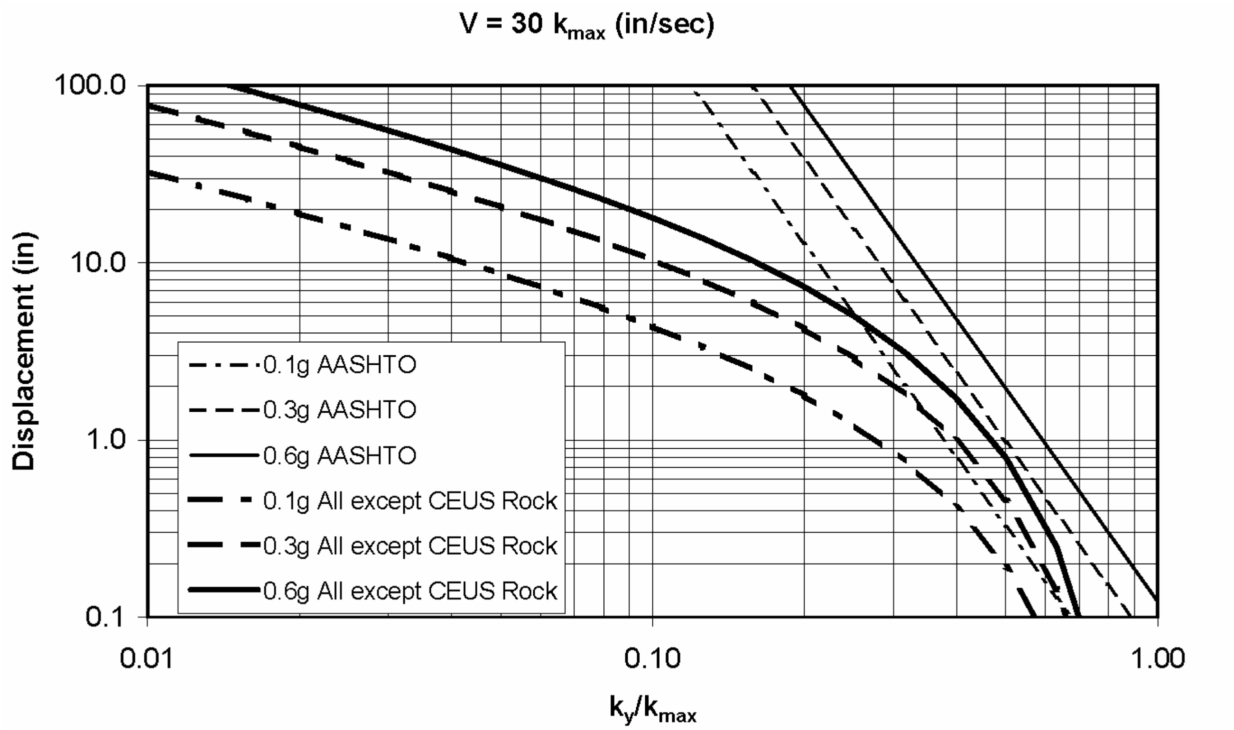


Figure 2-25: Comparison between AASHTO (2004) and Recommended Displacement Equation

The proposed Newmark equations given above represent a simplified method of estimating the displacements that will occur if the $^C/_D$ ratio for a limiting equilibrium stability analysis is less than 1.0. Alternate methods of analysis such as finite element or finite difference model can be used to calculate permanent wall displacements. Such models require considerable expertise in the set-up and interpretation of model results, particularly relative to the selection of strength parameters consistent with seismic loading.

2.6. SURCHARGE LOADS

2.6.1 Uniform Surcharge Loads

Where a uniform surcharge is present as shown in Figure 2-26, a constant horizontal earth pressure must be added to the basic lateral earth pressure. This constant earth pressure may be taken as:

$$\sigma_h = K\,Q. \tag{91}$$

Realistically, the lateral earth pressure due to surcharge loads will diminish with depth. The simplified stress distribution as shown in Figure 2-26 violates this rule of a thumb. Therefore, the constant earth pressure as suggested by Eqn. 91 should not extend indefinitely into below the ground surface. In design practice, the constant earth pressure is considered to be distributed only between the ground line and the excavation line.

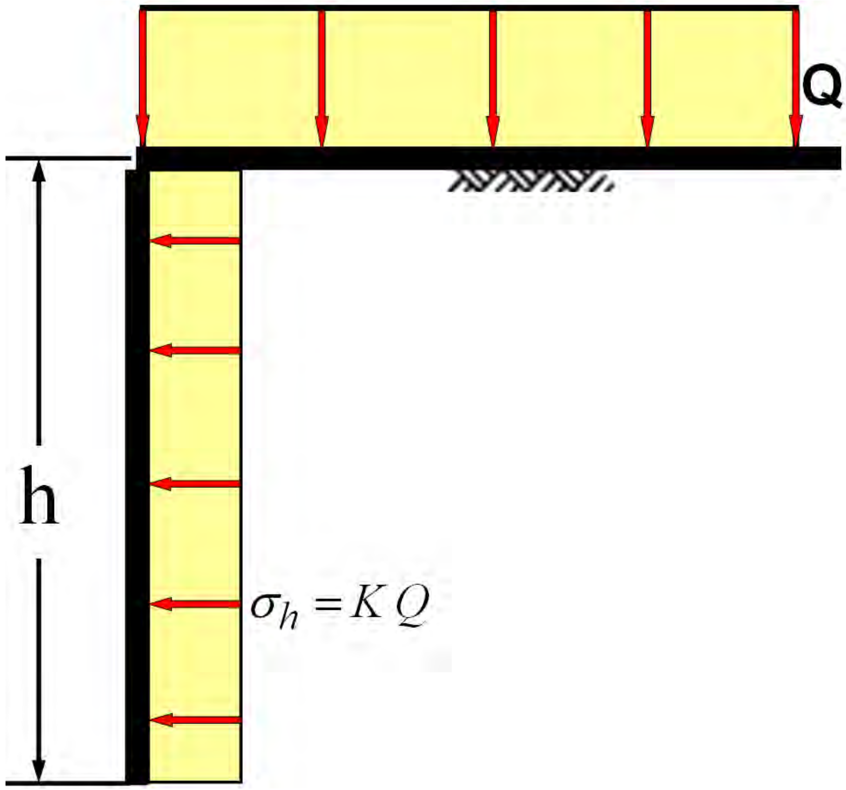


Figure 2-26: Lateral Pressure Due to Uniform Surcharge

2.6.2 Boussinesq Loads

Typically, there are three types of Boussinesq Loads. They are as follows:

2.6.2.1 *Strip Load*

Strip loads (see Figure 2-27(a)) are loads such as highways and railroads and are generally parallel to the wall. The general equation for determining the pressure at distance *h* below the ground line is:

$$\sigma_h = 2Q\,\frac{\beta - \sin\beta\cos 2\alpha}{\pi} \quad ; \ \alpha \text{ and } \beta \text{ in rad.} \tag{92}$$

2.6.3

2.6.3.2 *Line Load*

Line loads (see Figure 2-27(b)) are loads such as a continuous wall footing of narrow width or similar load generally parallel to the wall. K-railing could be considered to be a line load. The general equation for determining the pressure at distance h below the ground line is:

For $m \leq 0.4$

$$\sigma_h = \frac{0.2\ Q\ n}{\left(0.16 + n^2\right)^2 h} \tag{93}$$

For $m > 0.4$

$$\sigma_h = \frac{1.28\ Q\ m^2\ n}{\left(m^2 + n^2\right)^2 h} \tag{94}$$

2.6.3.3 ***Point Load***

Point loads (see Figure(c)) are loads such as a wheel load from a concrete truck. The general equation for determining the pressure at distance h below the ground line is:

For $m \leq 0.4$

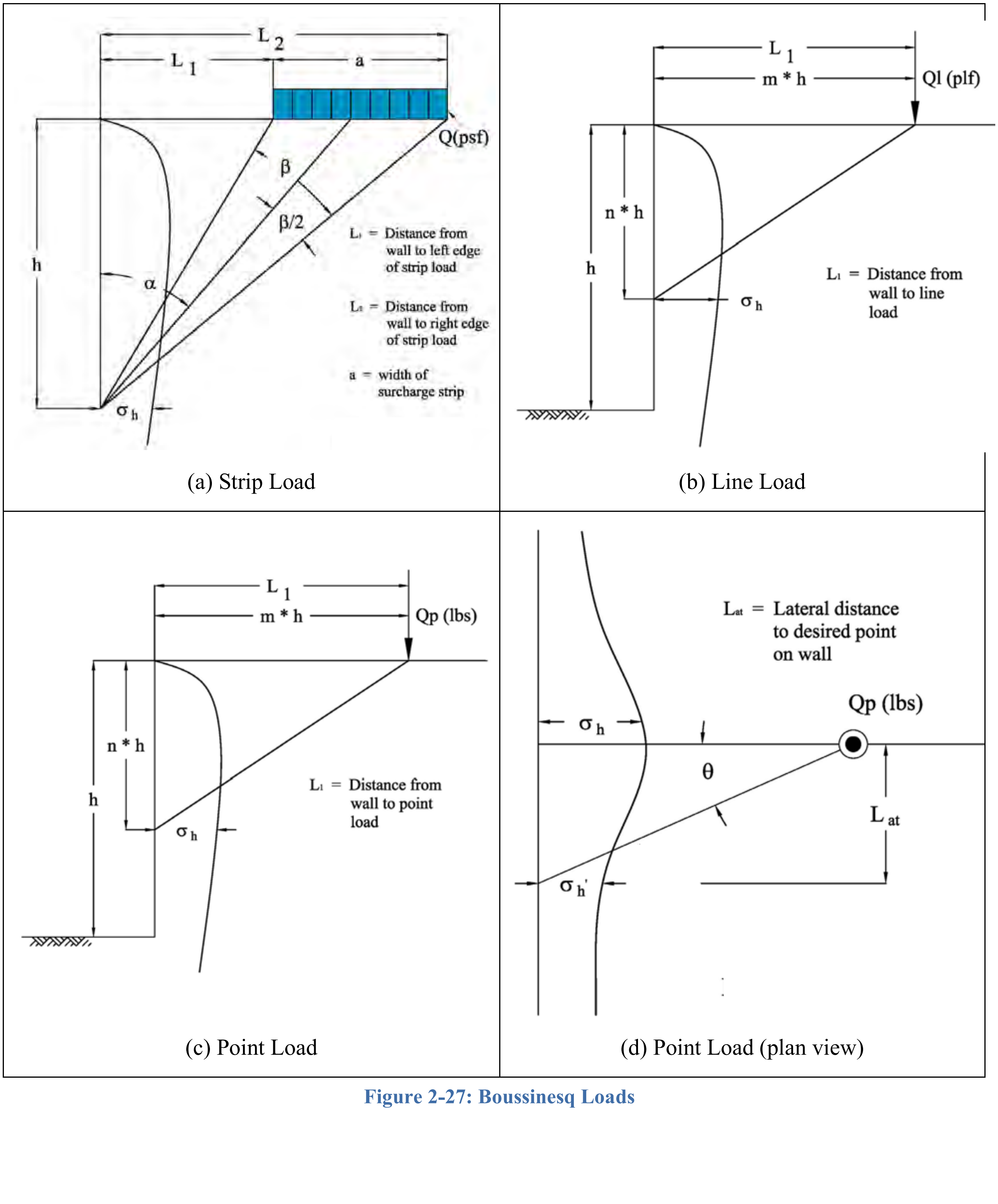
$$\sigma_h = \frac{0.28\ Q_p\ n^2}{\left(0.16 + n^2\right)^3 h^2} \tag{95}$$

For $m > 0.4$

$$\sigma_h = \frac{1.77\ Q_p\ m^2\ n^2}{\left(m^2 + n^2\right)^3 h^2} \tag{96}$$

In addition, σ_h is further adjusted by the following when the point is further away from the line closest to the point load (see, Figure 2-27 (d)):

$$\sigma_h' = \sigma_h \cos^2\left(1.1\theta\right) \tag{97}$$



(a) Strip Load

(b) Line Load

(c) Point Load

(d) Point Load (plan view)

Figure 2-27: Boussinesq Loads

2.7. SOIL PRESSURE DISTRIBUTION FOR LAYERED SOIL

When designing a shoring system in the layer soils, it is very important to develop appropriate soil pressure distribution for each individual soil layer as shown in Figure 2-28.

$$\sigma_1^+ = \gamma_1 y_1 k_{a1}$$
$$\sigma_1^- = \gamma_1 y_1 k_{a2}$$
$$\sigma_2^+ = \sigma_1^- + \gamma_2 y_2 k_{a2}$$
$$\sigma_2^- = (\gamma_1 y_1 + \gamma_2 y_2) k_{a3}$$
$$\sigma_3^+ = \sigma_2^- + \gamma_3 (y_3 + y_4) k_{a3}$$
$$\sigma_p = \gamma_3 y_4 k_{p3}.$$

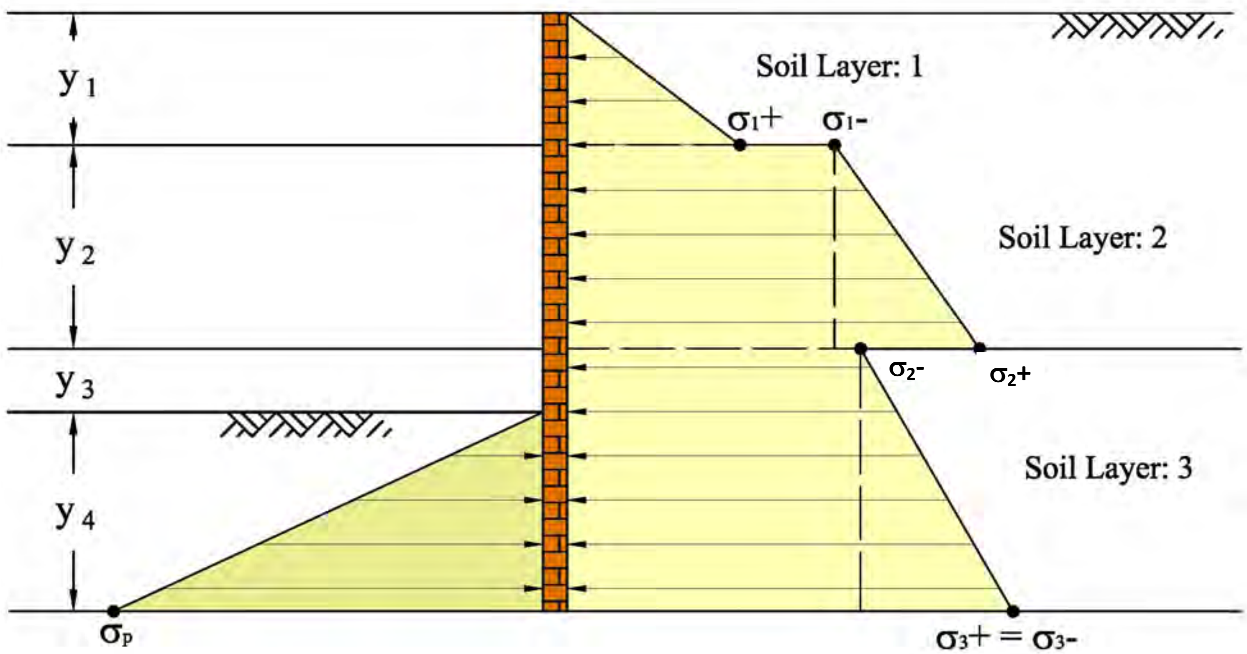


Figure 2-28: Earth Pressure Distribution

2.7.1 Example Problem 2-1: Earth Pressure Distribution in Layered Soil

For a shoring system subjected to the lateral load given below, calculate the horizontal earth pressure diagram.

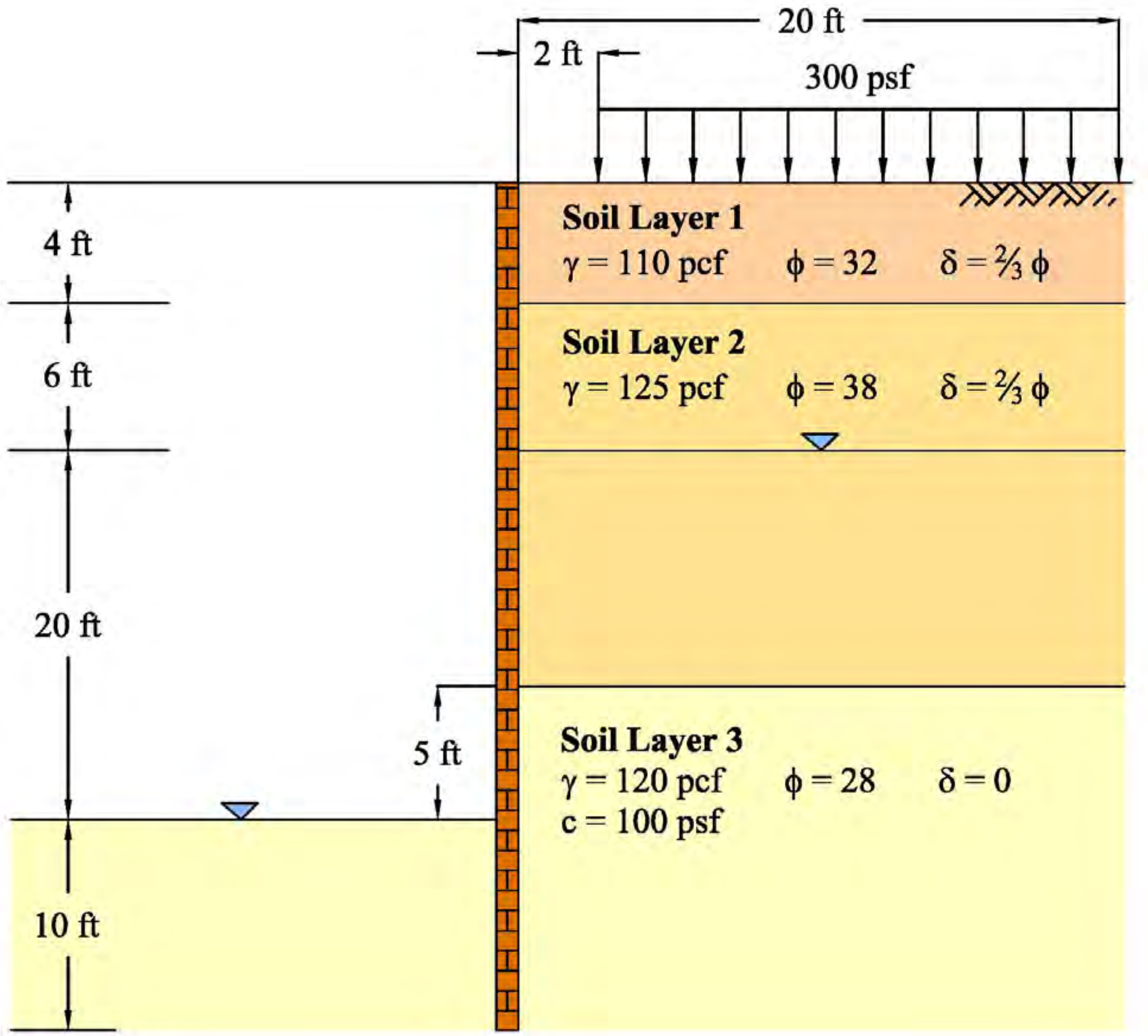


Figure 2-29: Earth Pressure Distribution in Layered Soil

Solution:

(i) Determination of earth pressure coefficients: (Coulomb equation can be used to calculate the earth pressure coefficients for soil layers 1 and 2, and Rankine equation for soil layer 3.)

$K_{a1} = 0.256$ (horizontal component)

$K_{a2} = 0.196$ (horizontal component)

$K_{a3} = 0.361$

$K_{p3} = 2.770$

Note: It is recommended to use the Log-Spiral-Rankine method to compute the passive earth pressure. Here the Rankine equation is applied; thus the passive pressure is underestimated (see Figure 2-14).

(ii) Sample calculation of surcharge stress at h = 3 ft. for strip load:

From Figure 2-27 (a), σ_h can be determined by applying Eqn. 92:

$$\sigma_h = 2\,Q\frac{\beta - \sin\beta\cos 2\alpha}{\pi} = 0.219 \text{ ksf}$$

where $\beta = \tan^{-1}\left(\frac{20}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right) = 0.834$ (rad)

$$\alpha = \tan^{-1}\left(\frac{2}{3}\right) + \frac{\beta}{2} = 0.588 + 0.417 = 1.005 \text{ (rad)}$$

$$Q = 0.3 \text{ ksf}$$

(iii) Determination of horizontal earth pressure distribution:

$$\sigma_1^+ = \gamma_1\ (h = 4\text{ft})K_{a1} = 110\ (4)\ (0.256) = 0.113 \text{ ksf}$$

$$\sigma_1^- = \gamma_1\ (h = 4\text{ft})K_{a2} = 110\ (4)\ (0.196) = 0.086 \text{ ksf}$$

$$\sigma_2 = \sigma_1^- + \gamma_2\ (h = 6\text{ft})K_{a2} = 86 + 125\ (6)\ (0.196) = 0.233 \text{ ksf}$$

$$\sigma_3^+ = \sigma_2 + (\gamma_2 - \gamma_w)\ (h = 15\text{ft})K_{a2} = 233 + 6\ (125 - 62.4)\ (0.196) = 0.417 \text{ ksf}$$

$$\sigma_3^- = \left[\gamma_1\ (4\text{ft}) + \gamma_2 (6\text{ft}) + (\gamma_2 - \gamma_w)(15\text{ft})\right] K_{a3} - 2\ C\sqrt{K_{a3}}$$
$$= \left[110(4) + 125(6) + 15\ (125 - 62.4)\right]\ (0.361) - 2\ (100)\ \sqrt{0.361} = 0.648 \text{ ksf}$$

$$\sigma_4 = \sigma_3^- + (\gamma_3 - \gamma_w)\ (h = 15\text{ft}) K_{a3} = 648 + 15\ (120 - 62.4)\ (0.361) = 0.960 \text{ ksf}$$

$$\sigma_{p1} = 2\ C\sqrt{K_{p3}} = 2\ (100)\ \sqrt{2.770} = 0.333 \text{ ksf}$$

$$\sigma_{p2} = \sigma_{p1} + (\gamma_3 - \gamma_w)\ (h = 10\text{ ft})\ K_{p3} = 333 + 10(120 - 62.4)(2.770) = 1.928 \text{ ksf}$$

$$\sigma_{aw} = \gamma_w (h = 30\text{ft}) = 1.872 \text{ ksf}$$
$$\sigma_{pw} = \gamma_w (h = 10\text{ft}) = 0.624 \text{ ksf}$$

The horizontal earth pressure distribution of this shoring system is illustrated in Figure 2-30.

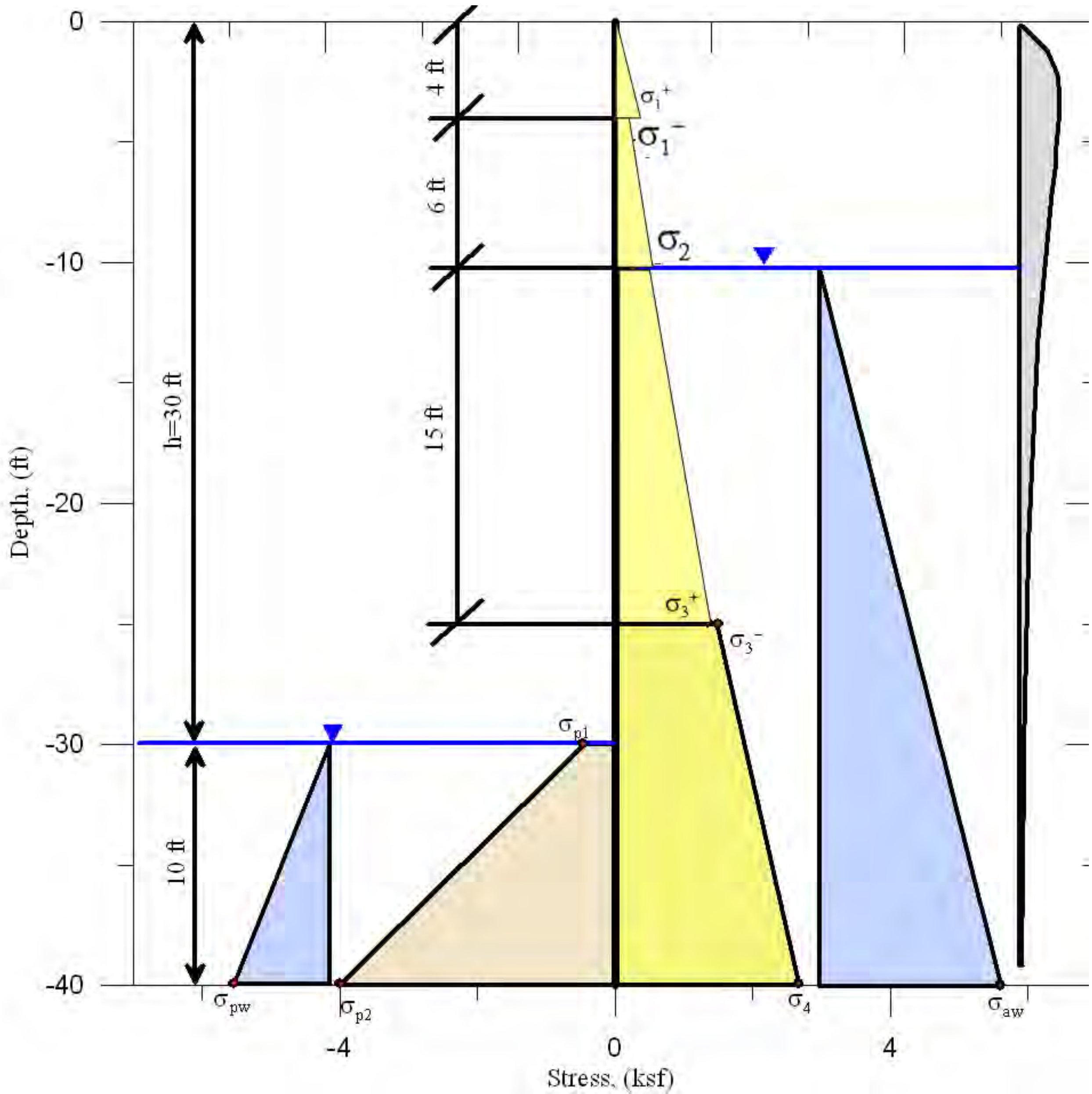


Figure 2-30: Pressure Loading Diagram

CHAPTER 3 GRAVITY AND SEMI-GRAVITY EARTH RETAINING STRUCTURES

Design of gravity walls must satisfy both external and internal stability, as well as integrity of the structural components of the wall. When designing a gravity retaining wall both external and internal stability analysis has to be performed in order to evaluate the ability of the wall to resist lateral total thrust loads. The lateral thrust load includes static active earth pressure, earth pressure surcharge loads, surcharge live loads, incremental seismic load, hydrostatic water pressure, and seepage force. Relevant modes of the failure for gravity and semi-gravity earth retaining systems supported on spread footing are depicted in Figure 3-1.

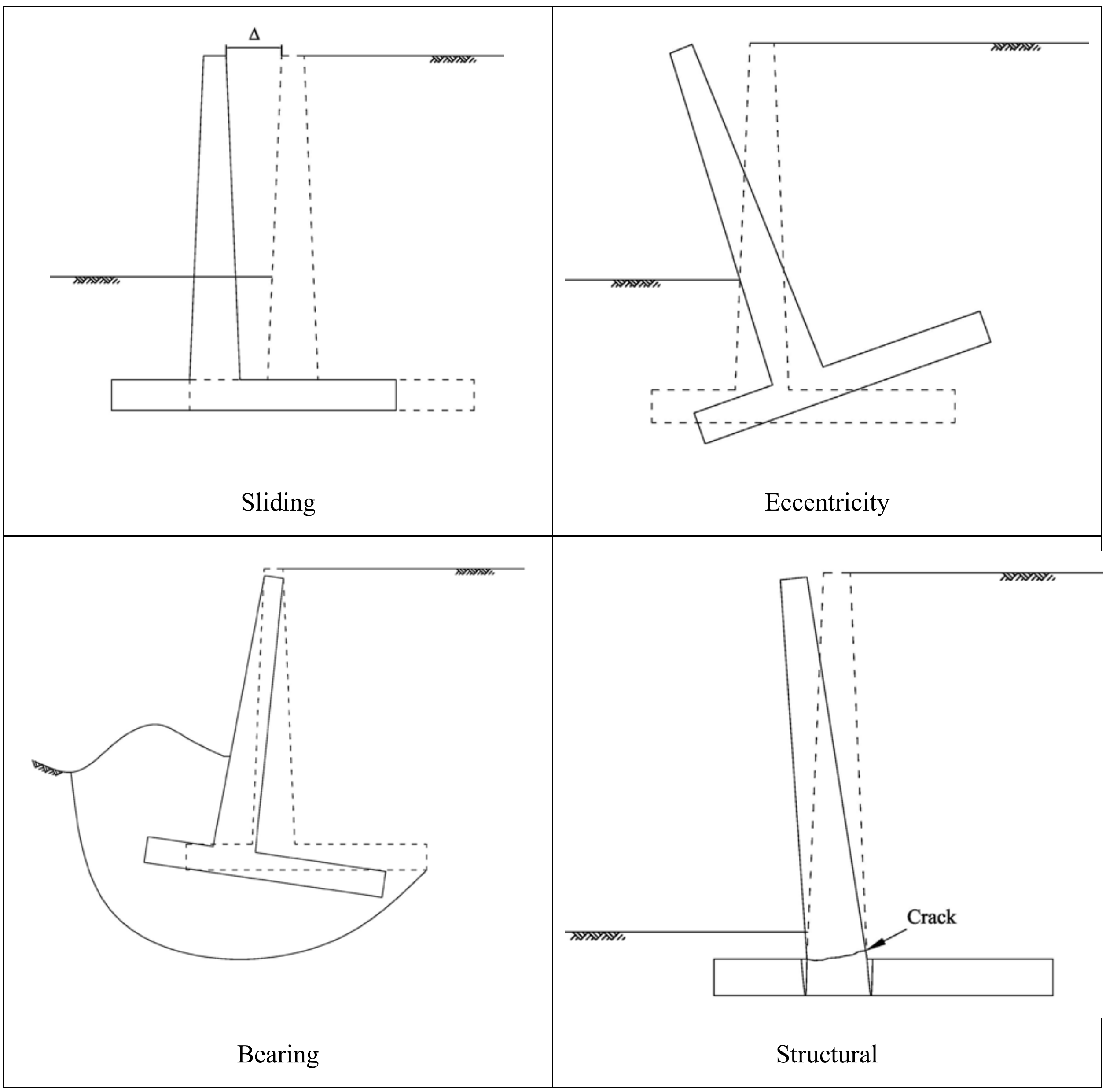


Figure 3-1: Modes of Failure of Semi-gravity Retaining Walls

The modes of the failure of semi-gravity retaining walls include:

- Sliding failure
- Eccentricity failure
- Bearing capacity failure
- Structural failure

Sliding failure of the wall is due to excessive horizontal thrust loads.

Eccentricity failure is a concern particularly for tall walls with narrow footings. Excessive lateral thrust loads and, in particular, incremental seismic loads will cause induced tilting and/or rotation of the wall during a seismic event.

Bearing capacity failure is due to excessive vertical live loads, surcharge loads, and induced incremental seismic stresses during a seismic event. Loss of bearing capacity will cause overturning and/or excessive tilting and settlement of the wall.

Structural failure of the walls is due to overstressing of the structural components including the stem and/or foundation of the earth retaining systems as a result of excessive vertical and horizontal loads.

All of these failure modes are driven by either the horizontal or the vertical component, or both of the active earth pressure imposed on the wall by the soil medium. Thus, an accurate estimation on the active pressure is essential to achieve a safe and economical design. To be conservative, unless the wall's foundation goes deep down below the excavation line or otherwise specified, the resistances provided by the passive earth pressure against the above failure modes are neglected. The stabilizing forces and moments acting on the semi-gravity wall are offered primarily by the self-weights of the RC structural components and by the soil mass sitting above the heel of foundation behind the wall. Step-by-step analytical and design procedures for the semi-gravity retaining walls can be found in §3.7.

3.1. RETAINING WALL ON SPREAD FOOTING

Majority of Cast-In-Place concrete retaining walls related to highway structures are of semi-gravity type. Lateral thrust load on this type of wall is a function of wall geometry, the material properties of backfill, the slope of the ground surface behind the wall, the friction between the wall and soil, and the ability of the wall to translate or rotate about its base. Rankine, Coulomb, Log-Spiral-Rankine and Trial Wedge methods may be used to calculate unfactored loads on gravity and semi-gravity earth retaining systems.

The Rankine theory is applicable for the semi-gravity cantilevered walls with long footing heels where the conjugate failure surface in the backfill soil does not interferes with the back face of the wall and is fully developed, as shown in Figure 3-2. The lateral earth pressure acts against a vertical plane, ab, inside of the soil mass. The position of the conjugate sliding plane, bc, can be determined using Eqn. 98.

$$\alpha_o = (45^\circ - \frac{\phi}{2}) - \frac{1}{2}\left[\sin^{-1}\left(\frac{\sin\beta}{\sin\phi}\right) - \beta\right] \qquad (98)$$

The Coulomb theory is applicable for the design of retaining walls for which the back face of the wall interferes with the full development of the conjugate failure surface in the backfill soil, as shown in

Figure 3-3. In general, the Coulomb theory applies for gravity, semi-gravity and prefabricated modular walls, which have relatively steep back faces, and semi-gravity cantilevered walls with short footing heels. Coulomb theory is also applicable for the semi-gravity cantilevered walls with long footing.

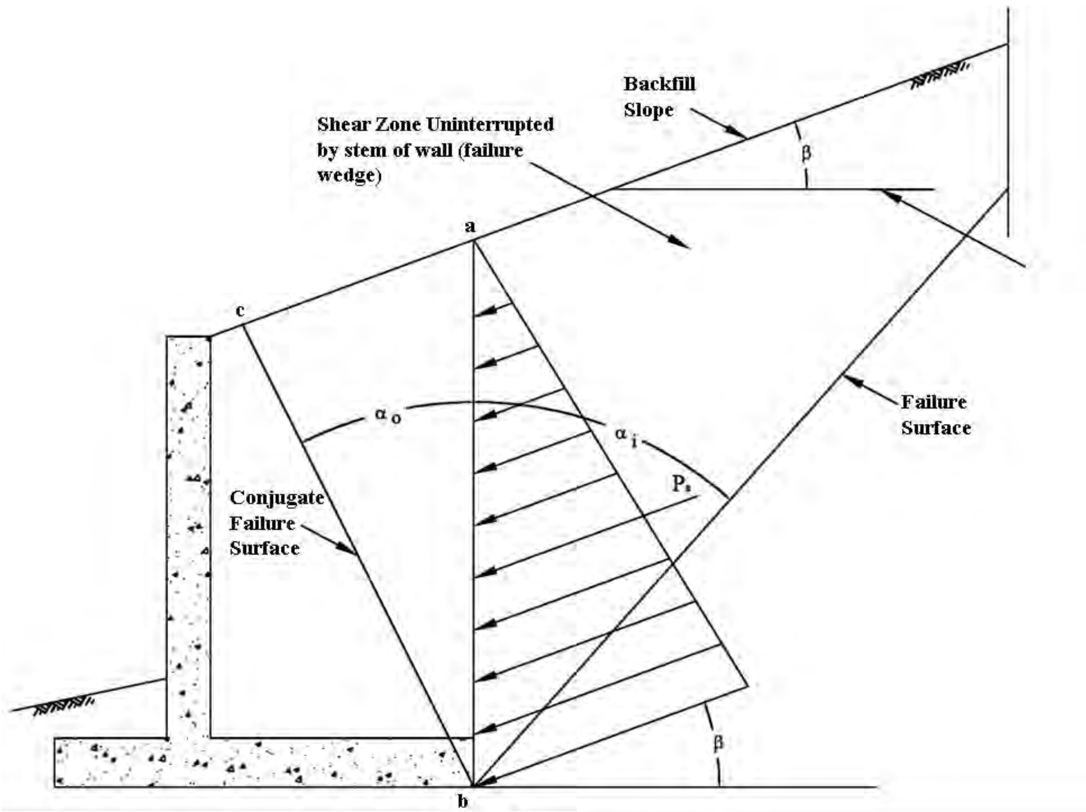
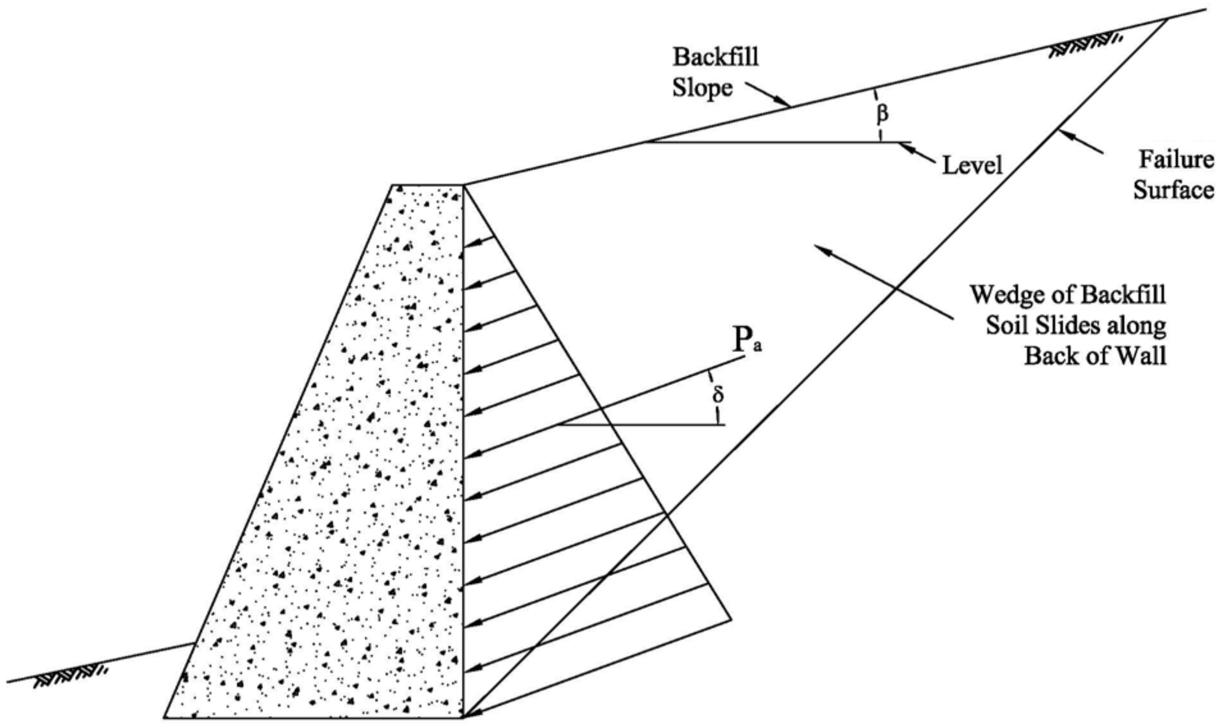


Figure 3-2: Application of the Rankine Earth Pressure Theory



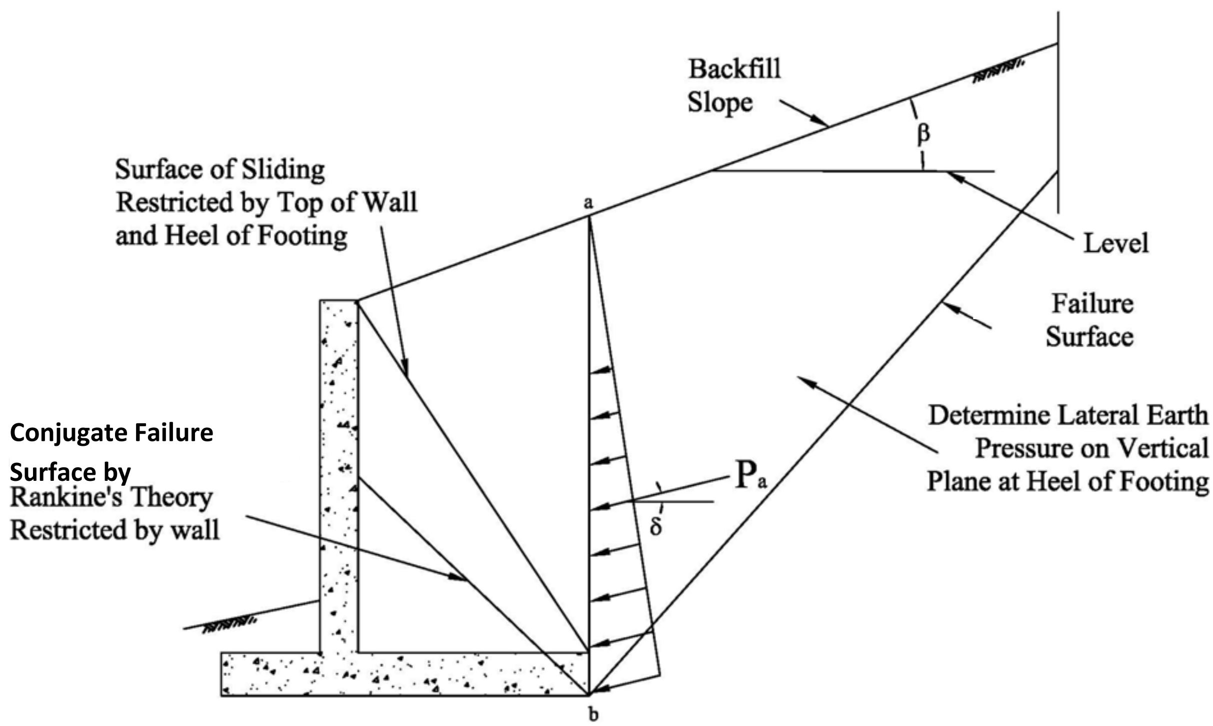
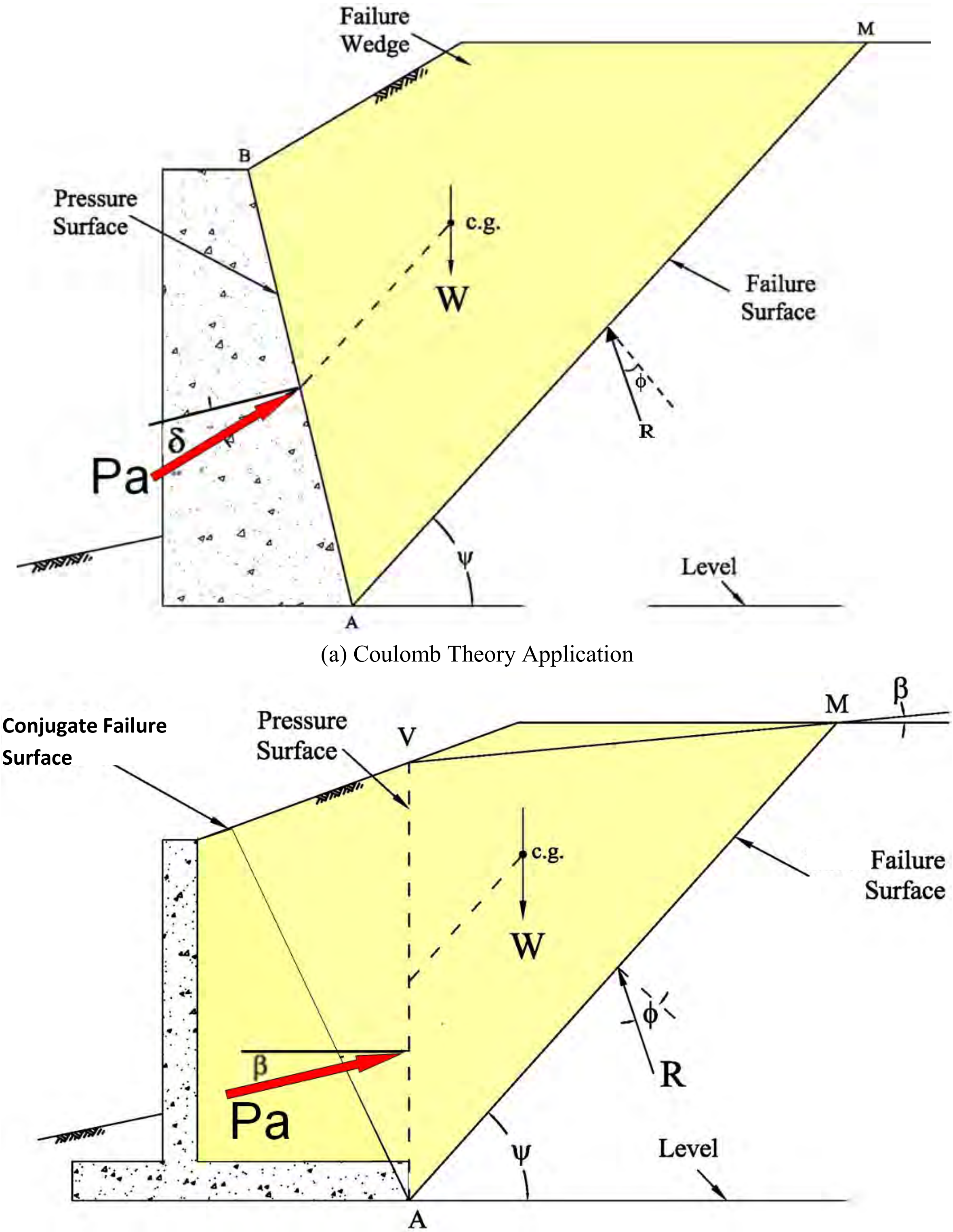


Figure 3-3: Application of Coulomb Earth Pressure Theory

The Trial Wedge method is applicable for the design of the gravity walls with either uniform or irregular backfill soil with application of both Rankine and Coulomb earth pressure theory, as shown in Figure 3-4.



(a) Coulomb Theory Application

(b) Rankine Theory Application

Figure 3-4: Application of the Wedge Theories

There are many situations, such as natural slopes, where the existing ground is stable and stands without sliding and/or caving. If a conventional retaining wall is to be constructed at the toe of the kind of slope, the backfill zone is limited and the failure surface is thus prescribed, as shown in Figure 3-5. In these cases, none of the classical earth pressure theories can estimate the lateral earth pressure by the limited backfill. However, since the weight of the backfill and the possible failure angle is known, Eqn. 61 can still be used to estimate the lateral earth pressure imposed from the limited backfill zone on the wall.

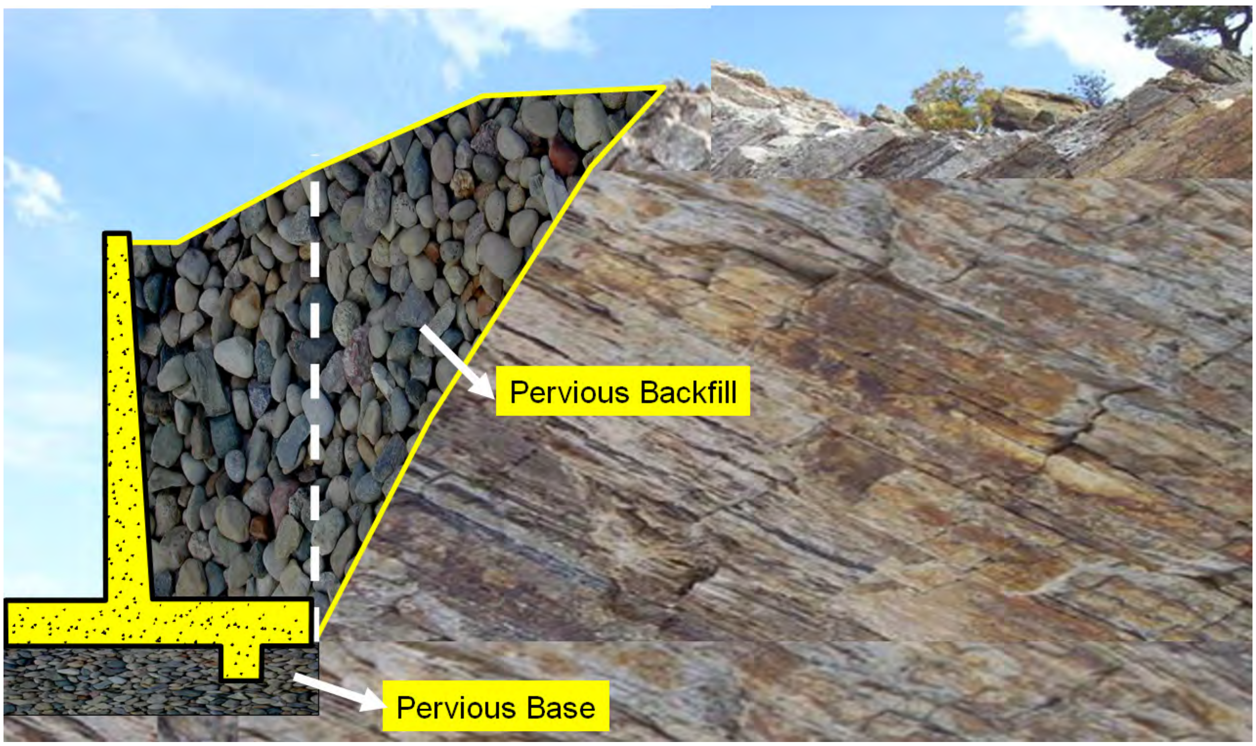


Figure 3-5: Retaining System subject to a Prescribed Failure Surface

3.2. DESIGN OF SEMI-GRAVITY CANTILEVER RETAINING WALLS

Semi-gravity cantilever retaining walls should be designed to satisfy both external and internal failure criteria. The wall of this type consists of two structural elements: the stem and the footing. The portion of the footing in the front of the stem is the toe and the portion in the rear of the stem is the heel of the wall. The footing must be wide and thick enough to provide adequate external and internal stability for the wall. When the lateral thrust load on the wall is large, a shear key may be added to stabilize the wall against sliding.

Both external and internal stabilities of the wall shall be achieved using trial sections of the wall until satisfactory proportions are obtained. The thickness of the stem and footing must be sufficient to resist the factored shears and factored moments due to earth thrust. The footing of the wall must be wide enough to provide adequate stability against sliding, overturning and bearing failure.

As a result of excessive earth thrust, it is assumed that the wall may experience horizontal displacement as well as rotational movement about the toe of the wall. As a result of rotational movement, the soil pressure increases under the toe and reduces under the footing, as shown in Figure 3-6 through Figure

3-8. The line **ci** in Figure 3-6 is drawn parallel to the average ground surface to calculate the earth thrust (i.e., area **fij** in Figure 3-6) acting on the stem. The passive earth pressure *Pp* acts against the vertical face of the toe; but unless otherwise specified, the passive force should be neglected.

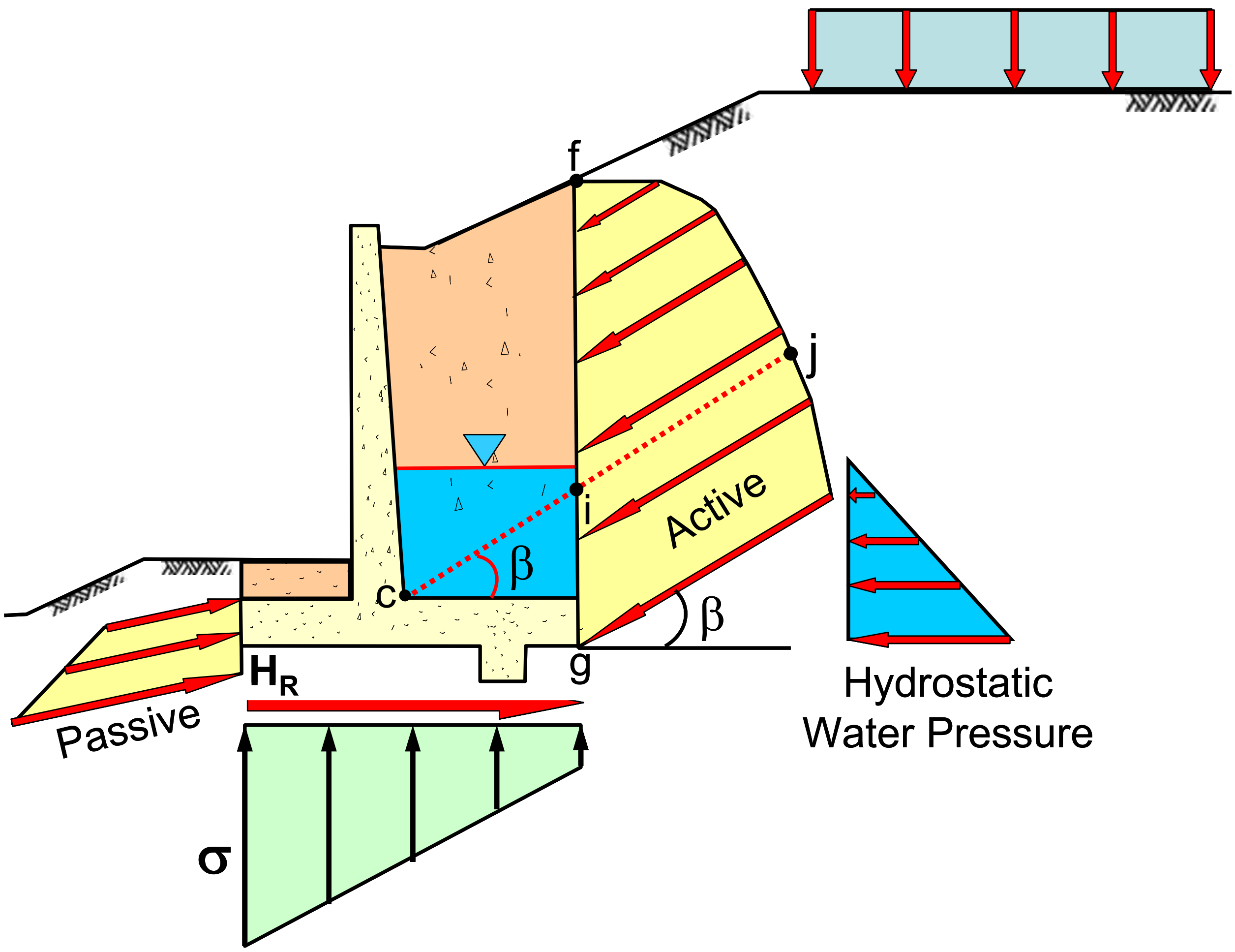


Figure 3-6: Generalized Lateral and Bearing Pressure Distribution

The sum total of the vertical loads (N) and the sum total of the horizontal resistance forces (H_R) act at the base of the wall to provide stability against sliding and overturning (see Figure 3-7). It is a good practice to design the footing such that the resultant force N is located within the middle third ($e \leq {}^{B}/_{6}$, cf. Figure 3-7) of the base. If N falls outside the middle third ($e > {}^{B}/_{6}$, case 3 in Figure 3-7) of the base, the heel width is not in full contact with the soil underneath the footing and a larger pressure distribution may result in a much larger settlement of the toe than the heel, with a corresponding rotation of the wall.

The resistance due to passive lateral earth pressure in front of the wall shall be neglected unless the wall extends well below the depth of frost penetration, scour or other types of disturbance. Development of passive lateral earth pressure in the soil in front of a rigid wall requires an outward rotation of the wall about its toe or other movement of the wall into the soil. The magnitude of movement required to mobilize passive pressure is a function of the soil type and condition in front of the wall.

When groundwater levels may exist above the bottom of wall footing elevation, consideration shall be given to the installation of a drainage blanket and piping at the wall excavation face to intercept the groundwater before it saturates the wall backfill. In general, all wall designs should allow for the thorough drainage of the back-filling material.

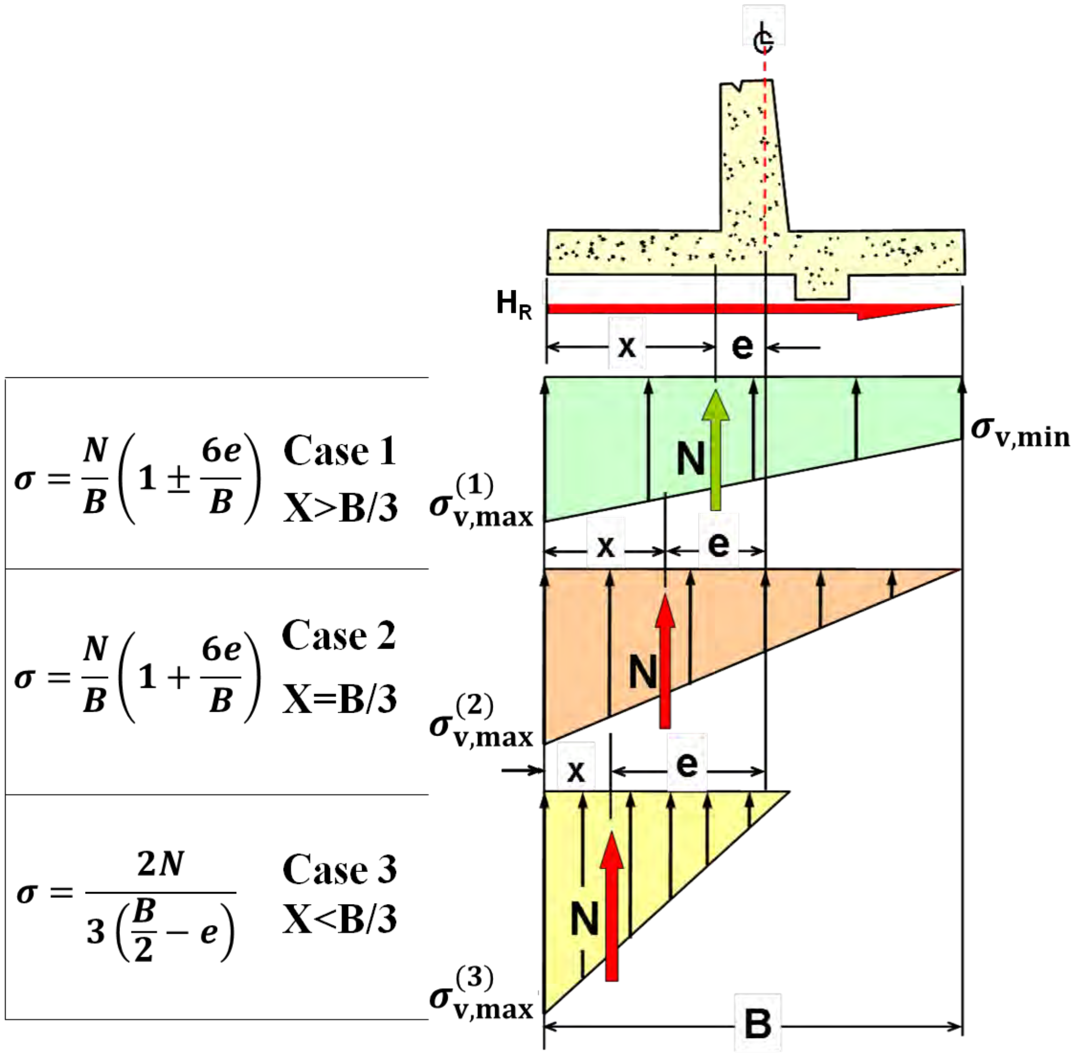


Figure 3-7: Bearing Pressures for Different Locations of Resultant Forces

3.3. STRUCTURAL DESIGN OF SEMI-GRAVITY CANTILEVER WALLS SUPPORTED ON SPREADING FOOTINGS

The lateral thrust load on the stem can cause the stem to bend away from the backfill, creating a tensile force on the back face of the stem and compressive forces in the front of the stem. The exaggerated deformed shape of the wall as a result of thrust loads is shown in Figure 3-8. The heel of the footing supports a large amount of backfill weight and vertical components of the thrust loads, while the soil bearing pressure acting at the bottom of the heel is very small. This will cause the footing to bend concave downward, creating tensile stresses at the top surface and compressive stresses at the bottom surface of the heel. The bearing pressure at the bottom surface of the toe is relatively large compared to the weight of the soil resting at the top surface of the toe. This causes the toe to bend concave upward, creating tensile stresses at the bottom surface and compressive stresses at the top surface of the toe.

Since the concrete has low tensile strength, reinforcement is required on the tension sides of all of the wall components. The flexural reinforcement is required at the backfill face of the stem, top of the heel and bottom face of the toe. It is assumed that all structural members, including the stem, the heel and the toe, behave like cantilever beams with fixed ends located at sections ac, ab and cd, as shown in Figure 3-8.

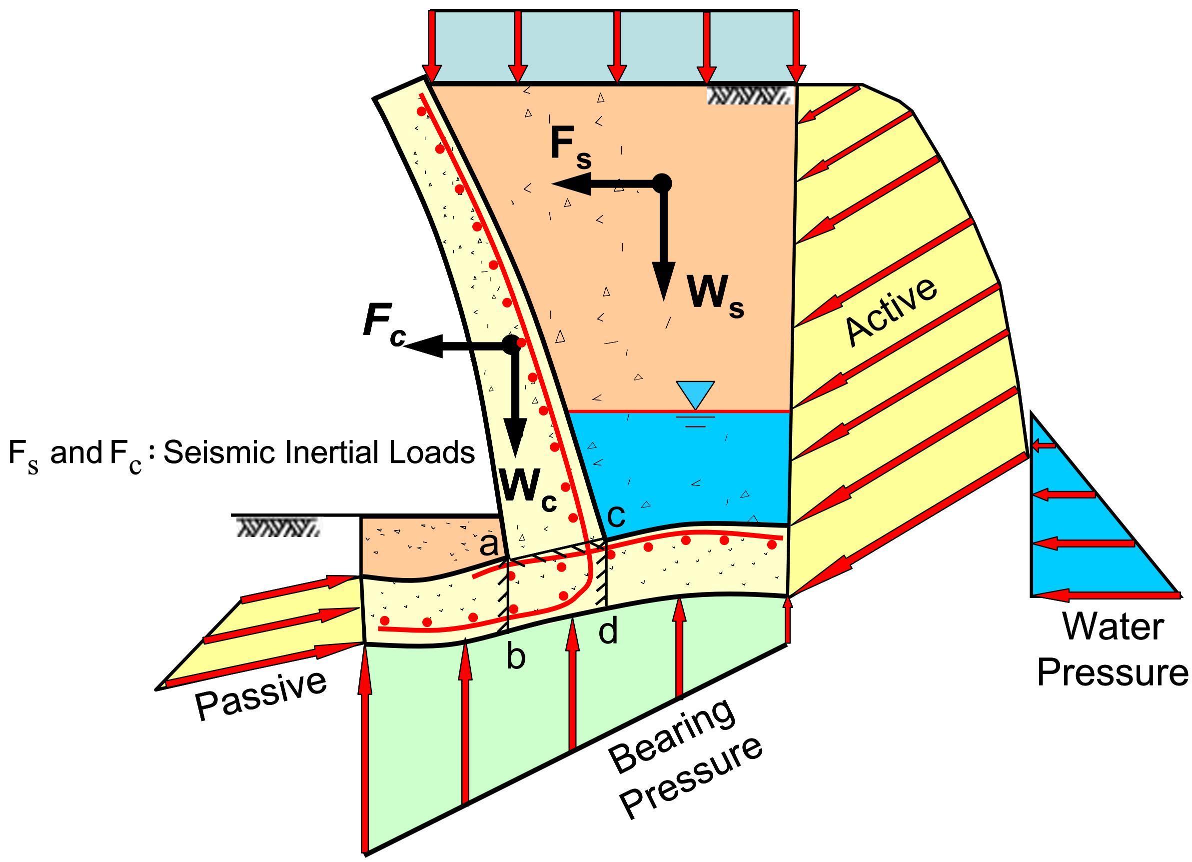


Figure 3-8: Combined Earth Thrusts and Deformed Shape of Cantilever Wall

The thickness and the required reinforcement of the stem, the heel and the toe is controlled by the induced shear and moment from the lateral thrust loads, the backfill vertical weight and vertical component of the lateral thrust loads, and the bearing pressure.

3.4. GRAVITY RETAINING WALL ON PILE FOUNDATION

Retaining structures are commonly constructed on pile foundations when the soil for a considerable depth is too week or compressible to provide adequate support for the wall as shown in Figure 3-9.

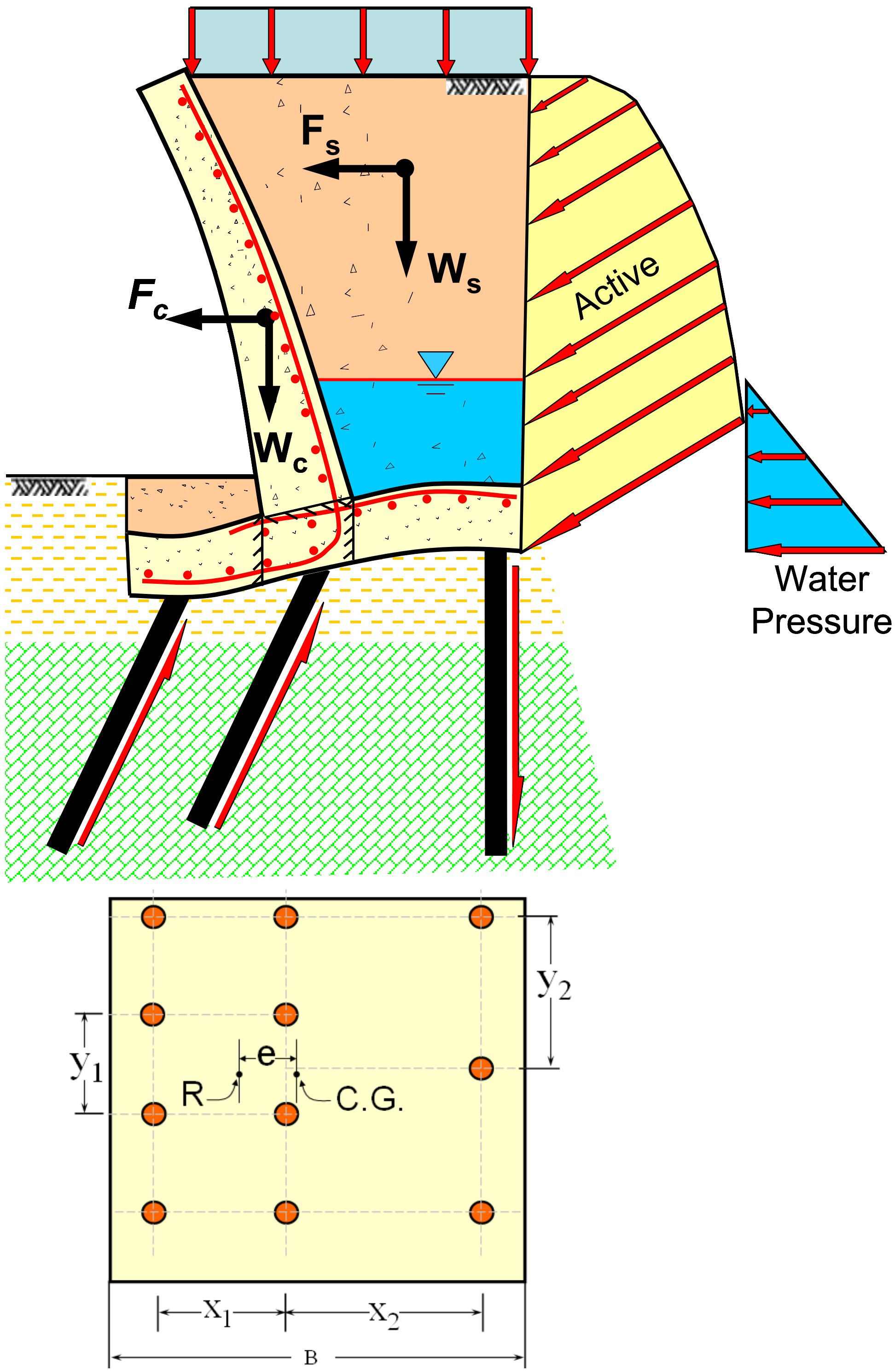


Figure 3-9: Pile Supported Retaining Wall

The piles in the front row are battered to resist the horizontal components of the lateral earth pressure. The pile layout should be arranged such that the center of gravity (C.G.) of the piles lies at the location to the right of the resultant forces R, as shown in Figure 3-9.

3.5. LOAD RESISTANCE FACTOR DESIGN (LRFD) FOR EARTH RETAINING SYSTEMS

Earth retaining systems must be designed to satisfy both ultimate limit states and serviceability limit states. Ultimate limit states are associated with the sliding (SL), overturning (EC), bearing capacity (BC) and structural failures. Serviceability limit states are related to excessive wall deformation and settlement.

Per AASHTO LRFD Specifications (2010), there are three distinct limit states for the design of earth retaining systems: (1) Service I Limit State, (2) Strength I Limit State, and (3) Extreme Event I Limit State. Earth retaining systems shall be designed to satisfy all three limit states. The LRFD factors for various types of permanent and transient loads and forces as illustrated in Figure 3-10 is listed in Table 3-1. In the evaluation for the Strength I Limit State, the load factors for permanent loads should consider both the minimum (i.e., Strength I (a)) and maximum (i.e., Strength I (b)) cases.

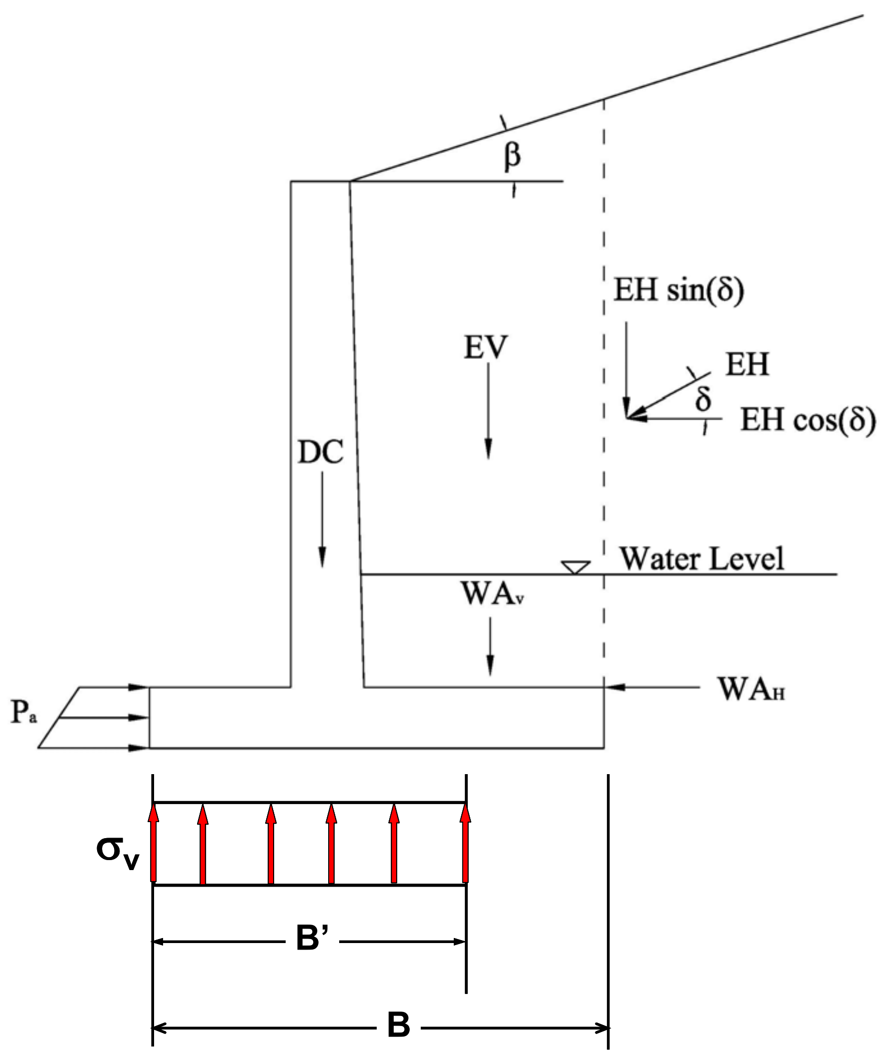


Figure 3-10: LRFD Factors for Rigid Retaining Walls

Table 3-1: LRFD Factors for Rigid Retaining Walls

Limit State	DC	EV	LS_v	LS_h	EH	Probable USE
Service I	1.00	1.00	1.00	1.00	1.00	Settlement
Strength I (a)	0.90	1.00	1.75	1.75	=1.50	BC/EC/SL
Strength I (b)	1.25	1.35	1.75	1.75	1.50	BC (max value)
Extreme Event I	1.00	1.00	0.00	0.00	1.00	BC/EC/SL

DC: Dead load of Concrete

EV: Vertical Pressure from Dead load of Earth Backfill

LS_v: Live Load Surcharge (Vertical Component)

LS_h: Live Load Surcharge (Horizontal Component)

EH: Active Earth Pressure

3.6. DESIGN STEPS FOR GRAVITY AND SEMI-GRAVITY RETAINING WALLS

Steps for solving gravity retaining walls on spread footings with shear key or on pile foundations are as follows:

1. Select an appropriate earth pressure theory to develop unfactored static and seismic load including water, surcharge, and compaction.
2. Apply appropriate load factors to develop loading groups for the four limit states.
3. Evaluate lateral sliding.
4. Evaluate excessive loss of base contact (i.e., eccentricity failure).
5. Evaluate bearing resistance failure.
6. Evaluate structural integrity of the wall.
7. Evaluate maximum wall displacement and settlement.

Lateral Sliding

Sliding of the wall is due to excessive horizontal driving forces. The driving forces in a sliding evaluation generally include factored horizontal loads due to earth pressure, surcharge load, hydrostatic water pressure and incremental seismic load. The factored resistance forces include shear resistance at the base of the wall and the factored passive resistance in front of the wall. The shear resistance capacity at the base of the pile-supported retaining walls is provided by the lateral capacity of the pile foundation while that of walls on spread footings is provided by the shear resistance between the foundation base and foundation soil.

Per AASHTO 2010 Section 10.6.3.4, the factored resistance of walls on spread footings against sliding can be written as:

$$R_R = \varphi\, R_n = \varphi_\tau\, R_\tau + \varphi_{ep}\, R_{ep} \qquad (99)$$

where R_n = nominal sliding resistance against sliding failure

R_τ = nominal sliding resistance between soil and foundation

R_{ep} = nominal passive resistance of soil available throughout the design life of the structure

φ_τ = resistance factor for shear resistance between soil and foundation specified in Table 3-2

φ_{ep} = resistance factor for passive resistance specified in Table 3-2

If the soil beneath the footing is cohesionless, the nominal sliding resistance between soil and foundation is given as:

$$R_\tau = N\ \tan\delta \qquad \textbf{(100)}$$

where $\tan\delta = \tan\phi_f$ (101)

For cohesive soil refer to the procedure in AASHTO (2010) Section 10.6.3.4.

Table 3-2 Table Resistance Factors for Geotechnical Resistance of Shallow Foundations

	Soil Condition	Resistance Factor
φ□	Precast concrete placed on sand	0.90
	Cast-in-Place Concrete on sand	0.80
	Cast-in-Place or precast Concrete on Clay	0.85
	Soil on soil	0.90
φ_{ep}	Passive earth pressure component of sliding resistance	0.50

Eccentricity Failure

Eccentricity or the wall rotation is a concern particularly for tall walls with narrow footings. Excessive lateral loads will cause induced tilting and/or rotation of the wall during a seismic event. The rotational stability of the wall is evaluated by comparing the factored moment, M_D, tending to rotate the earth retaining system to the factored moment, M_R, tending to resist the wall rotation. The rotation of the wall is a function of the driving force and its line of action. When checking the wall eccentricity, the location of the resultant force at the base of the footing shall be computed. The location of the resultant force, N, should be within B/4 of the foundation centroid for foundations on soil, and within 3B/8 of the foundation centroid for foundations on rock. Eqn. 102 may be use to calculated wall eccentricity.

$$e = \frac{B}{2} - \frac{M_R - M_D}{N} \qquad \textbf{(102)}$$

Bearing Resistance Failure

The bearing capacity of the foundation material supporting the wall footing with respect to the induced bearing pressure is evaluated by comparing the resultant vertical forces at the base of the wall to the allowable foundation material bearing capacity.

Generalized bearing pressure distribution for the wall base resting on the foundation material is shown in Figure 3-8. The procedure for evaluating bearing resistance is given in AASHTO (2010) Section 10.6.3.1 and 10.6.3.2. For walls on soil foundations, the vertical stress is calculated assuming a uniform distribution of pressure over an effective base width, B':

$$\sigma_v = \frac{N}{B'} \tag{103}$$

where

$$B' = B - 2e. \tag{104}$$

If e is computed to be less than zero, then assume e is zero.

For walls founded on rock, the vertical stress shall be calculated assuming a linearly distributed pressure over an effective base area as was illustrated in Figure 3-7. If the resultant vertical force is within the middle one-third of the wall base:

$$\sigma_{v,\max} = \frac{N}{B}\left(1+\frac{6e}{B}\right), \tag{105}$$

$$\sigma_{v,\min} = \frac{N}{B}\left(1-\frac{6e}{B}\right). \tag{106}$$

If the resultant is outside the middle one-third of the wall base, then:

$$\sigma_{v,\max} = \frac{2N}{3\left(\frac{B}{2}-e\right)}, \tag{107}$$

$$\sigma_{v,\min} = 0. \tag{108}$$

Structural Integrity

For the structural design of the wall, all of the LRFD load combinations should be considered. The structural design of the wall should compare the factored shear forces and factored bending moments computed at the critical sections of the wall. The critical sections for a cast-in-place concrete wall are shown in Figure 3-11. In RC design, the design shear and design moment are allowed to be taken at the cross section located at a distance (equal to the element's effective depth) from the critical section of the structural component. To be conservative, in the design examples provided in §3.7, all the structural components will be designed for the shear and moment demands estimated right at the critical sections (i.e., sections ab, ac, and cd in Figure 3-11).

Wall Displacement

Per AASHTO 10.5.2.2, foundation movement criteria should be consistent with the function and type of structure, anticipated service life, and consequences of unacceptable movements on structure performance. Foundation movement shall include vertical, horizontal, and rotational movements. The tolerable movement criteria shall be established by either empirical procedures or structural analyses, or by consideration of both.

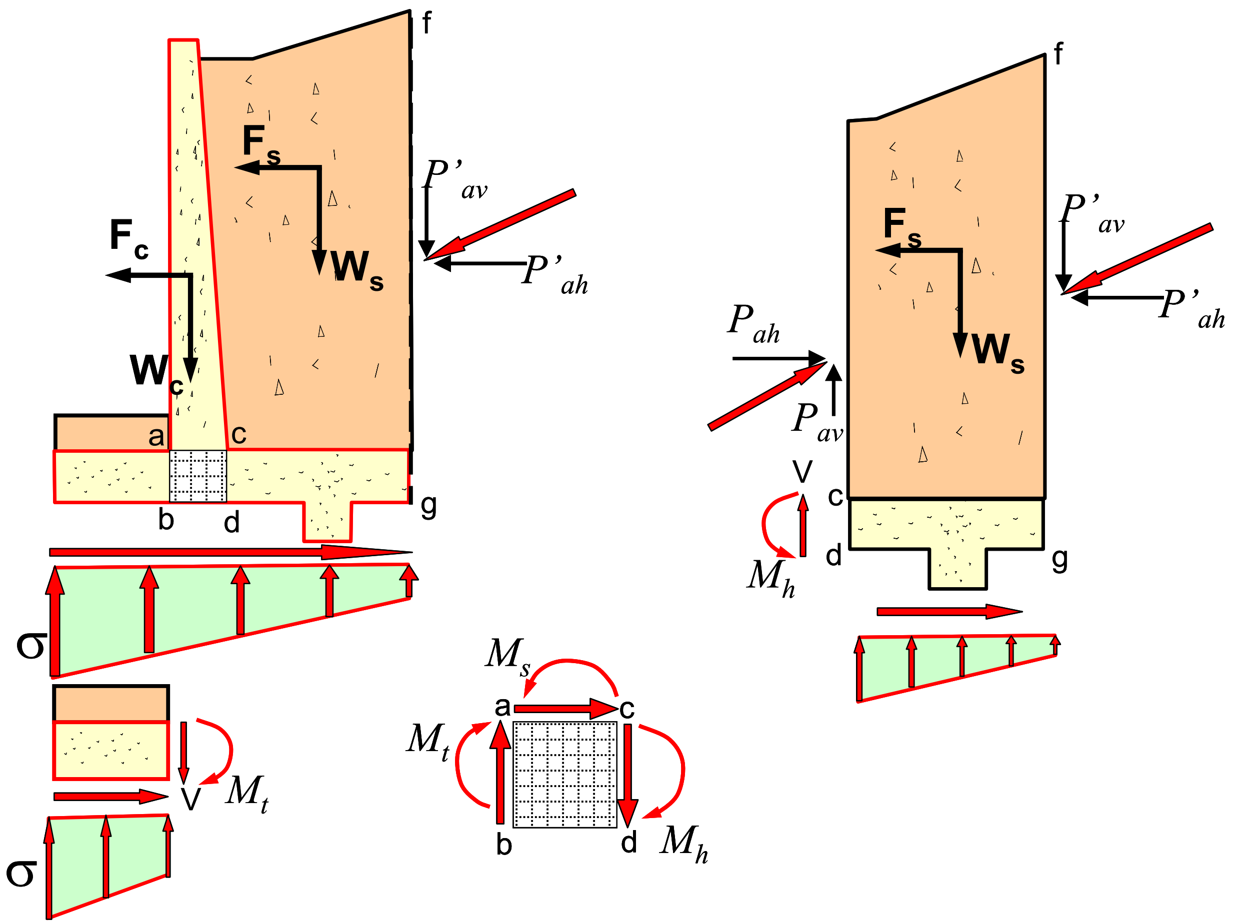


Figure 3-11: Forces Acting at the Critical Sections of the Wall

3.7. DESIGN EXAMPLES OF SEMI-GRAVITY CANTILEVER WALLS

3.7.1 Example 3-1: Cantilever with Toe

A cantilever reinforced cast-in-place concrete wall is to be constructed at the toe of the slope shown in Figure 3-12. The required height of the retaining wall is estimated 26 feet with 1 foot extended above the backfill. A 2-foot wide drainage ditch is required at the toe of the slope backfill for the surface water rub off. The slope angle of the backfill material in contact with the in-situ rock slope is approximately 56.0 degrees from the horizontal. The soils report recommends a spread footing with a maximum allowable bearing capacity of 6.5 ksf. There is no surcharge load behind the wall. The pervious wall backfill will be silty fine sand (USCS classification: SM) compacted to 95% relative compaction with a moist unit weight of 120 pcf, a friction angle of 34 degrees, and a cohesive strength of 300 psf. The maximum tolerable lateral wall yield displacement is 4” to preclude wall damage. The backfill will be drained using weepholes, which will not allow hydrostatic pressure behind the wall to develop. The horizontal seismic acceleration coefficient of 0.20g should be used for the Extreme Event limit state. Use compressive strength of concrete as 4.0 ksi and steel flexural strength of 60 ksi.

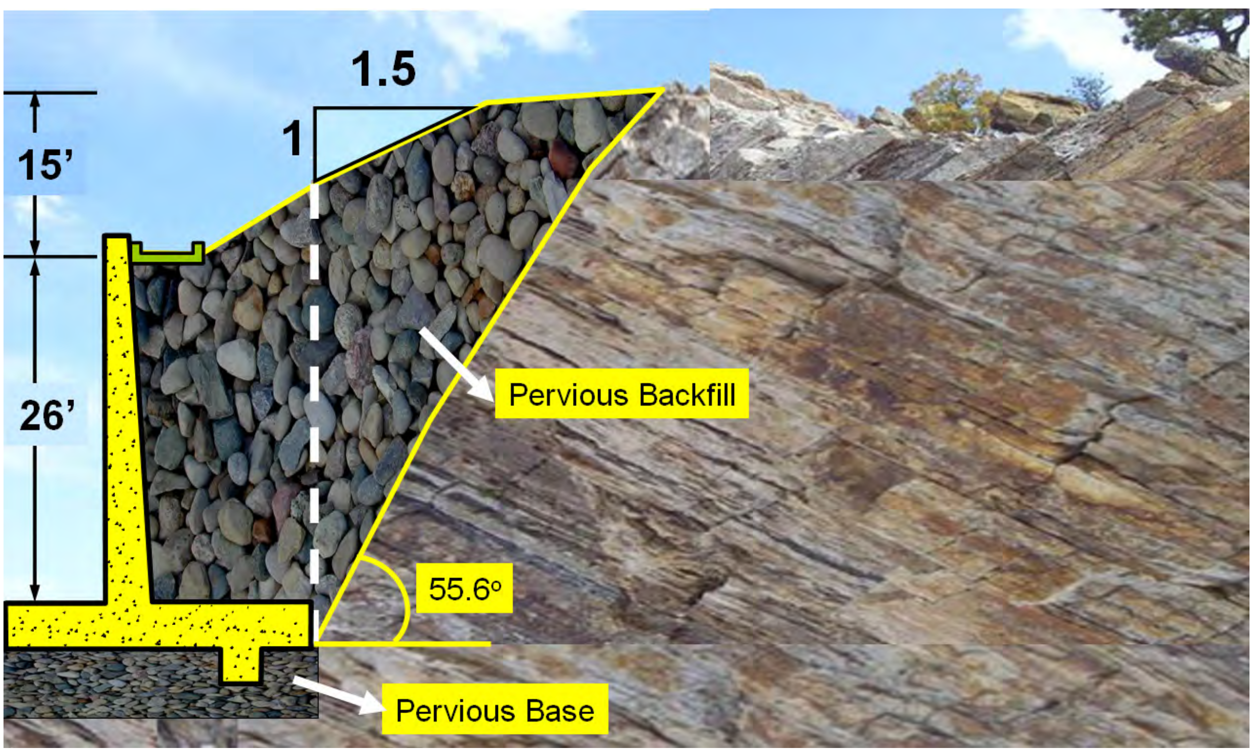


Figure 3-12: Cantilever Wall with Toe

Solution Procedure:

- Use Trial Wedge method (Eqn. 61) to calculate the active earth pressure due to backfill zone.
- Calculate LRFD load combination.
- Check sliding failure

- Check eccentricity failure
- Check bearing resistance failure
- Perform structural design for the stem, toe, and heel of wall
- Calculate wall displacement

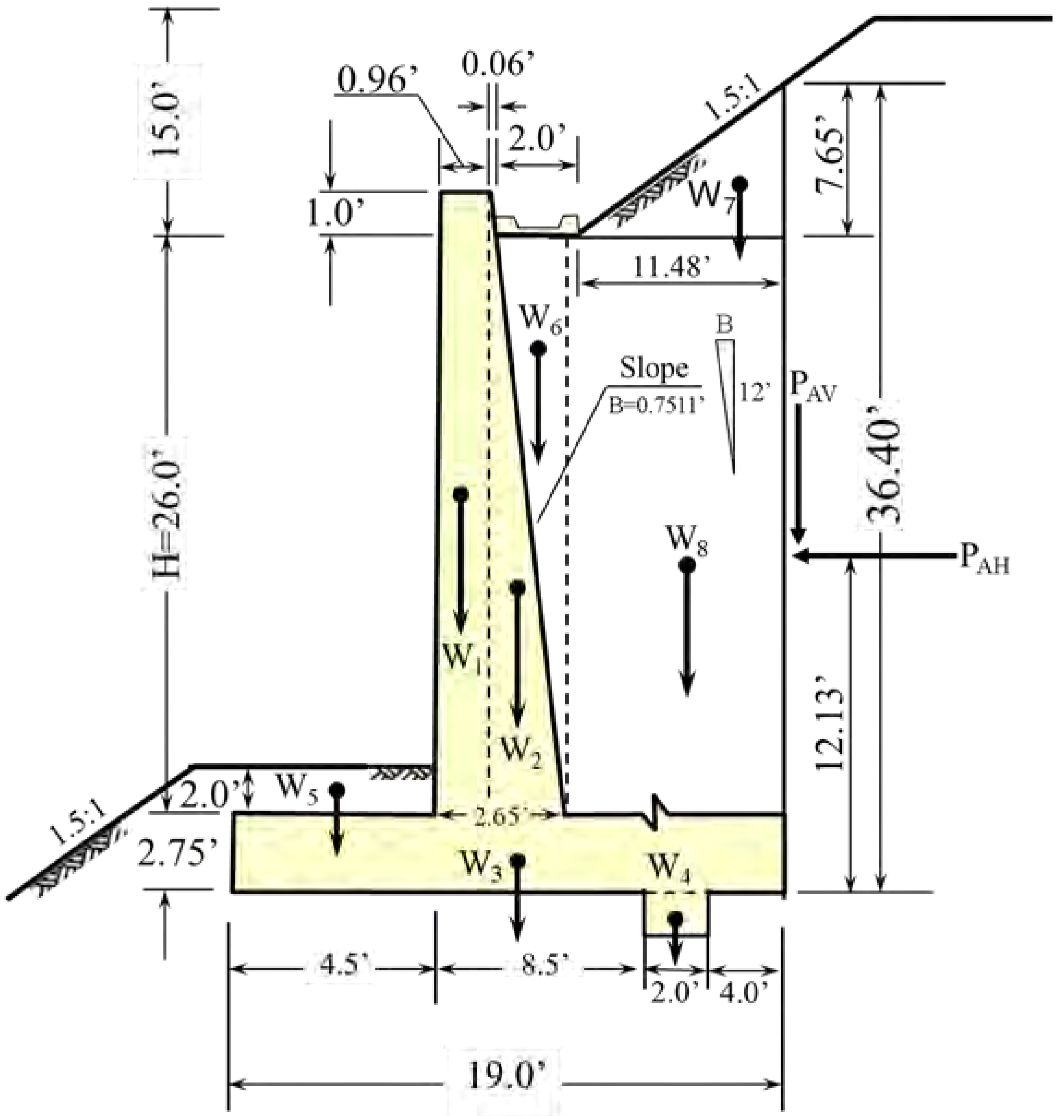


Figure 3-13: Forces Acting on the Wall

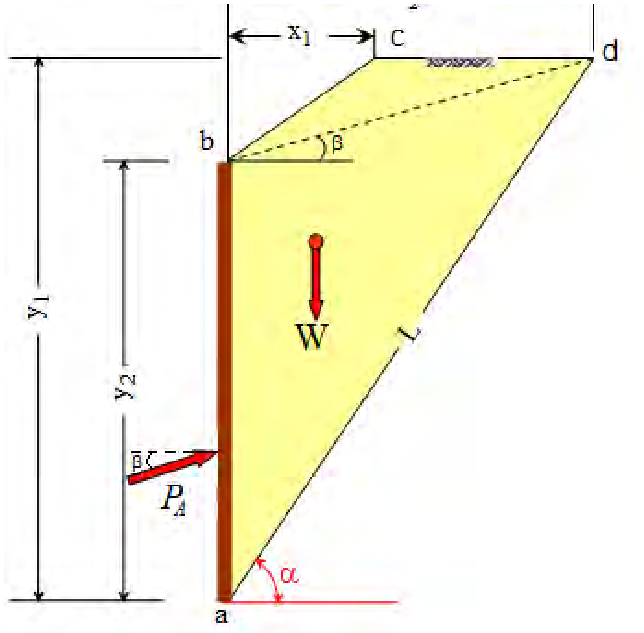


Figure 3-14: Active Wedge for Static Case

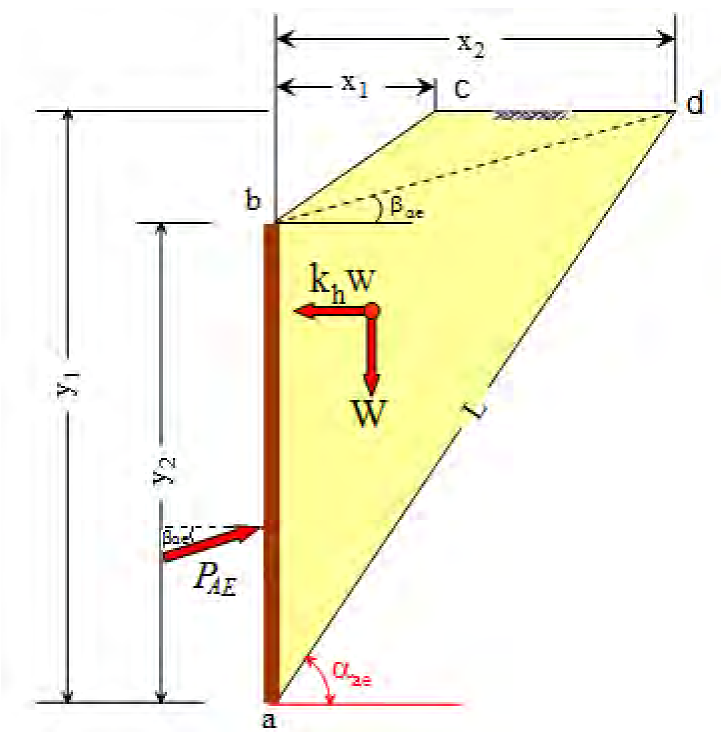


Figure 3-15: Active Wedge for Seismic Case

Step 1: Calculate active earth force due to backfill zone

The failure surface in the backfill in this example is prescribed (see Figure 3-12), so the inclined angle of the failure surface under both the static and seismic cases is:

$$\alpha = \alpha_{ae} = 56°$$

The geometry of the active wedge as shown in Figure 3-14 and Figure 3-15 can be determined as follows:

$$y_1 = 43.75\,\text{ft}$$
$$y_2 = 36.40\,\text{ft}$$
$$x_1 = 11.02\,\text{ft}$$
$$x_2 = 59.51\,\text{ft}$$
$$L = 52.77\,\text{ft}$$

$$\text{Area} = \frac{y_1(x_2 - x_1) + y_2 x_1}{2} = \frac{43.75(29.51 - 11.02) + 36.4(11.02)}{2} = 605.03\,\text{ft}^2/\text{ft}$$
$$W = 605.03(0.12) = 72.60\,\text{kips/ft}$$

The friction angle at the vertical face, ab, is assumed to be parallel to the average ground surface:

$$\delta = \beta = \beta_{ae} = 13.98°$$

Compute the unfactored horizontal static active earth force due to soil wedge behind the wall using Eqn. 61.

$$P_{AH} = \frac{W\left[\tan(\alpha - \phi)\right] - cL\left[\sin\alpha \tan(\alpha - \phi) + \cos\alpha\right]}{1 + \tan\delta \tan(\alpha - \phi)}$$
$$= \frac{72.60\left[\tan(56° - 34°)\right] - 0.3(52.77)\left[\sin(56°)\tan(56° - 34°) + \cos(56°)\right]}{1 + \tan(13.98°)\tan(56° - 34°)} = 13.792$$

$$P_{AV} = 13.792\tan(13.98°) = 3.434$$

$$P_A = \sqrt{P_{AH}^2 + P_{AV}^2} = 14.21\text{ kips/ft}$$

Compute unfactored horizontal seismic active earth force due to soil wedge behind the wall.

$$P_{AEH} = \frac{W\left[\tan(\alpha - \phi) + k_h\right] - cL\left[\sin\alpha \tan(\alpha - \phi) + \cos\alpha\right]}{1 + \tan\delta \tan(\alpha - \phi)}$$

$$= \frac{72.60\left[\tan(56° - 34°) + 0.2\right] - 0.3(52.77)\left[\sin(56°)\tan(56° - 34°) + \cos(56°)\right]}{1 + \tan(13.98°)\tan(56° - 34°)} = 26.986$$

$$P_{AEV} = 26.986\tan(13.98°) = 6.718$$

$$P_{AE} = \sqrt{P_{AEH}^2 + P_{AEV}^2} = 27.81\text{ kips/ft}$$

Compute the unfactored stabilizing moments due to vertical loads, as listed in Table 3-3.

Table 3-3: Unfactored Stabilizing Moments due to Vertical Loads

Item	Vertical Load, N (k/ft)	Moment Arm (ft)	Moment About Toe, M_{RES} (k-ft/ft)
W_1	0.96 (27) (0.15) = 3.888	4.980	19.362
W_2	0.5 (1.69) (27) (0.15) = 3.422	6.023	20.613
W_3	(2.75)(19) (0.15) = 7.837	9.500	74.456
W_4	2 (1) (0.15) = 0.300	14.000	4.200
W_5	2 (4.5) (.12) = 1.080	2.250	2.430
W_6	0.5 (1.627) (26) (0.12) = 2.539	6.608	16.775
W_7	0.5(7.65)(11.48)(0.12) = 5.269	15.174	79.956
W_8	11.85 (26) (0.12) = 36.972	13.075	483.409
P_{AV}	14.21 sin(13.98) = 3.433	19.000	65.225

$$\sum N = 64.7 \text{ kips/ft}, \quad \sum M_{RES} = 766.4 \text{ kips-ft / ft}$$

Compute the unfactored overturning moments due to horizontal loads, as listed in Table 3-4

Table 3-4: Unfactored Overturning Moments due to Horizontal Loads

Item	Horizontal (k/ft)	Moment Arm (ft)	Moment @ Toe (k-ft/ft)
P_{AH}	14.21 cos (13.98) = 13.789	12.134	167.315
Extreme Event Values			
P_{AEH}	27.81 cos (13.98) =26.986	12.134	327.448
$W_c K_h$	(3.888+3.422+7.837+0.300) (0.2) = 3.090	7.381	22.803
$W_s K_h$	(2.539+5.269+36.972) (0.2) = 8.956	17.826	159.645

$$\overline{Y_c} = \frac{W_1\overline{Y_1} + W_2\overline{Y_2} + W_3\overline{Y_3} + W_4\overline{Y_4}}{W_1 + W_2 + W_3 + W_4} = \frac{3.888(16.25) + 3.422(11.75) + 7.837(1.375) + 0.300(-0.5)}{3.888 + 3.422 + 7.837 + 0.300} = 7.381 \text{ ft}$$

$$\overline{Y}_s = \frac{W_6\overline{Y}_6 + W_7\overline{Y}_7 + W_8\overline{Y}_8}{W_6 + W_7 + W_8} = \frac{2.543(20.083) + 5.269(31.300) + 36.972(15.750)}{2.543 + 5.269 + 36.972} = 17.826 \text{ ft}$$

Next, the unfactored loads and moments tabulated in Table 3-3 and Table 3-4 will be multiplied by their associated LRFD Factors to determine the total factored loads for all the limit states considered.

Step 2: Calculate the LRFD load combinations.

The design of the wall shall be checked for all possible LRFD load combinations. The table below shows the LRFD Factors for all the relevant limit states considered in the retaining wall design.

Limit State	DC	EV	LS_v	LS_h	EH	Probable USE
Service I	1.00	1.00	1.00	1.00	1.00	Settlement
Strength I (a)	0.90	1.00	1.75	1.75	1.50	BC/EC/SL
Strength I (b)	1.25	1.35	1.75	1.75	1.50	BC (max value)
Extreme Event I	1.00	1.00	0.00	0.00	1.00	BC/EC/SL

DC: Dead load of Concrete

EV: Vertical Pressure from Dead load of Earth Backfill

LS_v: Live Load Surcharge (Vertical Component)

LS_h: Live Load Surcharge (Horizontal Component)

EH: Horizontal Earth Pressure

Compute the factored vertical loads and stabilizing moments for all limit states.

Table 3-5: Factored Vertical Loads

Group	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	P_{AV}	P_{AEV}	Total
Service	3.888	3.422	7.837	0.300	1.080	2.539	5.269	36.972	3.433	0.000	64.741
Strength I (a)	3.499	3.080	7.054	0.270	1.080	2.539	5.269	36.972	5.149	0.000	64.912
Strength I (b)	4.860	4.278	9.797	0.375	1.458	3.427	7.114	49.912	5.149	0.000	86.370
Extreme Event	3.888	3.422	7.837	0.300	1.080	2.539	5.269	36.972	0.000	6.718	68.026

Table 3-6: Factored Stabilizing Moments from Vertical Loads

Group	M_{W1}	M_{W2}	M_{W3}	M_{W4}	M_{W5}	M_{W6}	M_{W7}	M_{W8}	M_{PAV}	M_{PAEV}	Total
Service	19.362	20.613	74.456	4.200	2.430	16.775	79.956	483.409	65.225	0.000	766.427
Str. I (a)	17.426	18.552	67.011	3.780	2.430	16.775	79.956	483.409	97.838	0.000	787.176
Str. I (b)	24.203	25.767	93.070	5.250	3.281	22.648	107.941	652.602	97.838	0.000	1032.597
Extreme Event	19.362	20.613	74.456	4.200	2.430	16.775	79.956	483.409	0.000	127.650	828.852

Compute the factored horizontal loads and overturning moments for all limit states.

Table 3-7: Factored Horizontal Loads

Group	Service	Strength I (a)	Strength I (b)	Extreme Event
Total	13.789	20.684	20.684	39.032

Table 3-8: Factored Overturning Moments from Horizontal Loads

Group	Service	Strength I (a)	Strength I (b)	Extreme Event
Total	167.315	250.973	250.973	509.897

Step 3: Check sliding failure. (Only the Service Limit State is checked here as an example.)

Error! Objects cannot be created from editing field codes.

$$F_r = \varphi_t N \mu$$

$$\varphi_t = 0.85 \quad \text{(AASHTO Eq. 10.5.5.2.2, for static cases)}$$

$$\varphi_t = 1.0 \quad \text{(AASHTO Eq. 10.5.5.3.3, for seismic cases)}$$

$$\mu = 0.65 \quad \text{(Geotechnical Recommendation)}$$

$$\text{Sliding Capacity Demand Ratio } (CDR) = \frac{0.85(64.745)(0.65)}{13.789} = \frac{35.769}{13.789} = 2.594$$

Table 3-9: Stability analysis for sliding failure

Group	F_{DH}	F_{RH}	CDR
Service	13.789	35.769	2.594
Strength I (a)	20.684	35.864	1.734
Strength I (b)	20.684	47.719	2.307
Extreme Event	39.032	44.216	1.133

Step 4: Check eccentricity failure.

Calculate eccentricity (e) as shown in Table 3-10.

$$X = \frac{M_V - M_H}{N} = \frac{766.448 - 167.308}{64.741} = 9.254\,\text{ft}$$

$$e = \frac{B}{2} - X = \frac{19}{2} - 9.254 = 0.246\,\text{ft}$$

$$e_{\max} = \frac{B}{4} = \frac{19}{4} = 4.75\,\text{ft}$$

$$\text{ecc. ratio} = \frac{e}{e_{\max}} = \frac{0.246}{4.75} = 0.052 < 1.0 \ \text{(O.K.)}$$

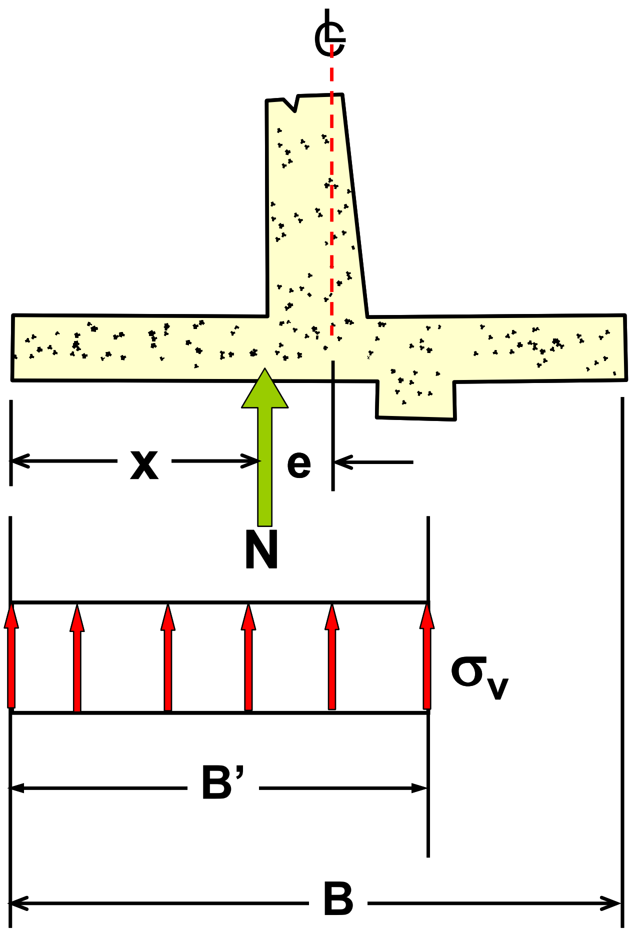


Figure 3-16: Free Body Diagram to Calculate Eccentricity

Step 5: Check bearing resistance failure.

Calculate Bearing Pressure σ_v, as shown in Table 3-10.

$$B' = B - 2e = 19 - 2 \times 0.246 = 18.508$$

$$\sigma_v = \frac{N}{B-2e} = \frac{N}{B'} = \frac{64.741}{18.508} = 3.498 \text{ ksf}$$

$$\text{Bearing Capacity / Demand Ratio } (CDR) = \frac{q_{all}}{\sigma_v} = \frac{6.5}{3.498} = 1.858 > 1.0 \quad \text{(O.K.)}$$

$$q_{all} = 6.5 \quad \text{(Geotechnical Recommendation)}$$

The bearing capacity recommended by geotechnical engineers is smaller than the bearing pressure in the Extreme Event Limit State. Use of a larger spread footing may be required.

Table 3-10: Stability Analysis for Eccentricity Failure and Bearing Resistance Failure

Group	N (k/ft)	B' (ft)	Bearing Pressure, σ_v (ksf)	CDR	e (ft)	e_{max} (ft)	ecc. Ratio
Service	64.741	18.508	3.498	1.858	0.246	4.750	0.052
Strength I (a)	64.916	16.521	3.929	1.654	1.240	4.750	0.261
Strength I (b)	86.370	18.099	4.772	1.362	0.450	4.750	0.095
Extreme Event	68.026	9.377	7.254	0.896	4.811	6.333	0.760

Determine Bearing Capacities for Structural Design

Per AASHTO Section 10.6.5, the structural design of eccentrically loaded foundation will use triangular or trapezoidal contact stress distributions. To determine σ_{toe} and σ_{heel} defined in Figure 3-18, Eqns. 105 to 108 and the stress diagram shown in Figure 3-7 need to be used (calculation for the Service Limit State is shown here as an example).

$$\sigma_{toe} = \frac{N}{B}\left(1+\frac{6e}{B}\right) = \frac{64.741}{19}\left(1+\frac{6\times 0.246}{19}\right) = 3.672$$

$$\sigma_{heel} = \frac{N}{B}\left(1-\frac{6e}{B}\right) = \frac{64.741}{19}\left(1-\frac{6\times 0.246}{19}\right) = 3.143$$

In order to determine σ_{toe} and f_1 for Extreme Event Limit State, Eqn. 107 must be applied.

$$\sigma_{toe} = \frac{2N}{3\left(\frac{B}{2}-e\right)} = \frac{2\times 68.03}{3\left(\frac{19}{2}-4.811\right)} = 9.672 \text{ ksf}$$

$$f_1 = \frac{2N}{\sigma_{toe}} = \frac{2\times 68.03}{9.672} = 14.066 \text{ ft}$$

The table below shows σ_{toe} and σ_{heel} for all the Limit States considered.

Table 3-11: Bearing Pressure Distribution for All Limit States

Limit State	σ_{toe}	σ_{heel}	f_1	f_2
Service	3.672	3.143	19.000	0.000
Strength I (a)	4.754	2.079	19.000	0.000
Strength I (b)	5.192	3.899	19.000	0.000
Extreme Event	9.672	0.000*	14.066	4.934

The figure below shows the bearing pressure distribution for all the limit states.

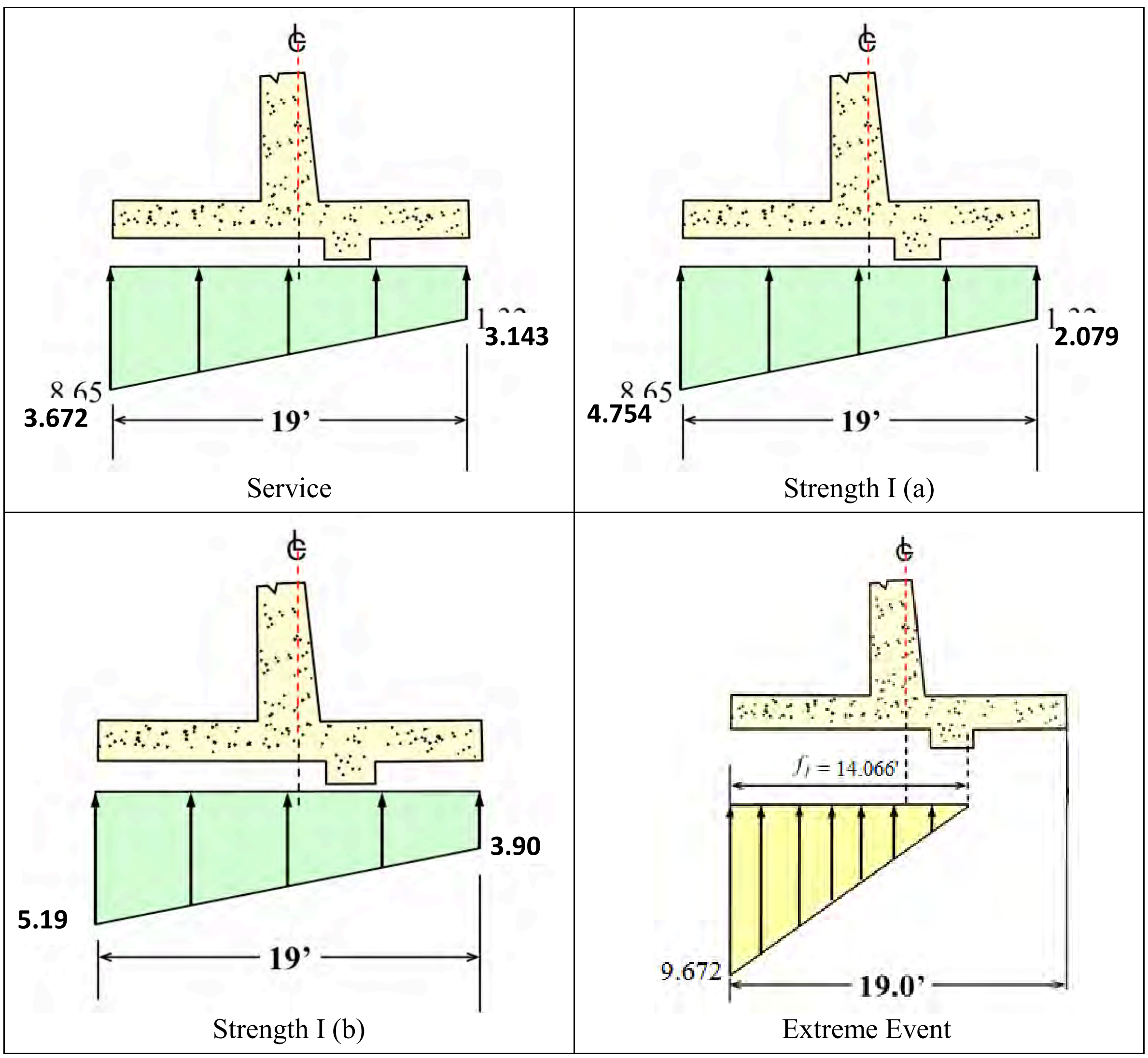
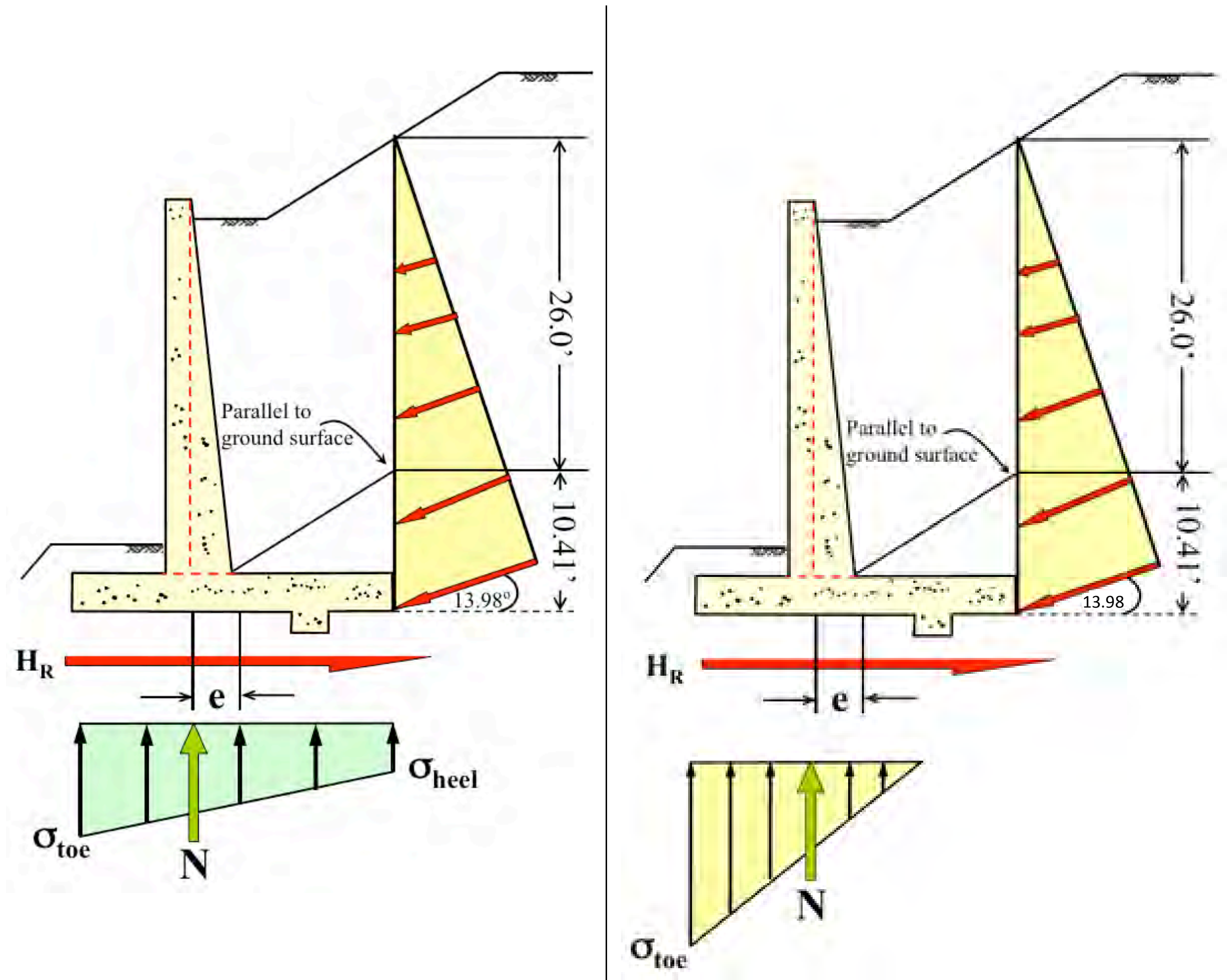


Figure 3-17: Bearing Pressure Distribution for all the Limit States



Free body diagram that will be used for Service, Strength I (a), and Strength I (b) Limit States

Free body diagram that will be used for Extreme Event Limit State

Figure 3-18: Free body diagrams of the Forces Acting on the Wall

Step 6: Structural Design

General Procedures:

(i) Draw the free body diagrams for various structural components of the wall.

(ii) Compute the unfactored stabilizing forces and moments and the overturning forces and moments acting on each component.

(iii) Apply LRFD Factors to obtain the design shear force and bending moment for each component under all limit states.

(iv) Design the flexural and shear reinforcements of all structural components and compute their nominal shear strengths and nominal bending capacities. Repeat this step until the capacities are greater than the demands for all structural components and for all limit states.

Stem design:

The values of shear and moment at the base of the stem due to lateral earth pressure are computed to check the stem thickness and necessary reinforcement. Only Service Limit State calculations are presented below.

(i) Draw the free body diagram.

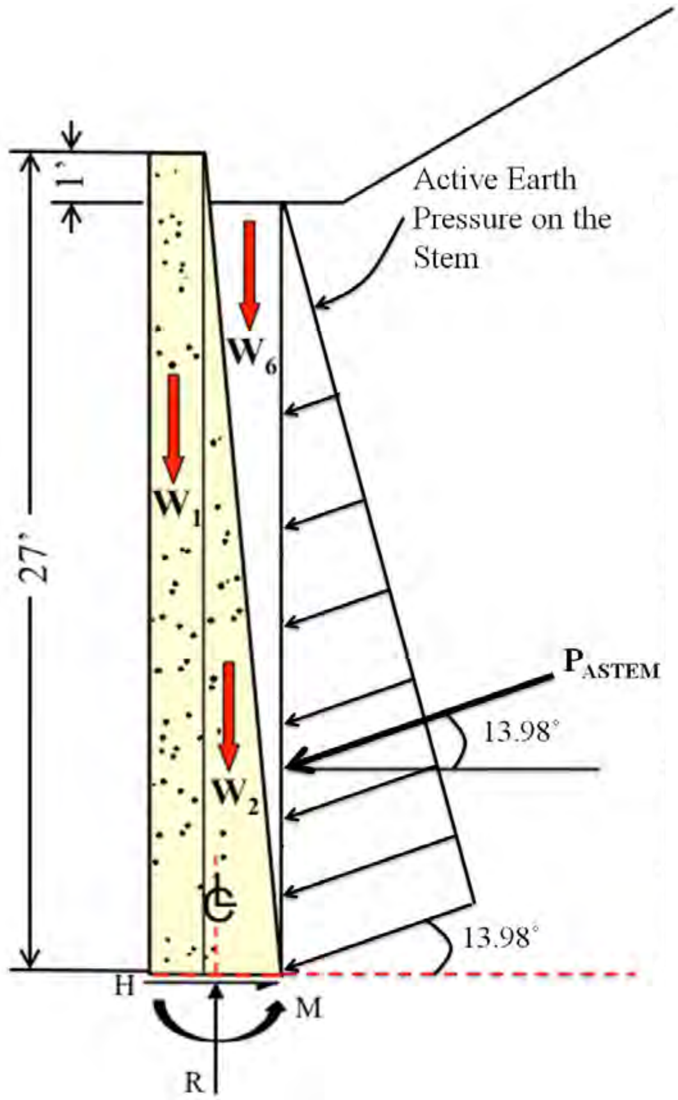


Figure 3-19: Forces Acting on the Stem

(ii) Compute the unfactored stabilizing forces and moments and the overturning forces and moments.

Take the center point at the bottom of stem as the moment center, and calculate the stabilizing moments generated by the vertical loads.

Table 3-12: Unfactored Stabilizing Moments due to Vertical Loads

Force	Vertical Load (kips/ft)	Arm (ft)	Resisting Moment (k-ft/ft)
W_1	0.96 (27) (0.15) = 3.888	0.845	3.285
W_2	0.5 (1.69) (27) (0.15) = 3.422	0.198	−0.679
W_6	0.5 (1.627) (26) (0.12) = 2.539	0.783	−1.987
P_{AV}	7.25 sin (13.98) =1.751	1.325	−2.320

$\Sigma M_{RES} = -1.700$ kips-ft/ft

Calculate the driving force (i.e., horizontal loads) and the overturning moment at the base of the stem. This can be calculated using proportions from the P_A and M determined for the entire height (36.40 ft) of the retaining wall.

$$P_{A_{STEM}} = P_A \frac{h'^2}{H^2} = 14.21 \frac{26^2}{36.40^2} = 7.25 \text{ kips / ft}$$

$$V_u = V_{AH_{STEM}} = P_{A_{STEM}} \cos(13.98) = 7.25\cos(13.98) = 7.035 \text{ kips / ft}$$

$$M_{OT} = P_{AH_{STEM}} \bar{Y} = 7.035\left(\frac{26}{3}\right) = 60.967 \text{ kips - ft / ft}$$

$$M_u = M_{OT} - M_{RES} = 60.967 - 1.700 = 59.267 \text{ kips - ft / ft}$$

Table 3-13: Unfactored Overturning Moments due to Horizontal Loads

Item	Horizontal Load (k/ft)	Moment Arm (ft)	Overturning Moment (k-ft/ft)
$V_{AH_{STEM}}$	7.25 cos(13.98) = 7.035	8.667	60.967
Extreme Event Values			
$V_{AEH_{STEM}}$	14.188 cos(13.98) =13.767	8.667	119.317
$W_c K_h$	(3.888+3.422) (0.2) = 1.462	11.393	16.658
$W_s K_h$	(2.539) (0.2) = 0.508	17.333	8.801

$V_u = \Sigma V_{driv}$ =15.737 $\Sigma M_{OT} = 144.775$ kips-ft/ft

$$M_u = M_{OT} - M_{RES} = 144.775 - 3.921 = 140.854 \text{ kips-ft/ft}$$

(iii) Apply LRFD Factors to obtain the design shear force and bending moment.

Table 3-14: Design Shear Force and Bending Moment for Stem

Group	M_{RES}	M_{OT}	M_u	V_u
Service	1.701	60.967	59.267	7.035
Strength I (a)	3.121	91.450	88.329	10.552
Strength I (b)	2.904	91.450	88.546	10.552
Extreme Event	3.921	144.775	140.854	15.737

(iv) Design the flexural and shear reinforcements.

Assuming #10 at 6 in.

The development length is l_d = 42 in. (For the equations of basic development length of deformed bars and the modification factors associated with various conditions, please refer to AASHTO 5.11.2.)

$$A_s = 2.54 \text{ in}^2$$
$$h = 2.65 \text{ ft} = 31.800 \text{ in}$$
$$d = h - \text{clear cover} - \frac{d_b}{2} = 31.800 - 2 - \frac{1.270}{2} = 29.165 \text{ in}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.54(60{,}000)}{0.85(4{,}000)(12)} = 3.735 \text{ in}$$

$$d_v = d - \frac{a}{2} = 29.165 - \frac{3.735}{2} = 27.297 \text{ in}$$
$$d_v > 0.9 \times d = 0.9 \times 29.165 = 26.248 \text{ in (O.K.)}$$
$$d_v > 0.72 \times h = 0.72 \times 31.800 = 22.896 \text{ in (O.K.)}$$

In order to determine the required area of steel, the design moment, M_u, will be compared to the cracking moment (AASHTO 5.7.3.3.2).

$$M_{cr} = S_c f_r = \frac{2022.484(0.74)}{12} = 124.717 \text{ kips-ft/ft}$$

$$\text{where} \quad S_c \text{ (Section Modulus)} = \frac{1}{6} b h^2 = \frac{1}{6} 12(31.800)^2 = 2022.48$$

$$f_r = 0.37\sqrt{f_c'} \text{ ksi} = 0.74 \text{ ksi (AASHTO 5.4.2.6)}$$

The calculation for Extreme Event Limit State is presented below as an example.

Check the flexural design:

$M_u = 140.854 \text{ kips - ft / ft}$

(a) General requirement on factored flexural resistance (AASHTO 5.7.3.2)
The factored flexural resistance M_r shall be taken as:

$M_r = \varphi M_n$

where: M_n = nominal flexural resistance (kip-in.)

φ = resistance factor as listed in Table 3-15 (AASHTO 5.5.4.2 and AASHTO 11.5.7).

Table 3-15: Resistance Factors for Tension-, Shear-, and Axial-Controlled RC Members

Limit State	φ_f (tension-controlled)	φ_v (shear)	φ_c (bearing on concrete)
Strength	0.9	0.9	0.7
Extreme Event	1.0	1.0	1.0

Check $M_r = \varphi M_n \geq M_u$:

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right) = \frac{1.0(2.54)(60)\left(29.165 - \frac{3.735}{2}\right)}{12}$$
$$= 346.673 \text{ kips-ft/f} > M_u = 140.854 \text{ kips-ft/ft (O.K.)}$$

(b) Minimum reinforcement (AASHTO 5.7.3.3.2)
The factored flexure resistance must be greater than or equal to the lesser of $1.2M_{cr}$ or $1.33M_u$.

$$1.2M_{cr} = 1.2(124.717) = 149.661 \text{ kips - ft / ft}$$

$$1.33M_u = 140.854(1.33) = 187.335 \text{ kips - ft / ft}$$

$$1.2M_{cr} < 1.33M_u \Rightarrow \quad \text{check} \quad \phi M_n = 346.673 \geq 1.2M_{cr} \text{ (O.K.)}$$

(c) Additional requirement on longitudinal reinforcement (AASHTO 5.8.3.5)
At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_{ps}f_{ps} + A_s f_y = \frac{|M_u|}{d_v \varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta \quad \text{(AASHTO Eq. 5.8.3.5-1)}$$

where θ = angle of crack (degrees)

$$\theta = 29 + 3500\varepsilon_s \qquad \text{(AASHTO 5.8.3.4)}$$

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{\left(E_s A_s + E_p A_{ps}\right)} \qquad \text{(AASHTO Eq.5.8.3.4.2-4)}$$

$$M_u = 140.854 \text{ kips-ft/ft}$$
$$d_v = 27.297 \text{ in}$$
$$N_u = -\left(W_1 + W_2 + W_6 + P_{AV}\right) = -13.276 \text{ kips/ft (compression is negative)}$$
$$V_u = 15.737 \text{ kips/ft}$$

$$V_p = A_{ps} = 0$$

$$E_s = 29000 \text{ ksi}$$
$$A_s = 2.54 \text{ in}^2$$

$$\varepsilon_s = \frac{\frac{140.854(12)}{27.297} - 0.5(13.276) + (15.737)}{(29000)(2.54)} = 0.000964$$

$$\theta = 29 + 3500\varepsilon_s = 32.378°$$

$$A_{s,\min} = \left[\frac{|M_u|}{d_v \varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta\right] \Big/ f_y$$

$$= \left[\frac{140.854 \times 12}{27.297 \times 1.0} + 0.5\frac{-13.276}{1.0} + \left(\left|\frac{15.737}{1.0} - 0\right| - 0\right)\cot(32.378°)\right] \Big/ 60 = 1.335$$

$$A_s = 2.54 \text{ in}^2 > A_{s,\min} = 1.335 \text{ in}^2 \quad \text{(O.K.)}$$

Table 3-16: Checklist for the Flexural Design of Stem

Limit State	φ	φM_n	M_u	$1.2M_{cr}$	$1.33M_u$	A_s	$A_{s,min}$
Service	0.9	312.005	59.266	149.661	78.824	2.540	0.567
Strength I (a)	0.9	312.005	88.329	149.661	117.477	2.540	0.904
Strength I (b)	0.9	312.005	88.546	149.661	117.766	2.540	0.865
Extreme Event I	1.0	346.673	140.854	149.661	187.335	2.540	1.335

Check the shear design:

$V_u = 15.737$ kips/ft

Per AASHTO 5.8.2.1, the factored shear resistance V_r shall be taken as:

$V_r = \varphi V_n$

where: V_n = nominal shear resistance (kip)

φ = resistance factor as listed in Table 3-15 (AASHTO 5.5.4.2).

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3)

$$V_c = 0.0316\beta\sqrt{f'_c}\,b\,d_v$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$

where d_v = effective shear depth as determined in AASHTO 5.8.2.9 (in.)

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

θ = angle of inclination of diagonal compressive stresses (degrees)

α = angle of inclination of transverse reinforcement to longitudinal axis (degrees)

A_v = area of shear reinforcement within a distance s (in^2)

s = spacing of transverse reinforcement (in.)

β is determined by the following equations (AASHTO 5.8.3.4):

$$\beta = \frac{4.8}{1+750\varepsilon_s} \quad \text{(if minimum shear reinforcement is provided)}$$

$$\beta = \frac{4.8}{1+750\varepsilon_s}\frac{51}{39+S_{xe}} \quad \text{(otherwise)}$$

where $S_{xe} = S_x\left(\frac{1.38}{a_g+0.63}\right)$ is the crack spacing parameter (AASHTO Eq. 5.8.3.4.2-5).

$$12 \text{ in} \le S_{xe} \le 80 \text{ in}$$

$$S_x = d - \frac{a}{2} = 29.165 - \frac{3.735}{2} = 27.297 \text{ in}$$

a_g (maximum aggregate size) = 1.5 in.

$$12 \le S_{xe} = 27.297\left(\frac{1.38}{1.5+0.63}\right) = 17.685 \le 80 \quad \text{(O.K.)}$$

$$\beta = \frac{4.8}{(1+750\times 0.000965)}\frac{51}{(39+17.685)} = 2.506$$

$$\Rightarrow \quad V_c = (0.0316)(2.506)\sqrt{4}(12)(27.297) = 51.885 \text{ kips/ft}$$

$$A_v = 0 \quad \Rightarrow \quad V_s = 0$$

Check $V_r = \varphi V_n \ge V_u$:

$$\varphi V_n = \varphi(V_c + V_s + V_p) = 1.0(51.885+0+0) = 51.885 \text{ kips/ft} \;>\; V_u = 15.737 \text{ kips/ft (O.K.)}$$

Table 3-17: Checklist for the Shear Design of Stem

Limit State	φ	N_u	ε_s	β	V_c	φV_n	V_u
Service	0.9	11.600	0.000370	3.3796	69.964	62.968	7.035
Strength I (a)	0.9	11.745	0.000591	2.9928	61.956	55.761	10.552
Strength I (b)	0.9	15.192	0.000569	3.0275	62.676	56.409	10.552
Extreme Event I	1.0	13.276	0.000964	2.5063	51.885	51.885	15.737

Toe Design:

(i) Draw the free body diagram.

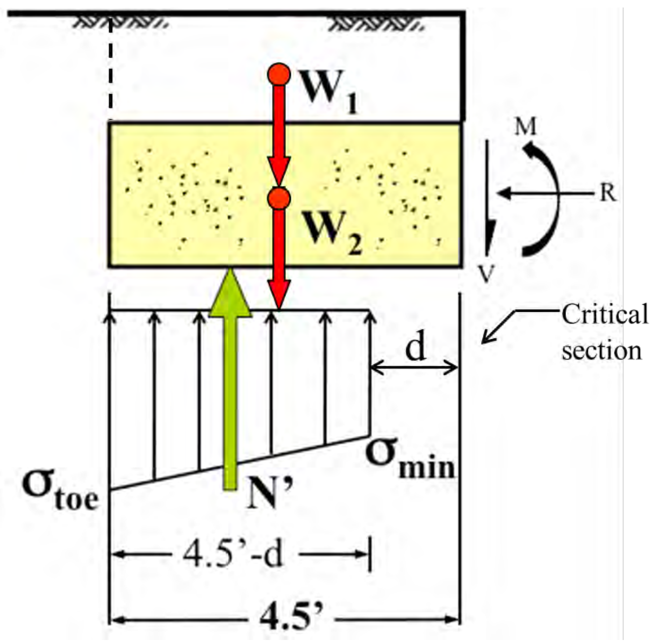


Figure 3-20: Forces Acting on the Toe for the Extreme Event Limit State

Remark:

In RC design, the design shear and the design moment are allowed to be taken at the cross-section located at a distance *d* (equal to the element's effective depth) from the critical section of the structural component. To be conservative, all the structural components in this example are designed for the shear and moment demands estimated right at the critical sections (i.e., the σ_{min} in Figure 3-20 is taken at the right face of toe).

(ii) Compute the unfactored design shear force and design bending moment.

The calculations below are for the Extreme Event Limit State.

By using similar triangles, σ_{min} can be calculated, as in

$$\sigma_{min} = \frac{\sigma_{toe}(f_1 - 4.5)}{f_1} = \frac{9.673(14.066 - 4.5)}{14.066} = 6.578$$

$$N' = \frac{(\sigma_{min} + \sigma_{toe})}{2}(4.5) = \frac{(9.673 + 6.578)}{2}(4.5) = 36.563 \text{ kips/ft}$$

$$W_1 = 4.5(2)(0.12) = 1.08 \text{ kips/ft}$$

$$W_2 = 4.5(2.75)(0.15) = 1.856 \text{ kips/ft}$$

Design shear force (unfactored):

$$V_u = N' - W_1 - W_2 = 36.563 - 1.08 - 1.856 = 33.627 \text{ kips/ft}$$

Design bending moment (unfactored):

Evaluate the moment at the stem face,

$$M_u = N'\overline{Y_1} - W_1\overline{Y_2} - W_2\overline{Y_3}$$

$$\overline{Y_1} = \frac{(\sigma_{toe} - \sigma_{min})\left(4.5/3\right) + \sigma_{min}(4.5/2)}{(\sigma_{toe} + \sigma_{min})/2} = \frac{(9.673 - 6.578)\left(4.5/3\right) + 6.578(4.5/2)}{(9.673 + 6.578)/2} = 2.393 \text{ ft}$$

$$\overline{Y_2} = \overline{Y_3} = \frac{4.5}{2} = 2.25 \text{ ft}$$

$$M_u = 36.563(2.393) - 1.08(2.25) - 1.856(2.25) = 80.882 \text{ kips-ft/ft}$$

(iii) Apply LRFD Factors to obtain the design shear force and bending moment.

Table 3-18: Design Shear Force and Bending Moment for Toe

Group	σ_{toe}	σ_{min}	W_1	W_2	N'	V_u	M_u
Service	3.672	3.547	1.08	1.856	16.242	13.306	30.150
Strength I (a)	4.754	4.120	1.08	1.670	19.967	17.216	39.805
Strength I (b)	5.192	4.886	1.458	2.32	22.676	18.898	43.036
Extreme Event	9.673	6.578	1.08	1.856	36.563	33.627	80.882

(iv) Design the flexural and shear reinforcements.

Assuming #10 at 6”

$$A_s = 2.54 \text{ in}^2$$
$$h = 2.75 \text{ ft} = 33 \text{ in}$$
$$d = h - \text{clear cover} - \frac{d_b}{2} = 33 - 3 - \frac{1.270}{2} = 29.365 \text{ in}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.54(60{,}000)}{0.85(4{,}000)(12)} = 3.735 \text{ in}$$

$$d_v = d - \frac{a}{2} = 29.365 - \frac{3.735}{2} = 27.497,$$
$$d_v > 0.9 \times d = 0.9 \times 30.365 = 27.328 \text{ in (O.K.)}$$
$$d_v > 0.72 \times h = 0.72 \times 33 = 23.760 \text{ in (O.K.)}$$

$$M_{cr} = S_c f_r = \frac{2178 \times 0.74}{12} = 134.31 \text{ kips-ft/ft}$$

where S_c (Section Modulus) $= \frac{1}{6} bh^2 = \frac{1}{6} 12(33)^2 = 2178 \text{ in}^3$

and $f_r = 0.37\sqrt{f_c'}$ ksi $= 0.74$ ksi (AASHTO 5.4.2.6)

The calculation for Extreme Event Limit State is presented below as an example.

Check the flexural design:

$M_u = 80.882$ kips-ft/ft

(a) General requirement on factored flexural resistance (AASHTO 5.7.3.2)
Check $M_r = \varphi M_n \geq M_u$:

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right)$$

$$= \frac{1.0(2.54)(60)\left(29.365 - \frac{3.735}{2}\right)}{12}$$

$$= 349.216 \text{ kips-ft/f} \; > \; M_u = 80.882 \text{ kips-ft/ft (O.K.)}$$

(b) Minimum reinforcement (AASHTO 5.7.3.3.2)
The factored flexure resistance must be greater than or equal to the lesser of $1.2M_{cr}$ or $1.33M_u$.

$$1.2M_{cr} = 1.2(134.31) = 161.172 \text{ kips-ft/ft}$$

$$1.33M_u = 80.882(1.33) = 107.573 \text{ kips-ft/ft}$$

$$1.2M_{cr} < 1.33M_u \quad \Rightarrow \quad \text{check } \; \varphi M_n = 349.216 \; \geq \; 1.2M_{cr} \text{ (O.K.)}$$

(c) Additional requirement on longitudinal reinforcement (AASHTO 5.8.3.5)

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{(E_s A_s + E_p A_{ps})} \quad \text{(AASHTO Eq. 5.8.3.4.2-4)}$$

$M_u = 80.882$ kips-ft/ft
$d_v = 27.497$ in
$N_u = 0$ kips/ft
$V_u = 33.627$ kips/ft

$V_p = A_{ps} = 0$

$E_s = 29000$ ksi
$A_s = 2.54$ in^2

$$\varepsilon_s = \frac{\frac{80.881(12)}{27.497} - 0.5(0) + (33.626)}{(29000)(2.54)} = 0.000936$$

$$\theta = 29 + 3500\varepsilon_s = 32.276°$$

$$A_{s,\min} = \frac{1}{f_y}\left[\frac{|M_u|}{d_v\varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta\right]$$

$$= \frac{1}{60}\left[\frac{80.881\times 12}{27.497\times 1.0} + 0.5\frac{0}{1.0} + \left(\left|\frac{33.626}{1.0} - 0\right| - 0\right)\cot(32.276^{\circ})\right] = 1.476$$

$$A_s = 2.54\ in^2 > A_{s,\min} = 1.476\ in^2\ \text{(O.K.)}$$

Table 3-19: Checklist for the Flexural Design of Toe

Limit State	φ	φM_n	M_u	$1.2M_{cr}$	$1.33M_u$	A_s	$A_{s,min}$
Service	0.9	314.295	30.150	161.172	40.100	2.540	0.666
Strength I (a)	0.9	314.295	39.805	161.172	52.941	2.540	0.860
Strength I (b)	0.9	314.295	43.036	161.172	57.238	2.540	0.935
Extreme Event I	1.0	349.216	80.882	161.172	107.573	2.540	1.476

Check the shear design:

$V_u = 33.627$ kips/ft

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3)

$$V_c = 0.0316\beta\sqrt{f_c'}\,b\,d_v$$

$$\beta = \frac{4.8}{1+750\varepsilon_s}\,\frac{51}{39+S_{xe}}$$

where $S_{xe} = S_x\left(\frac{1.38}{a_g+0.63}\right)$ (AASHTO Eq. 5.8.3.4.2-5)

$$12\text{ in} \le S_{xe} \le 80\text{ in}$$

$$S_x = d - \frac{a}{2} = 29.365 - \frac{3.735}{2} = 27.497\text{ in}$$

a_g (maximum aggregate size) = 1.5 in

$$12 \le S_{xe} = 27.497\left(\frac{1.38}{1.5+0.63}\right) = 17.815 \le 80\ \text{(O.K.)}$$

$$\beta = \frac{4.8}{(1+750\times 0.000936)}\frac{51}{(39+17.815)} = 2.5319$$

$$\Rightarrow \quad V_c = (0.0316)(2.5319)\sqrt{4}(12)(27.497) = 52.800 \text{ kips/ft}$$

$$A_v = 0 \quad \Rightarrow \quad V_s = 0$$

Check $V_r = \varphi V_n \geq V_u$:

$$\varphi V_n = \varphi\left(V_c + V_s + V_p\right) = 1.0(52.800+0+0) = 52.800 \text{ kips/ft} \; > \; V_u = 33.627 \text{ kips/ft (O.K.)}$$

Table 3-20: Checklist for the Shear Design of Toe

Limit State	φ	ε_s	β	V_c	φV_n	V_u
Service	0.9	0.000359	3.3942	70.782	63.703	13.306
Strength I (a)	0.9	0.000470	3.1865	66.452	59.807	17.216
Strength I (b)	0.9	0.000512	3.1140	64.940	58.446	18.898
Extreme Event I	1.0	0.000936	2.5319	52.800	52.800	33.627

Remark:

Flexural reinforcement in toe is extended from that in the stem. Therefore the bar # used in toe should be the same as that in stem, and the spacing of rebar in toe should be taken as a multiple of that in stem. Table 3-19 and Table 3-20 indicate that with spacing equal to 6", the flexural and shear capacities of toe are much greater than the demands. To achieve a more economical design, the spacing of the rebars in toe may be doubled, i.e., assuming #10 at 12". In this case, the same analytical procedure demonstrated in this section should be repeated again to ensure that the demands are smaller than the capacities in all limit states. Furthermore, even if the spacing is allowed to be doubled in the toe (i.e., cut off one bar in every two rebars), the required development length should still be maintained in toe for the cut-off bars.

Heel Design:

(i) Draw the free body diagram.

The lateral forces applied to the heel of the wall are shown below.

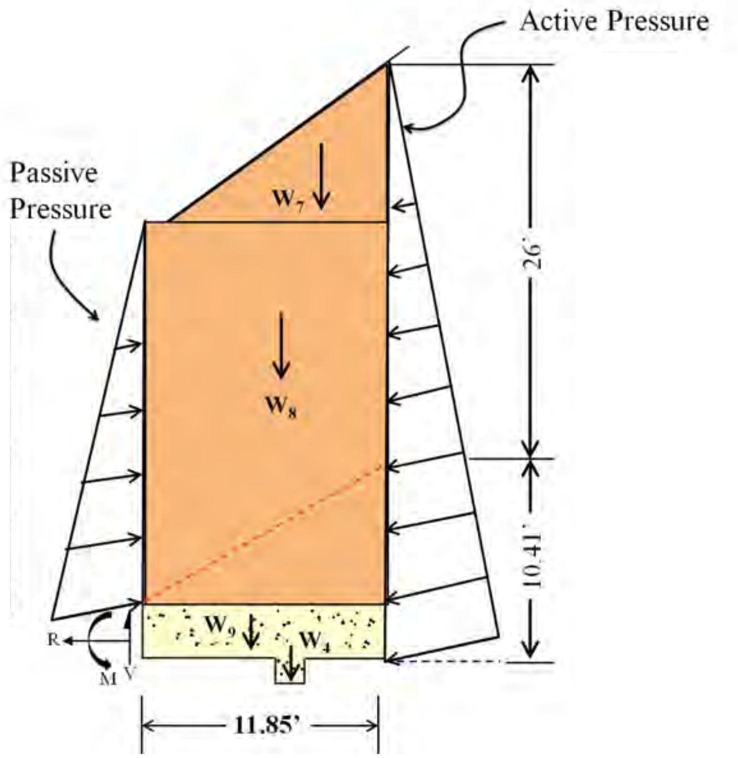


Figure 3-21: Lateral Forces Acting on the Heel

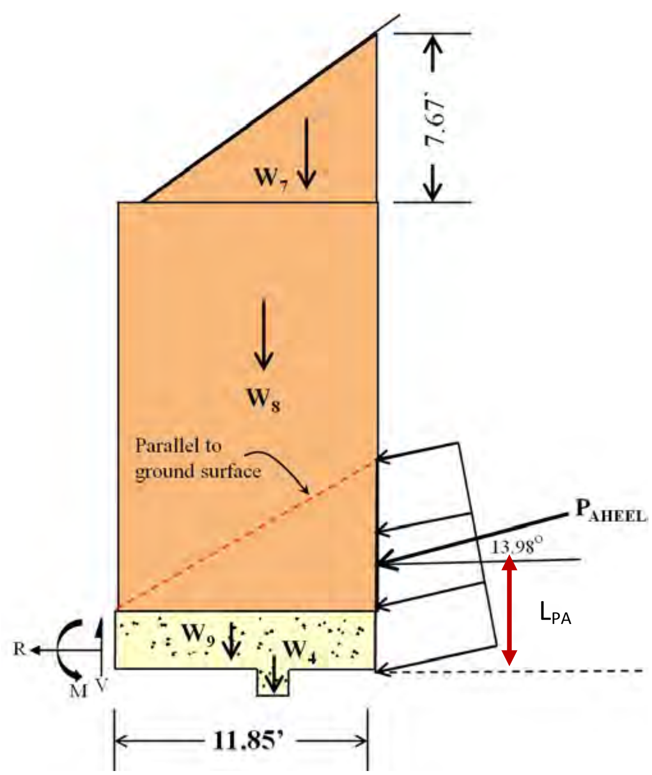
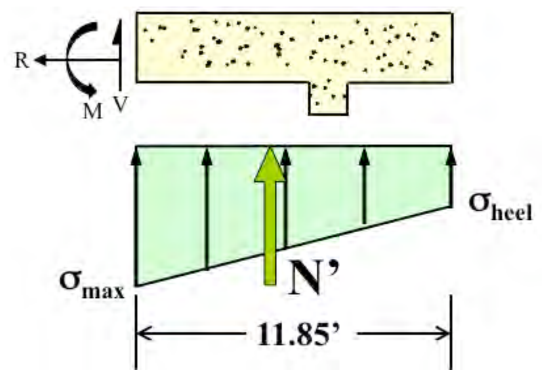
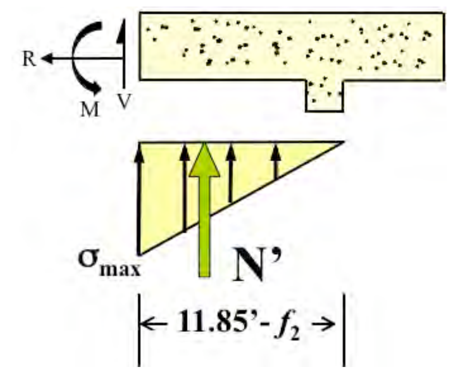


Figure 3-22: Lateral Pressure Distribution for the Heel Design

The figures below display the two possible bearing pressure distributions for the heel design. The bearing pressure diagrams for the entire spread footing were shown in Figure 3-17 (Note: Only positive bearing pressure is used in design).



Bearing Pressure Diagram that will be used for Service, Strength I (a), and Strength I (b)



Bearing Pressure Diagram that will be used for Extreme Event Limit State

Figure 3-23: Bearing Pressure Diagrams for the Heel Design

(ii) Compute the unfactored design shear force and design bending moment.

For the heel design calculations, it is assumed that all calculations are made from the face of the stem. Sample calculations are shown below for Service Limit State.

Table 3-21: Vertical Loads and Resulted Moments Acting on Heel

Force	Vertical Load (kips/ft)	Arm (ft)	Moment (kips-ft/ft), M_D
W_7	0.5 (11.48) (7.65) (0.12) = 5.269	8.024	-42.282
W_8	11.85 (26) (0.12) = 36.972	5.925	-219.060
W_9	11.85 (2.75) (0.15) = 4.888	5.926	-28.962
W_4	1 (2) (0.15) = 0.30	6.850	-2.055
P_{AV}	$\left(P_A - P_{A_{STEM}}\right)\sin(13.98)$ = 1.682	11.850	-19.927
N'	$0.5(\sigma_{max} + \sigma_{heel}) \times 11.85$ = 39.198	5.826*	228.383

*Note: $\frac{(\sigma_{\max}-\sigma_{heel})(11.85/3)+\sigma_{heel}(11.85)}{\sigma_{\max}+\sigma_{heel}}=\frac{(3.473-3.143)(11.85/3)+3.143(11.85)}{3.473+3.143}=5.826$

Design shear force (unfactored):

$$V_u = P_v + \sum W - N' = 9.913 \text{ kips/ft}$$

Design bending moment (unfactored):

$$P_{A_{STEM}} = P_A \frac{h'^2}{H^2} = 14.21\frac{26^2}{36.40^2} = 7.25 \text{ kips/ft}$$

$$P_{A_{HEEL}} = P_A - P_{A_{STEM}} = 14.21 - 7.25 = 6.96 \text{ kips/ft}$$

$$P_{AH,HEEL} = 6.96 \times \cos(13.98^o) = 6.754 \text{ kips/ft}$$

$$P_A\left(\frac{2}{3}H\right) = P_{A_{STEM}}\left(\frac{2}{3}h'\right) + P_{A_{HEEL}}(H - L_{PA})$$

$$L_{PA} = \frac{1}{P_{A_{HEEL}}}\left[P_{A_{STEM}}\left(\frac{2}{3}h'\right) + P_{A_{HEEL}}H - P_A\left(\frac{2}{3}H\right)\right]$$

$$= \frac{1}{6.96}\left[7.25\left(\frac{2}{3}26\right) + 6.96(36.4) - 14.21\left(\frac{2}{3}36.4\right)\right] = 4.911 \text{ ft} \quad \text{(cf. Figure 3-22)}$$

$$M_u = \sum M_D - P_{AH,HEEL}\left(L_{PA} - D_{HEEL}/2\right) = 83.903 - (6.754)(4.911 - 2.75/2) = 60.013 \text{ kips-ft/ft}$$

(iii) Apply LRFD Factors to obtain the design shear forces and design bending moments.

Table 3-22: Factored Design Shear and Design Moment of Heel for All Limit States

Group	$\sigma_{\max}$	σ_{heel}	M_{W7+W8}	M_{W9+W4}	M_{PAV}	$M_{N'}$	V_u	M_u
Service	3.473	3.143	261.342	31.017	19.927	-228.383	9.913	60.013
Str. I (a)	3.747	2.079	261.342	27.916	29.89	-185.018	14.912	98.295
Str. I (b)	4.706	3.899	352.811	38.772	29.89	-292.653	15.048	92.985
Ext. Event	4.756	0.000	261.342	31.017	39.853	-37.914	34.274	246.689

Note: For the Extreme Event Limit State, only the positive bearing pressure is used in the design calculations. M_{W7+W8} = Sum of the Soil Moments. M_{W9+W4} = Sum of the Concrete Moments.

(iv) Design the flexural and shear reinforcements.

Assuming #10 at 6 in

Required development length = 53 in

Available length = 52 in (adequately close)

(For the equations of basic development length of deformed bars and the modification factors associated with various conditions, please refer to AASHTO 5.11.2.)

$$A_s = 2.54\ in^2$$

$$h = 2.75\ \text{ft} = 33\ \text{in}$$

$$d = h - \text{clear cover} - \frac{d_b}{2} = 33 - 2 - \frac{1.270}{2} = 30.365\ \text{in}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.54(60{,}000)}{0.85(4{,}000)(12)} = 3.735\ \text{in}$$

$$d_v = d - \frac{a}{2} = 30.365 - \frac{3.735}{2} = 28.947,$$

$$d_v > 0.9 \times d = 0.9 \times 30.365 = 27.328\ \text{in (O.K.)}$$

$$d_v > 0.72 \times h = 0.72 \times 33 = 23.760\ \text{in (O.K.)}$$

$$M_{cr} = S_c f_r = \frac{2178(0.74)}{12} = 134.310\ \text{kips-ft/ft}$$

where S_c (Section Modulus) $= \frac{1}{6} b h^2 = \frac{1}{6} 12(33)^2 = 2178\ \text{in}^3$

$$f_r = 0.37\sqrt{f_c'}\ \text{ksi} = 0.74\ \text{ksi (AASHTO 5.4.2.6)}$$

The calculation for Extreme Event Limit State is presented below as an example.

Check the flexural design:

$$M_u = 246.689\ \text{kips-ft/ft}$$

(a) General requirement on factored flexural resistance (AASHTO 5.7.3.2)
Check $M_r = \varphi M_n \geq M_u$:

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2}\right) = 1.0(1.5)(60)\left(30.365 - \frac{3.735}{2}\right)\frac{1}{12}$$
$$= 361.916\ \text{kips-ft/ft} > M_u = 246.689\ \text{kips-ft/ft (O.K.)}$$

(b) Minimum reinforcement (AASHTO 5.7.3.3.2)
The factored flexure resistance must be greater than or equal to the lesser of $1.2M_{cr}$ or $1.33M_u$.

$$1.2M_{cr} = 1.2(134.31) = 161.172 \text{ kips-ft/ft}$$

$$1.33M_u = 246.689(1.33) = 328.097 \text{ kips-ft/ft}$$

$$1.2M_{cr} < 1.33M_u \quad \Rightarrow \quad \text{check} \;\; \varphi M_n = 361.916 \;\geq\; 1.2M_{cr} \text{ (O.K.)}$$

(c) Additional requirement on longitudinal reinforcement (AASHTO 5.8.3.5)

$$\varepsilon_s = \frac{\dfrac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{\left(E_sA_s + E_pA_{ps}\right)} \qquad \text{(AASHTO Eq. 5.8.3.4.2-4)}$$

$M_u = 246.689$ kips-ft/ft
$d_v = 28.497$ in
$N_u = 0$ kips/ft
$V_u = 34.274$ kips/ft

$V_p = A_{ps} = 0$

$E_s = 29000$ ksi
$A_s = 2.54 \text{ in}^2$

$$\varepsilon_s = \frac{\dfrac{246.689(12)}{28.497} - 0.5(0) + (34.274)}{(29000)(2.54)} = 0.001876$$

$$\theta = 29 + 3500\varepsilon_s = 35.566°$$

$$A_{s,\min} = \frac{1}{f_y}\left[\frac{|M_u|}{d_v\varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta\right]$$

$$= \frac{1}{60}\left[\frac{246.689\times 12}{28.497\times 1.0} + 0.5\frac{0}{1.0} + \left(\left|\frac{34.274}{1.0} - 0\right| - 0\right)\cot(35.366°)\right] = 2.530$$

$$A_s = 2.54 \text{ in}^2 > A_{s,\min} = 2.530 \text{ in}^2 \quad \text{(O.K.)}$$

Table 3-23: Checklist for the Flexural Design of Heel

Limit State	φ	φM_n	M_u	$1.2M_{cr}$	$1.33M_u$	A_s	$A_{s,min}$
Service	0.9	325.725	60.013	161.172	79.818	2.54	0.778
Strength I (a)	0.9	325.725	98.295	161.172	130.732	2.54	1.214
Strength I (b)	0.9	325.725	92.985	161.172	123.670	2.54	1.178
Extreme Event I	1.0	361.916	246.689	161.172	328.097	2.54	2.530

Check the shear design:

$V_u = 34.274$ kips/ft

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3)

$$V_c = 0.0316\beta\sqrt{f_c'}\,b\,d_v$$

$$\beta = \frac{4.8}{1+750\varepsilon_s}\frac{51}{39+S_{xe}}$$

where $S_{xe} = S_x\left(\frac{1.38}{a_g+0.63}\right)$ (AASHTO Eq. 5.8.3.4.2-5)

$$12 \text{ in} \le S_{xe} \le 80 \text{ in}$$

$$S_x = d - \frac{a}{2} = 30.365 - \frac{3.735}{2} = 28.497 \text{ in}$$

a_g (maximum aggregate size) = 1.5 in.

$$12 \le S_{xe} = 28.497\left(\frac{1.38}{1.5+0.63}\right) = 18.463 \le 80 \quad \text{(O.K.)}$$

$$\beta = \frac{4.8}{(1+750\times 0.001876)}\frac{51}{(39+18.463)} = 1.770$$

$$\Rightarrow \quad V_c = (0.0316)(1.770)\sqrt{4}(12)(28.497) = 38.257 \text{ kips/ft}$$

$$\mathrm{A}_v = 0 \quad \Rightarrow \quad V_s = 0$$

Check $V_r = \varphi V_n \ge V_u$:

$$\varphi V_n = \varphi\left(V_c + V_s + V_p\right) = 1.0\left(38.257+0+0\right) = 38.257 \text{ kips/ft} \;>\; V_u = 34.274 \text{ kips/ft} \quad \text{(O.K.)}$$

Table 3-24: Checklist for the Shear Design of Heel

Limit State	φ	ε_s	β	V_c	φV_n	V_u
Service	0.9	0.000478	3.137	67.787	61.008	9.913
Strength I (a)	0.9	0.00764	2.708	58.522	52.670	14.912
Strength I (b)	0.9	0.000736	2.745	59.329	53.396	15.048
Extreme Event I	1.0	0.001876	1.770	38.257	38.257	34.274

Step 7: Check Wall Displacement

Calculate wall displacement using Eqn. 88

$PGA = 0.45$

$S_1 = 0.35$

$F_{PGV} = F_v = 1.1$

$$PGV = 55F_V S_1 = 55(1.1)(0.35) = 21.175$$

$$k_{max} = F_{PGA}\, PGA = 1.1(0.45) = 0.495$$

$$k_y = 0.2$$

$$\log(d) = -1.51 - 0.74\log\left(\frac{k_y}{k_{max}}\right) + 3.27\log\left(1 - \frac{k_y}{k_{max}}\right) - 0.81\log(k_{max}) + 1.59\log(PGV)$$

$$\log(d) = -1.51 - 0.74\log\left(\frac{0.2}{0.495}\right) + 3.27\log\left(1 - \frac{0.2}{0.495}\right) - 0.81\log(0.495) + 1.59\log(21.175)$$

$$\log(d) = -1.51 + 0.291 - 0.735 + 0.247 + 2.10 = 0.394$$

$$d = 2.5 \text{ in} < 4.0 \text{ in} \quad \text{(O.K.)}$$

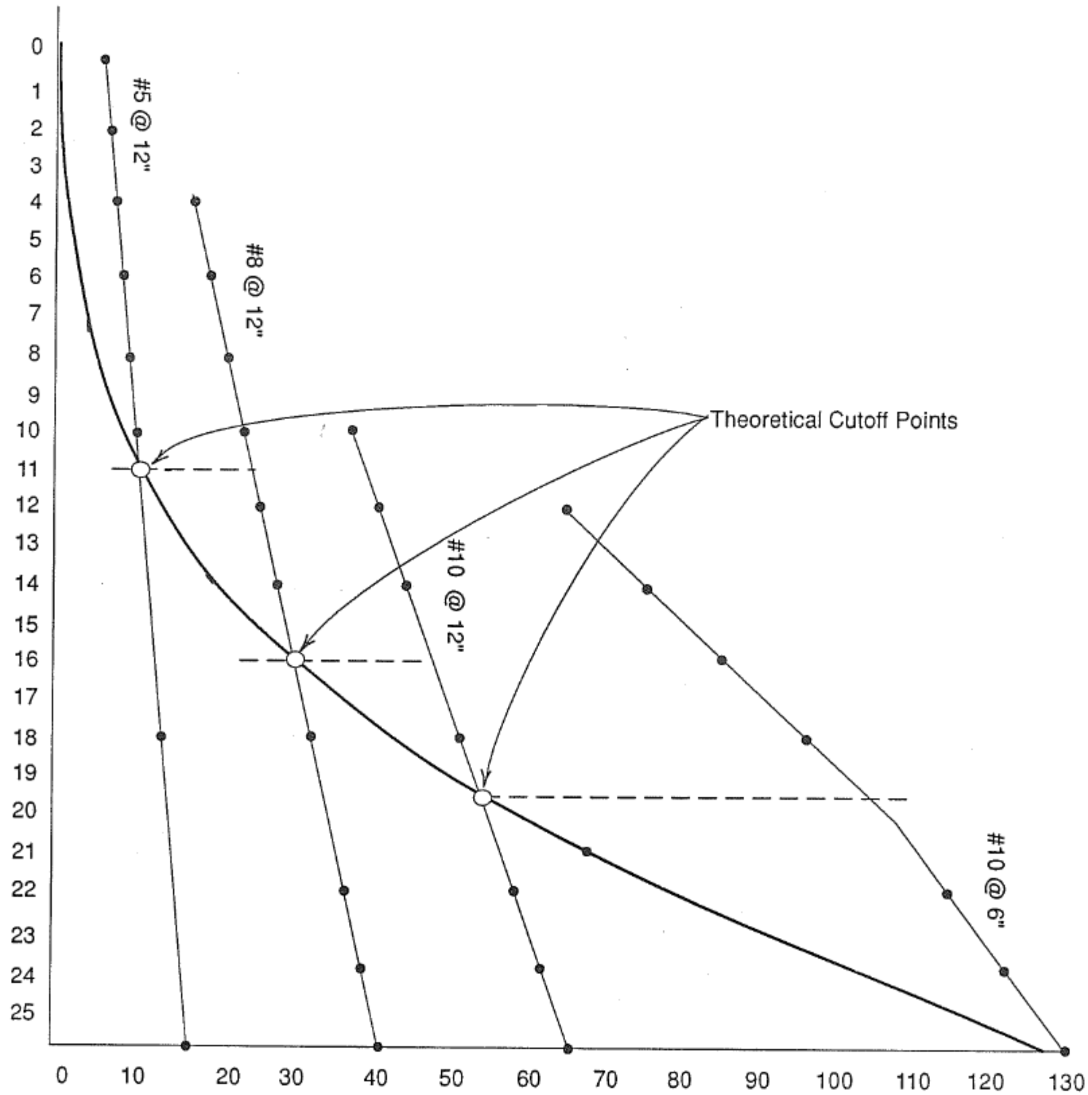


Figure 3-24: Flexural Demand and Capacity Diagram of Stem

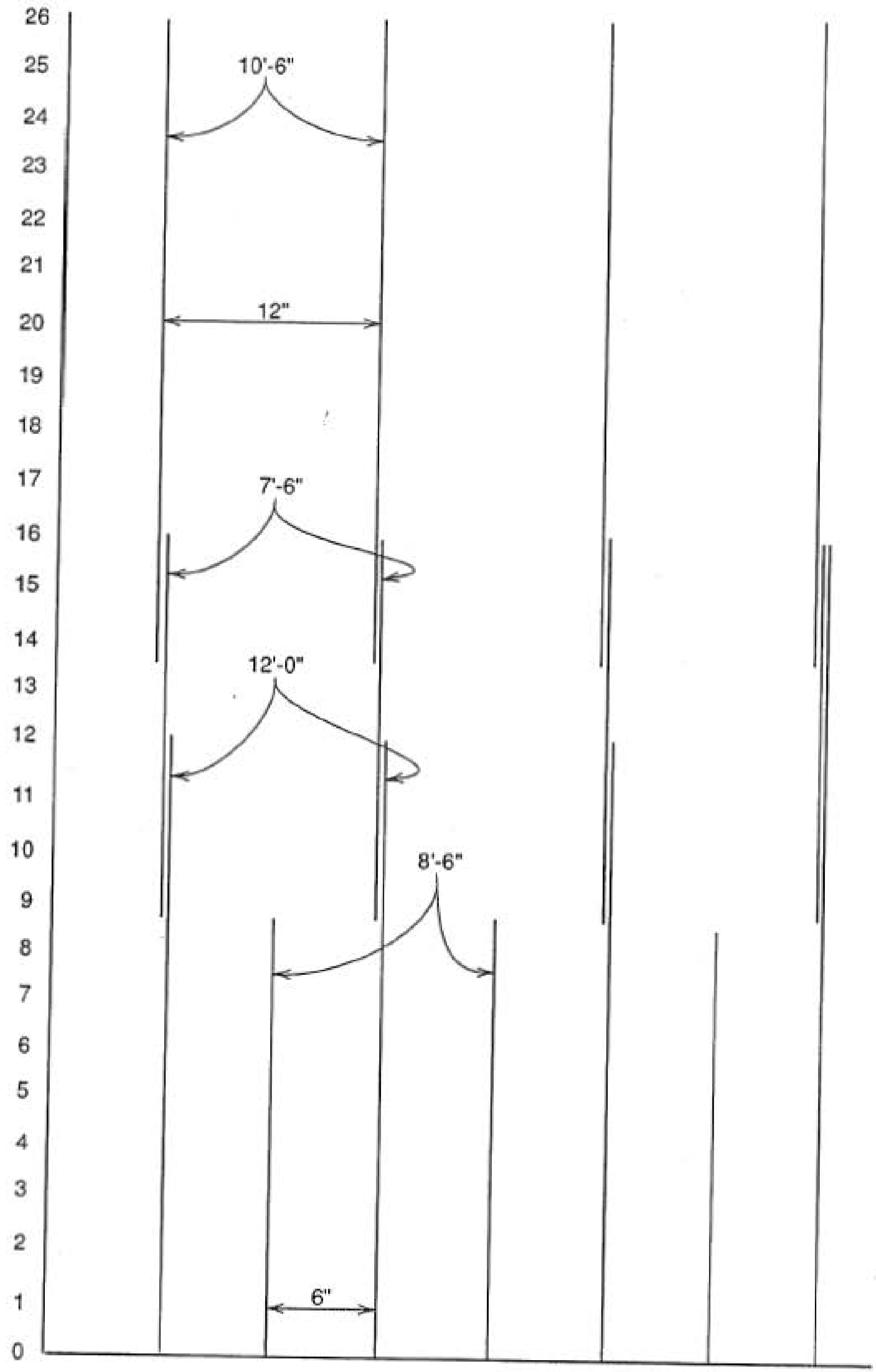


Figure 3-25: Elevation View of Flexural Reinforcement in Stem

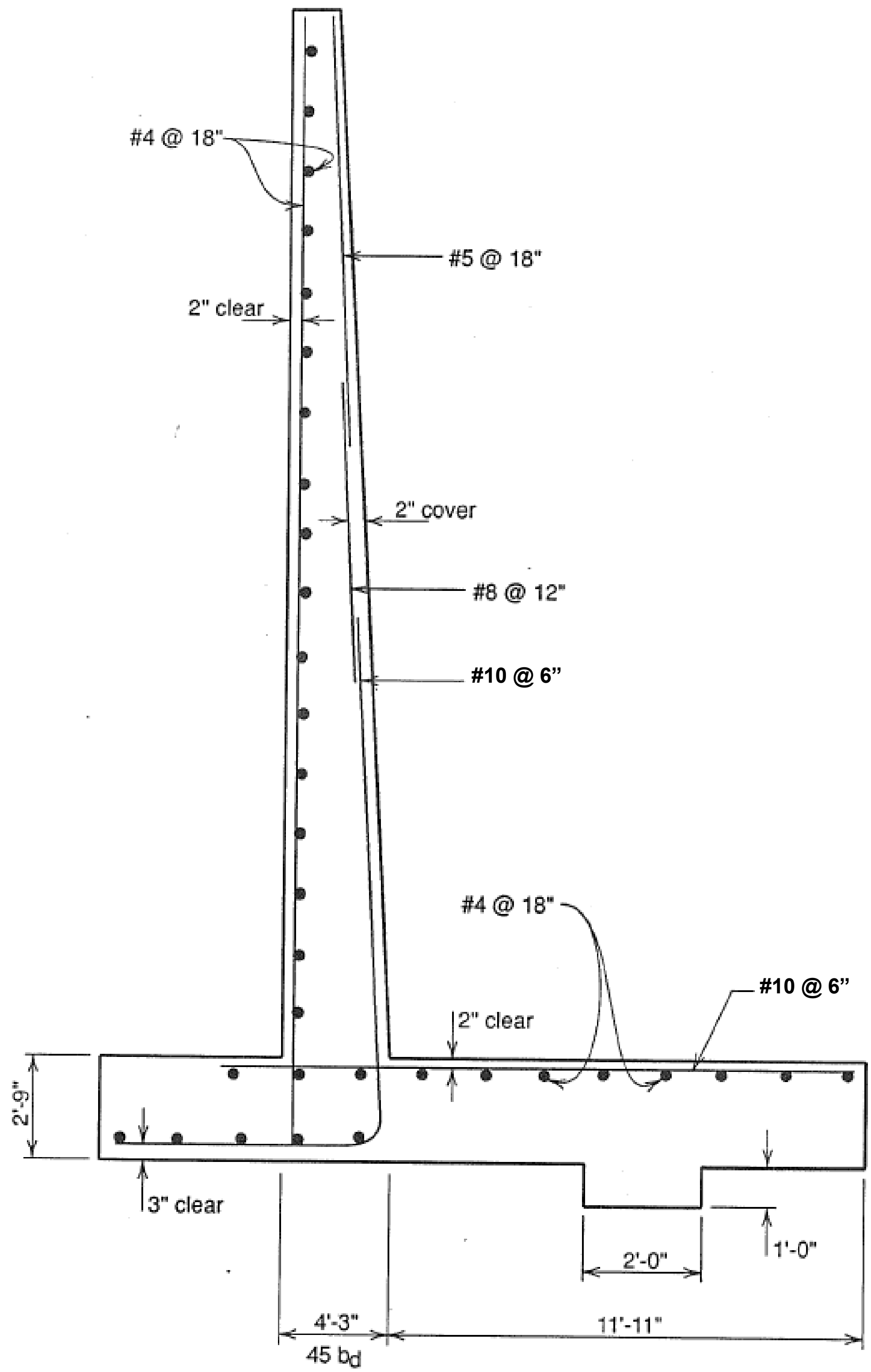


Figure 3-26: Final Cross-section Design for the Semi-Gravity Retaining Wall

3.7.2 Example 3-2: Pile Supported Cantilever Wall

A pile supported cast-in-place cantilever reinforced concrete wall is to be constructed at the toe of the slope shown in Figure 3-27. The height of the retaining wall is estimated 24 feet with 1 foot extended above the backfill. A 2-foot wide drainage ditch is required at the toe of the slope backfill for the surface water rub off. There is a 2 ksf in magnitude, 10-ft wide strip live load surcharge at a distance of 30 ft behind the wall. The pervious wall backfill will be silty fine sand (USCS classification: SM) compacted to 95% relative compaction with a moist unit weight of 120 pcf, friction angle of 34 degrees, and a cohesive strength of 300 psf. The backfill will be drained using weepholes, which will not allow hydrostatic pressure behind the wall. The horizontal seismic acceleration coefficient of 0.15g shall be used for the extreme event. Use compressive strength of concrete as 4.0 ksi and steel flexural strength of 60 ksi.

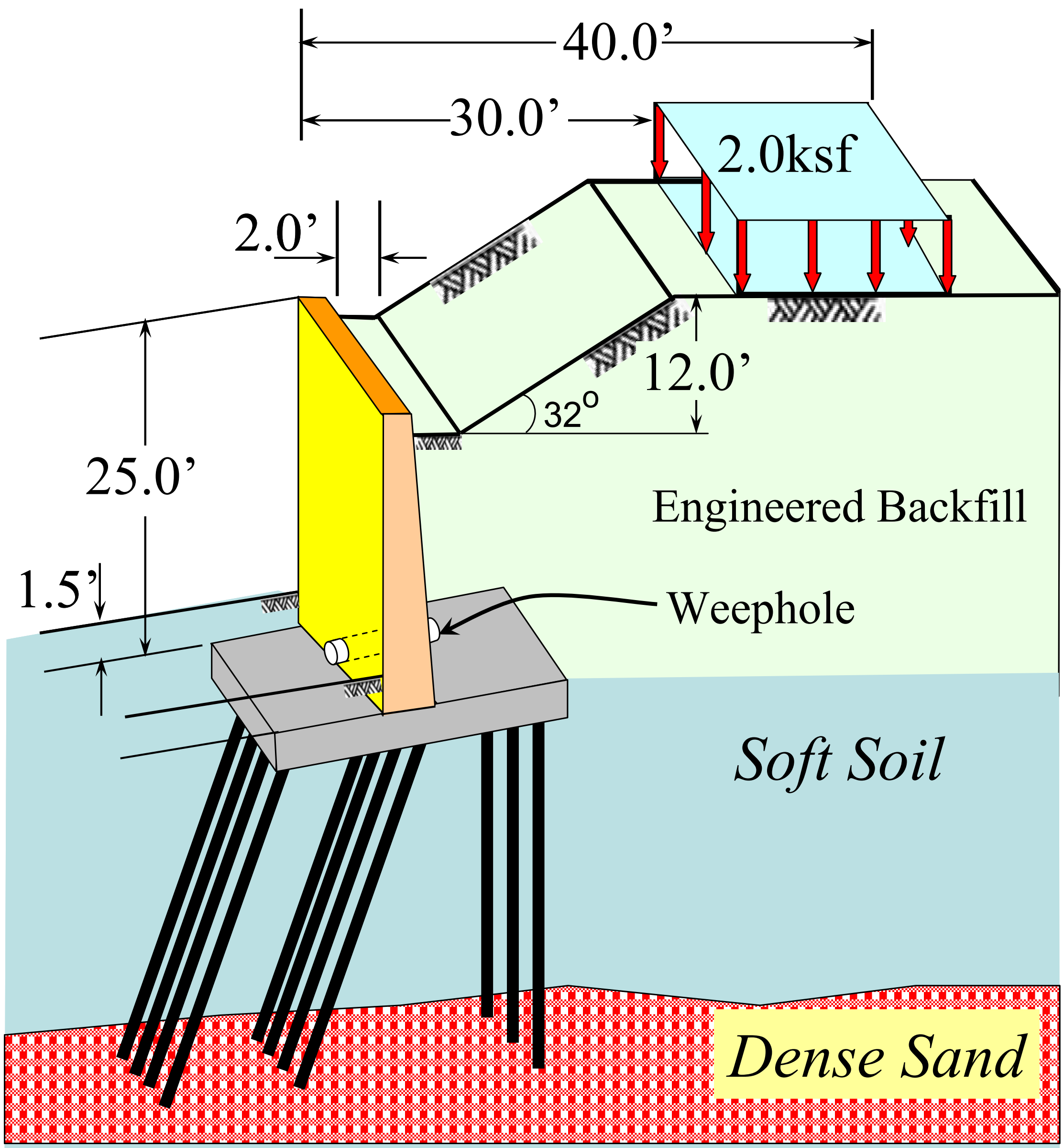


Figure 3-27: Pile Supported Retaining Wall

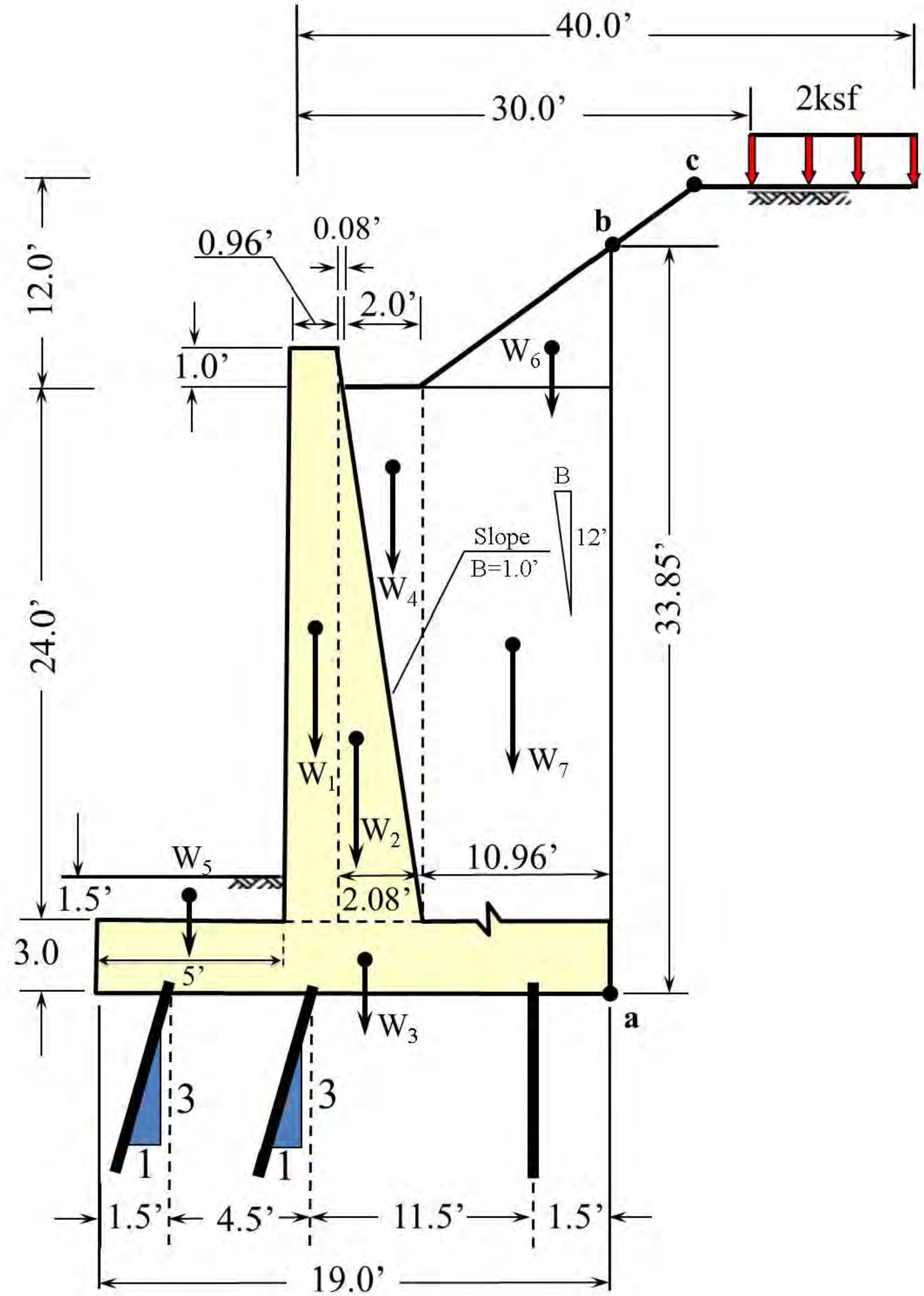


Figure 3-28: Forces Acting on Pile Supported Wall

In order to design the retaining wall system, two sets of calculations will need to be made. The first set will be for the static loading condition. The second set of calculations will be for the extreme event calculations. For each set of calculations, a five-step process will be used in order to obtain all the values needed to design the retaining wall. The steps are shown below:

1. Determine the Earth Pressure Distribution
2. Calculate Stabilizing and Overturning Forces and Moments, and Pile Reactions
3. Stem Design Calculations
4. Toe Design Calculations
5. Heel Design Calculations

Once the calculations are made, all the limit states will be compared and the largest values will control the design.

Step 1(a): Calculate active earth force due to backfill zone

The failure surface in the broken backfill in this example is unknown. It is estimated by adopting the Trial Wedge method in conjunction with an iterative approach. The wall-soil interface friction angle, δ, and the inclined angle of failure surface, α, are determined in such a way that the average backfill slope, β, defined by the critical Trial Wedge (see Figure 3-29 and Figure 3-30) being equal to δ. The computer software—CT-Rigid, developed by the Caltrans—is utilized to perform the computation. The converged critical Trial Wedge yields the following geometry:

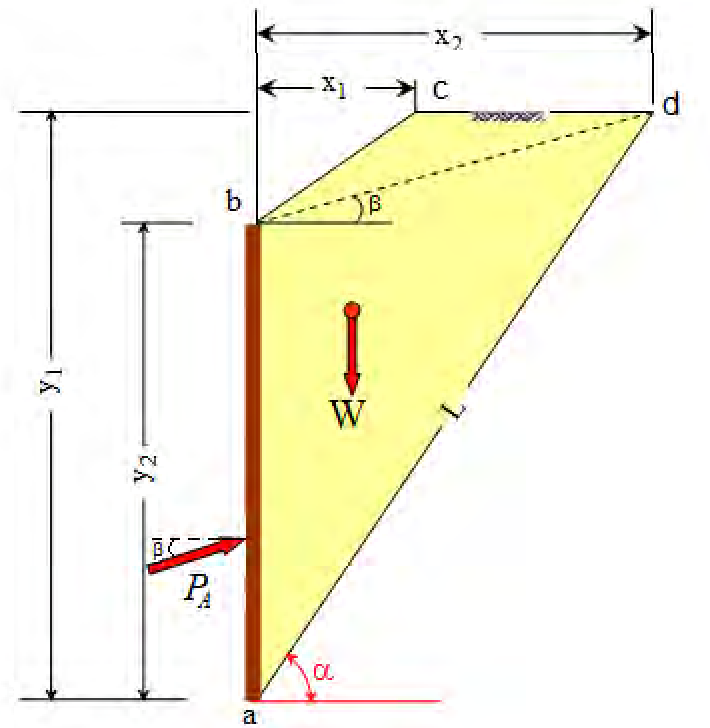


Figure 3-29: Critical Trial Wedge for Static Limit States

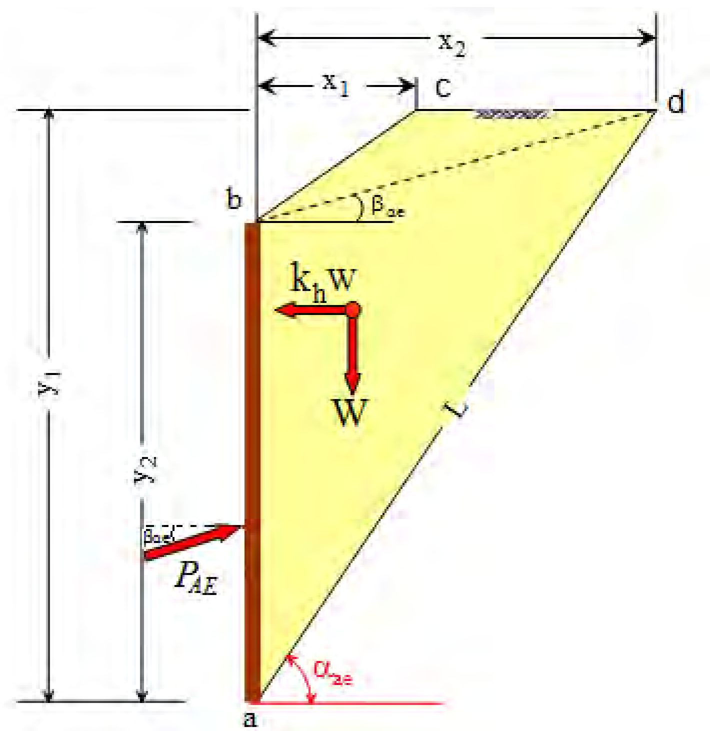


Figure 3-30: Critical Trial Wedge for Extreme Event Limit State

Static Case Calculations:

$\alpha = 60.20°, \quad \beta = 12.99° \; (= \delta).$

$y_1 = 39$ ft, $\quad y_2 = 33.85$ ft, $\quad x_1 = 8.24$ ft, $\quad x_2 = 22.32$ ft, $\quad L = 44.94$ ft.

$$\text{Area} = \frac{y_1(x_2 - x_1) + y_2 x_1}{2} = \frac{39(22.32 - 8.24) + 33.85(8.24)}{2} = 414.022 \text{ ft}^2/\text{ft}$$

$W = 414.022 \times 0.12 = 49.683$ kips/ft

Compute unfactored static active earth force due to soil wedge behind the wall using Eqn. 61.

$$P_{AH} = \frac{W\left[\tan(\alpha - \phi)\right] - cL\left[\sin\alpha \tan(\alpha - \phi) + \cos\alpha\right]}{1 + \tan\delta \tan(\alpha - \phi)}$$

$$= \frac{49.683\left[\tan(60.20° - 34°)\right] - 0.3(44.94)\left[\sin(60.20°)\tan(60.20° - 34°) + \cos(60.20°)\right]}{1 + \tan(12.99°)\tan(60.20° - 34°)} = 10.787$$

$$P_{AV} = 10.787 \tan(12.99°) = 2.484$$

$$P_A = \sqrt{P_{AH}^2 + P_{AV}^2} = 11.07 \text{ kips/ft}$$

Seismic Case Calculations: ($k_h = 0.15$)

Likewise, for the Extreme Event Limit State the failure surface of the critical Trial Wedge and its average backfill slope are determined in the same iterative approach, which yields:

$\alpha_{ae} = 54.35°, \quad \beta_{ae} = 10.44° \; (= \delta_{seismic}).$

$y_1 = 39$ ft, $\quad y_2 = 33.85$ ft, $\quad x_1 = 8.24$ ft, $\quad x_2 = 27.95$ ft, $\quad L = 47.98$ ft.

$$\text{Area} = \frac{y_1(x_2 - x_1) + y_2 x_1}{2} = \frac{39(27.95 - 8.24) + 33.85(8.24)}{2} = 523.811 \text{ ft}^2/\text{ft}$$

$W = 523.811 \times 0.12 = 62.857$ kips/ft

Compute unfactored horizontal seismic active earth force due to soil wedge behind the wall.

$$
\begin{aligned}
P_{AEH} &= \frac{W\left[\tan(\alpha-\phi)+k_h\right]-cL\left[\sin\alpha\tan(\alpha-\phi)+\cos\alpha\right]}{1+\tan\delta\tan(\alpha-\phi)} \\
&= \frac{62.857\left[\tan(54.35^\circ-34^\circ)+0.15\right]-0.3(47.98)\left[\sin(54.35^\circ)\tan(54.35^\circ-34^\circ)+\cos(54.35^\circ)\right]}{1+\tan(10.44^\circ)\tan(54.35^\circ-34^\circ)} \\
&= 18.754
\end{aligned}
$$

$$P_{AEV} = 18.754\tan(10.44^\circ) = 3.456$$

$$P_{AE} = \sqrt{P_{AEH}^2 + P_{AEV}^2} = 19.07 \text{ kips/ft}$$

Step 1(b): Calculate horizontal earth pressure due to surcharge load

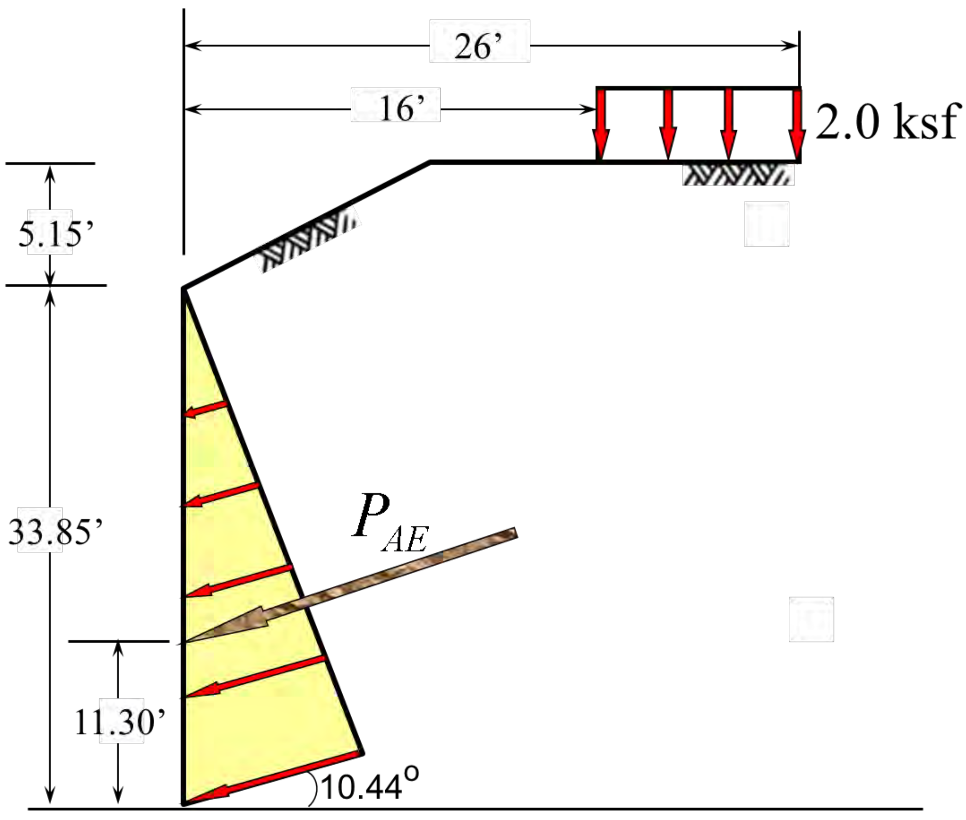


Figure 3-31: Lateral Forces Acting on Pile Supported Wall under Static Limit States

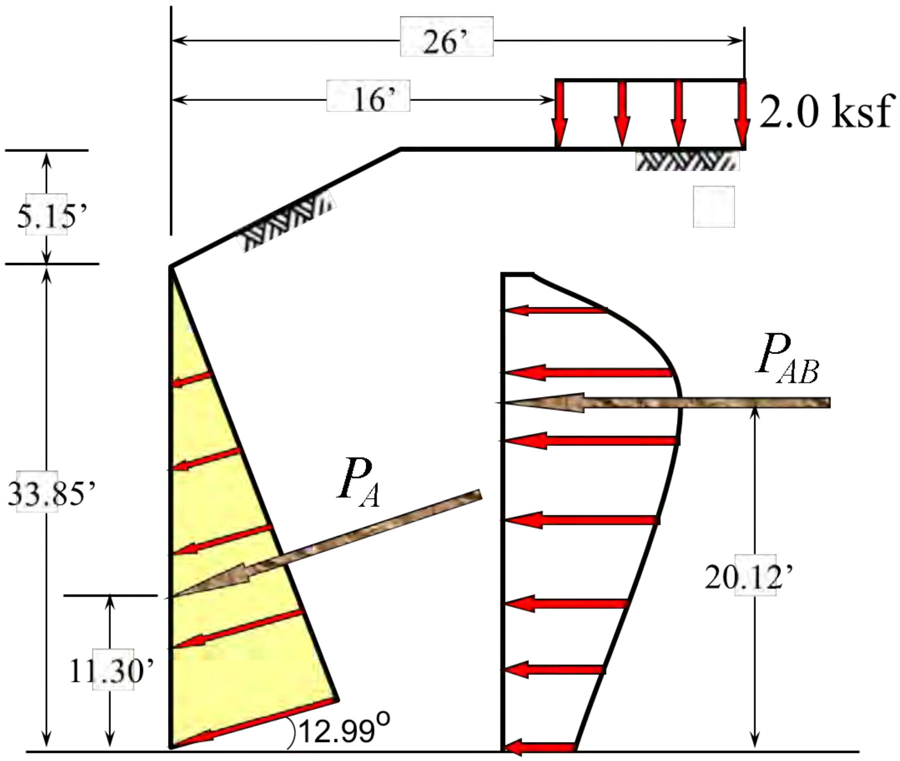


Figure 3-32: Lateral Forces Acting on Pile Supported Wall under Extreme Event Limit State

Boussinesq loading (Figure 2-27) is used to calculate the surcharge load due to strip load placed at the top of the structure.

Static Case Calculations:

$P_{AB} = 9.104$ kips/ft

Seismic Case Calculations:

From the statistic point of view, the maximum seismic earth pressures will rarely take place at the same time as the maximum live load. Therefore, the effects of live load surcharge are neglected in the calculation of the Extreme Event Limit State, i.e., $P_{AB}^{seismic} = 0$.

Table 3-25: Earth Pressure Distribution for Limit States

Group	P_A	P_{AV}	P_{AH}	P_{AB}
Service	11.070	2.488	10.787	9.104
Strength I (a)	16.605	3.732	16.180	15.932
Strength I (b)	16.605	3.732	16.180	15.932
Extreme Event	11.070	2.488	10.787	0

Step 1(c): Calculate unfactored stabilizing and overturning moments and forces

Static Case Calculations:

Compute the unfactored stabilizing moments due to vertical loads. The Service Limit State calculations are shown in Table 3-26 as the example.

Table 3-26: Unfactored Stabilizing Moments due to Vertical Loads

Item	Vertical Load, N (k/ft)	Moment Arm (ft)	Stabilizing Moment, M_{RES} (k-ft/ft)
W_1	0.96 (25) (0.15) = 3.600	5.480	19.728
W_2	0.5 (2.083) (25) (0.15) = 3.906	6.654	25.994
W_3	19 (3) (0.15) = 8.550	9.500	81.225
W_4	0.5 (2.0) (24) (0.12) = 2.880	7.377	21.245
W_5	5.0 (1.5) (0.12) = 0.900	2.500	2.250
W_6	0.5 (10.957) (6.85) (0.12) = 4.501	15.348	69.078
W_7	10.957 (24) (0.12) = 31.555	13.522	426.679
P_{AV}	11.07 sin(12.99) = 2.488	19.000	47.278

$$N = \sum W + P_{AV} = \mathbf{58.381\ kips/ft}$$

$$M_{RES} = \sum M_{RES} = \mathbf{693.477\ kips/ft}$$

Compute the unfactored overturning moments due to horizontal loads:

$$M_{OT} = 10.787 \times 11.282 + 9.104 \times 20.116 = \mathbf{304.822\ kips/ft}$$

Seismic Case Calculations:

Computation of the unfactored stabilizing moments due to vertical loads for the Extreme Event Limit State is shown in Table 3-27. The only difference in Table 3-26 and Table 3-27 is the vertical component of the active earth thrust.

Table 3-27: Unfactored Stabilizing Moments due to Vertical Loads

Item	Vertical Load, N (k/ft)	Moment Arm (ft)	Stabilizing Moment, M_{RES} (k-ft/ft)
W_1	0.96 (25) (0.15) = 3.600	5.480	19.728
W_2	0.5 (2.083) (25) (0.15) = 3.906	6.654	25.994
W_3	19 (3) (0.15) = 8.550	9.500	81.225
W_4	0.5 (2.0) (24) (0.12) = 2.880	7.377	21.245

W_5	5.0 (1.5) (0.12) = 0.900	2.500	2.250
W_6	0.5 (10.957) (6.85) (0.12) = 4.501	15.348	69.078
W_7	10.957 (24) (0.12) = 31.555	13.522	426.679
P_{AEV}	19.07 sin(10.44) = 3.456	19.000	65.656

$$N = \sum W + P_{AEV} = 59.348 \text{ kips/ft}$$

$$M_{RES} = \sum M_{RES} = 711.855 \text{ kips/ft}$$

Compute the unfactored overturning moments due to horizontal loads as shown in Table 3-28.

Table 3-28: Unfactored Overturning Moments due to Horizontal Loads for Extreme Event Limit State

Force	Horizontal Load (k/ft)	Arm (ft)	Overturning Moment, M_{OT} (k-ft/ft)
P_{AEH}	18.754	11.282	211.589
W_c (k_h)	2.408	7.031	16.934
W_s (k_h)	5.840	16.947	98.977

$$M_{OT} = \sum M_{OT} = 327.500 \text{ kips/ft}$$

Step 2(a): Calculate LRFD load combination for stabilizing and overturning forces and moments

The design of the wall shall be checked for all possible LRFD load combinations. The table below shows the LRFD Factors for all the relevant limit states considered in the retaining wall design.

Limit State	DC	EV	LS_v	LS_h	EH	Probable USE
Service I	1.00	1.00	1.00	1.00	1.00	Settlement
Strength I (a)	0.90	1.00	1.75	1.75	1.50	BC/EC/SL
Strength I (b)	1.25	1.35	1.75	1.75	1.50	BC (max. value)
Extreme Event I	1.00	1.00	0.00	0.00	1.00	BC/EC/SL

DC: Dead load of Concrete

EV: Vertical Pressure from Dead load of Earth Backfill

LS_v: Live Load Surcharge (Vertical Component)

LS_h: Live Load Surcharge (Horizontal Component)

EH: Horizontal Earth Pressure

The unfactored loads and moments should be multiplied by their associated LRFD Factors to determine the total factored loads for all the limit states considered.

Table 3-29: Factored Overturning and Stabilizing Forces and Moments for All Limit States

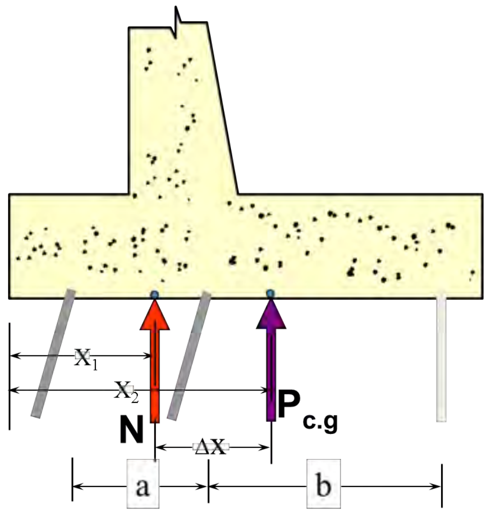
Group	Overturning Forces & Moments			Stabilizing Forces & Moments		
	P_{AH} (k/ft)	P_{AB} (k/ft)	M_{OT} (k-ft/ft)	P_{AV} (k/ft)	N (k/ft)	M_{RES} (k-ft/ft)
Service	10.787	9.104	304.822	2.488	58.381	693.477
Strength I (a)	16.180	15.932	503.014	3.732	58.019	704.422
Strength I (b)	16.180	15.932	503.014	3.732	77.582	930.591
Extreme Event	18.754	0	327.500	3.456	59.348	711.855

Step 2(b): Calculate Pile Reactions

Find the location of N (Figure 3-33) and determine the Pile Vertical and Horizontal Loads. These values will be needed when determining the structural design for the stem, toe and heel. The Service Limit State calculation is shown below as an example.

$$\bar{X}_1 = \frac{693.477 - 304.822}{58.381} = 6.657 \text{ ft}$$

The final pile layout is determined by trial and error method. After many trials the final layout is shown in Figure 3-33, with a = 4.5 ft, b = 11.5 ft, c = 3.0 ft, and d = 4.5 ft.



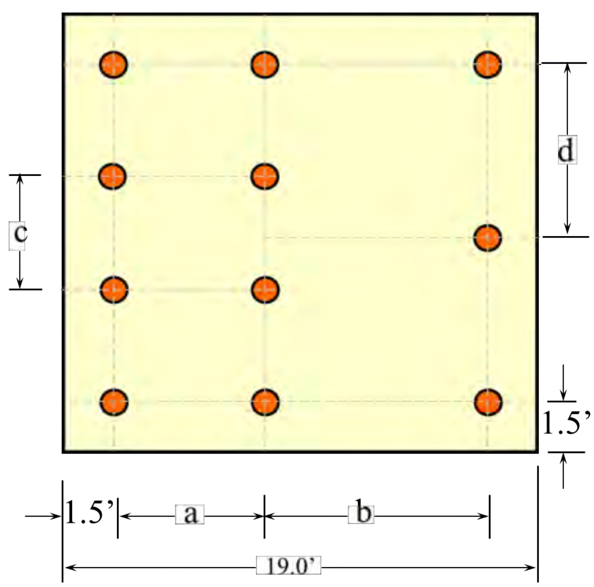


Figure 3-33: Pile Layout for the Pile Supported Wall

Table 3-30: Pile Foundation Profile

	Lateral Spacing (in)	Number of piles per ft	Orientation
Row #1	3.0'	(4−1)/(12ft-1.5ft×2) = 0.333	Battered
Row #2	3.0'	(4−1)/(12ft-1.5ft×2) = 0.333	Battered
Row #3	4.5'	(3−1)/(12ft-1.5ft×2) = 0.222	Vertical
Σ = 0.889 piles / ft			

Calculate Pile Center of Gravity (C.G.):

$$\overline{X}_2 = \left(0.333 \times 1.5 + 0.333 \times 6.0 + 0.222 \times 17.5\right)/0.889 = 7.188\, ft \quad \textbf{(from the toe)}$$

Calculate Moment of Inertia:

$$I = 0.333(7.188 - 1.5)^2 \left(\ \right)^2 + 0.333(7.188 - 6.0)^2 + 0.222(7.188 - 17.5)^2 = 34.885 \text{ pile-ft}^2\text{/ft}$$

Take Moment about piles Center of Gravity (using Service Limit States for example calculations):

$$M = N\Delta\overline{X}, \text{ where } \Delta\overline{X} = \overline{X}_2 - \overline{X}_1 = 7.188 - 6.657 = 0.530 \text{ ft}$$

$$M = 58.381 \times 0.530 = 30.956 \text{ kips-ft/ft}$$

Calculate the Piles' Vertical Reactions:

$$R_V = \frac{N}{A} \pm \frac{M\overline{X}}{I} \text{ (per pile)}$$

$$R_{V1} = \frac{58.381}{0.889} + \frac{30.956 \times 5.688}{34.885} = 70.725 \text{ kips/pile}$$

$$R_{V2} = \frac{58.381}{0.889} + \frac{30.956 \times 1.188}{34.885} = 66.732 \text{ kips/pile}$$

$$R_{V3} = \frac{58.381}{0.889} + \frac{30.956 \times -10.312}{34.885} = 56.527 \text{ kips/pile}$$

Calculate the Horizontal Resistance (assume that the shear resistance of a single pile is 18 kips):

$$R_{H1} = \frac{70.725}{3} \times 0.333 + 18 \times 0.333 = 13.858 \text{ kips/ft}$$

$$R_{H2} = \frac{66.732}{3} \times 0.333 + 18 \times 0.333 = 13.415 \text{ kips/ft}$$

$$R_{H3} = 0 + 18 \times 0.222 = 4.00 \text{ kips/ft}$$

Total Resisting Force:

$$F_{RES} = \sum R_H = 31.273 \text{ kips/ft}$$

Total Driving Force:

$$F_{DR} = P_{AH} + P_{AB} = 10.787 + 9.104 = 19.890 \text{ kips/ft}$$

Table 3-31: Values Required to Determine Pile Reactions

Group	N	M_{RES}	M_{OT}	X_1	X_2	ΔX	I	M
Service	58.381	693.477	304.822	6.657	7.188	0.530	34.885	30.956
Strength I (a)	58.019	704.422	503.014	3.471	7.188	3.716	34.885	215.606
Strength I (b)	77.582	930.591	503.014	5.511	7.188	1.676	34.885	130.040
Extreme Event	59.348	711.855	327.500	6.476	7.188	0.711	34.885	42.208

Table 3-32: Pile Reactions for All Limit States

Group	R_{V1}	R_{V2}	R_{V3}	R_{H1}	R_{H2}	R_{H3}	F_{RES}	F_{DR}
Service	70.725	66.732	56.527	13.858	13.415	4.000	31.273	19.890
Strength I (a)	100.423	72.611	1.536	17.158	14.068	4.000	35.226	32.111
Strength I (b)	108.480	91.706	48.838	18.053	16.190	4.000	38.243	32.111
Extreme Event	73.848	68.203	54.289	14.183	13.578	4.000	31.761	27.003

Pile Spacing will work for both the static and seismic conditions since $F_{DR} < F_{RES}$ for all Limit States.

For all the design calculations it is assumed that all calculations are made from the face of the stem. This is done in order to design a more conservative section.

Step 3: Stem Design Calculations

The values of shear and moment at the stem due to lateral earth pressure are computed to check the stem thickness and necessary reinforcement. Only the Service Limit State calculations are presented below.

General Procedures:

(i) Draw the free body diagrams for various structural components of the wall.

(ii) Compute the unfactored stabilizing forces and moments and the overturning forces and moments acting on each component.

(iii) Apply LRFD Factors to obtain the design shear force and bending moment for each component under all limit states.

(iv) Design the flexural and shear reinforcements of all structural components and compute their nominal shear strengths and nominal bending capacities. Repeat this step until the capacities are greater than the demands for all structural components and for all limit states.

Application of the procedures above:

(i) Draw the free body diagram.

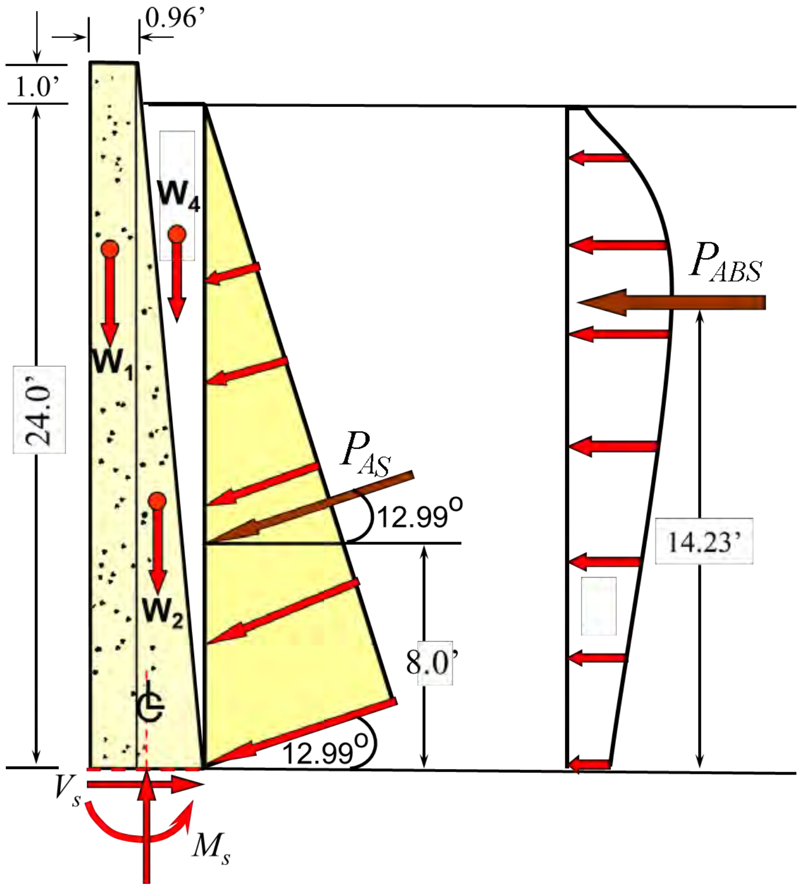


Figure 3-34: LRFD Stem Design for Static Condition

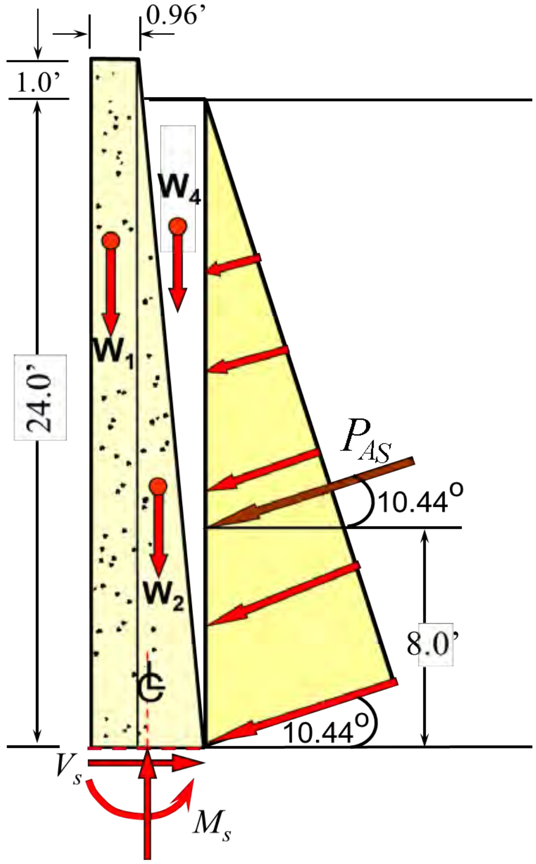


Figure 3-35: LRFD Stem Design for Seismic Condition

(ii) Compute the unfactored stabilizing forces and moments and the overturning forces and moments.

Check shear and moment for the static cases using Figure 3-34. For the moment, calculations are made along the centerline at the base of the stem. The centerline is located 1.52 inches from the left side of the stem. The calculation for the Service Limit State is demonstrated below as an example.

Static Case Calculations:

Compute unfactored static active earth forces acting on the stem.

$$P_{AS} = \left(\frac{h}{H}\right)^2 P_A = \left(\frac{24}{33.85}\right)^2 11.07 = 5.565 \text{ kips/ft}$$

$$P_{ASH} = 5.565 \ \cos(12.99^o) = 5.424 \text{ kips/ft}$$

$$P_{ASV} = 5.565 \ \sin(12.99^o) = 1.251 \text{ kips/ft}$$

The unfactored lateral earth force acting on the stem due to live load surcharge is estimated by using Eqn. 92 and Figure 2-27 (a). Integration of the stress should include only the part of horizontal stress that passes through the stem. The computer program CT-Rigid is used to perform the calculation, which yielded, $P_{ABS} = 6.275\ \textbf{kips/ft}$.

The stabilizing forces and moments acting on the stem are provided mostly by the self-weight of the concrete and soil. The overturning forces and moments include the horizontal forces due to active earth pressure and live load surcharge. They are summarized in Table 3-33.

Table 3-33: Unfactored stabilizing and overturning forces and moments acting on the stem

Stabilizing Forces and Moments					Overturning Forces and Moments			
Force	Vertical Component	Arm	Moment, M_{RES}		Force	Horizontal Component	Arm	Moment, M_{OT}
W_1	3.600	1.042	3.750		P_{ABS}	6.275	14.226	89.264
W_2	3.906	0.133	-0.519					
W_4	2.880	0.855	-2.462					
P_{ASV}	1.251	1.522	-1.904		P_{ASH}	5.424	8	43.388

Compute design shear and design moment:

$$V_u = P_{ASH} + P_{ABS} = 5.424 + 6.275 = 11.698\ \textbf{kips/ft}$$

$$M_u = \sum M_{RES} + \sum M_{OT} = -1.135 + 132.652 = 131.517\ \textbf{kips-ft/ft}$$

Seismic Case Calculations:

Compute unfactored seismic active earth forces acting on the stem:

$$P_{AES} = \left(\frac{h_s}{H}\right)^2 P_{AE} = \left(\frac{24}{33.85}\right)^2 19.07 = 9.586$$

$$P_{AESH} = 9.586\ \cos(10.44^o) = 9.430\ \textbf{kips/ft}$$
$$P_{AESV} = 9.586\ \sin(10.44^o) = 1.737\ \textbf{kips/ft}$$

The unfactored lateral earth force acting in the stem due to live load surcharge for the Extreme Event Limit State is neglected: $P_{ABS}^{seismic} = 0$. The calculation of the stabilizing forces and moments acting on the stem for the Extreme Event Limit State are tabulated in Table 3-34.

Table 3-34: Unfactored stabilizing and overturning forces and moments acting on the stem

Stabilizing Forces and Moments					Overturning Forces and Moments			
Force	Vertical Component	Arm	Moment, M_{RES}		Force	Horizontal Component	Arm	Moment, M_{OT}
W_1	3.600	1.042	3.750		$W_1\ K_H$	0.540	12.500	6.750
W_2	3.906	-0.133	-0.519		$W_2\ K_H$	0.586	8.333	4.883
W_4	2.880	-0.855	-2.462		$W_4\ K_H$	0.432	16.000	6.912
P_{AESV}	1.737	-1.522	-2.644		P_{AESH}	9.430	8.000	75.437

Compute design shear and design moment:

$$V_u = \sum \text{Horizontal Components} = 9.430 + 0.540 + 0.586 + 0.432 = 10.988 \text{ kips/ft}$$

$$M_u = \sum M_{RES} + \sum M_{OT} = -1.875 + 93.982 = 92.107 \text{ kips-ft/ft}$$

(iii) Apply LRFD Factors to obtain the design shear force and bending moment for all limit states.

Table 3-35: Ultimate Shear and Moment for Stem Design Calculations

Group	P_{ASH}	P_{ABS}	V_u		□M_{RES}	□M_{OT}	M_u
Service	5.424	6.275	11.698		-1.135	132.652	131.517
Strength I (a)	8.135	10.981	19.116		-2.410	221.294	218.884
Strength I (b)	8.135	10.981	19.116		-2.141	221.294	219.153
Extreme Event	9.430	0	10.988		-1.875	93.982	92.107

Step 4: Toe Design Calculations

For the toe design calculations, two sets of calculations will be made: first, for the structural element and second, for the pile connection. The calculations below are for the Service Limit State.

(i) Draw the free body diagram.

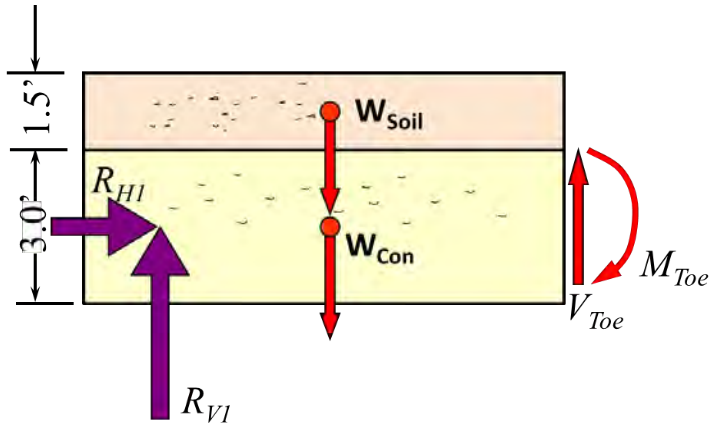


Figure 3-36: Free Body Diagram for Toe Design under both Static and Seismic Conditions

(ii) Compute the shear force and bending moment at the critical section of toe.

The shear force and bending moment at the critical section of toe is determined by examining the equilibrium of the factored forces and moments acting on the toe as shown in Figure 3-36. The calculation for the Service Limit State is demonstrated below as an example.

Check Shear of toe at the face of the stem:

$$W = W_{soil} + W_{conc} = 0.9 + 2.25 = 3.15 \text{ kips/ft}$$

$$V_u = \frac{R_{V1}}{3} - W = 23.575 - 3.15 = 20.425 \text{ kips/ft}$$

R_{v1} is the vertical component of the reaction force in each pile and is determined from Table 3-32.

Check moment of the Toe calculated at the face of the stem:

Force	Vertical Component	Moment Arm (ft)	Moment, M
W_{soil}	0.9	2.5	-2.25
W_{conc}	2.25	2.5	-5.625
$R_{v1}/3$	23.575	3.5	82.513

$M_u = \Sigma\, M = 74.638$

(iii) Obtain the design shear force and bending moment for all limit states.

Table 3-36: Toe Design Calculations for the Structural Element

Group	W_{soil}	W_{conc}	$R_{v1}/3$	V_u		M_{Wsoil}	M_{Wconc}	$M_{Rv1/3}$	M_u
Service	0.900	2.250	23.575	20.425		-2.250	-5.625	82.513	74.638
Strength I (a)	0.900	2.025	33.474	30.549		-2.250	-5.063	117.160	109.847
Strength I (b)	1.215	2.813	36.160	32.133		-3.037	-7.031	126.560	116.491
Extreme Event	0.900	2.250	24.549	21.399		-2.250	-5.625	85.922	78.047

(iv) Pile Connection Design Calculations

Check the toe pile spacing, which is 3.0 ft:

$$R_{V1} = 70.725 \quad \Rightarrow \quad w = \frac{70.725}{3.0} = 23.575 \text{ kips/ft}$$

$$V_u = 70.725/2 = 35.363 \text{ kips/ft}$$

$$M_u = -\frac{1}{10} w l_n^2 = -\frac{1}{10} 23.575 \times 3^2 = 21.218 \text{ kips/ft}$$

Table 3-37: Toe Design Calculations for Pile Connection Design

Group	R_{v1}	w	V_u	M_u
Service	70.725	23.575	35.363	21.218
Strength I (a)	100.423	33.474	50.212	30.127
Strength I (b)	108.480	36.160	54.245	32.544
Extreme Event	73.648	24.549	36.824	22.094

Step 5: Heel Design Calculations

For the heel design calculations, two different calculations will be made: first, the structural element calculation, and the second, the pile connection calculation. The calculations below are for the Service Limit State.

(i) Draw the free body diagram.

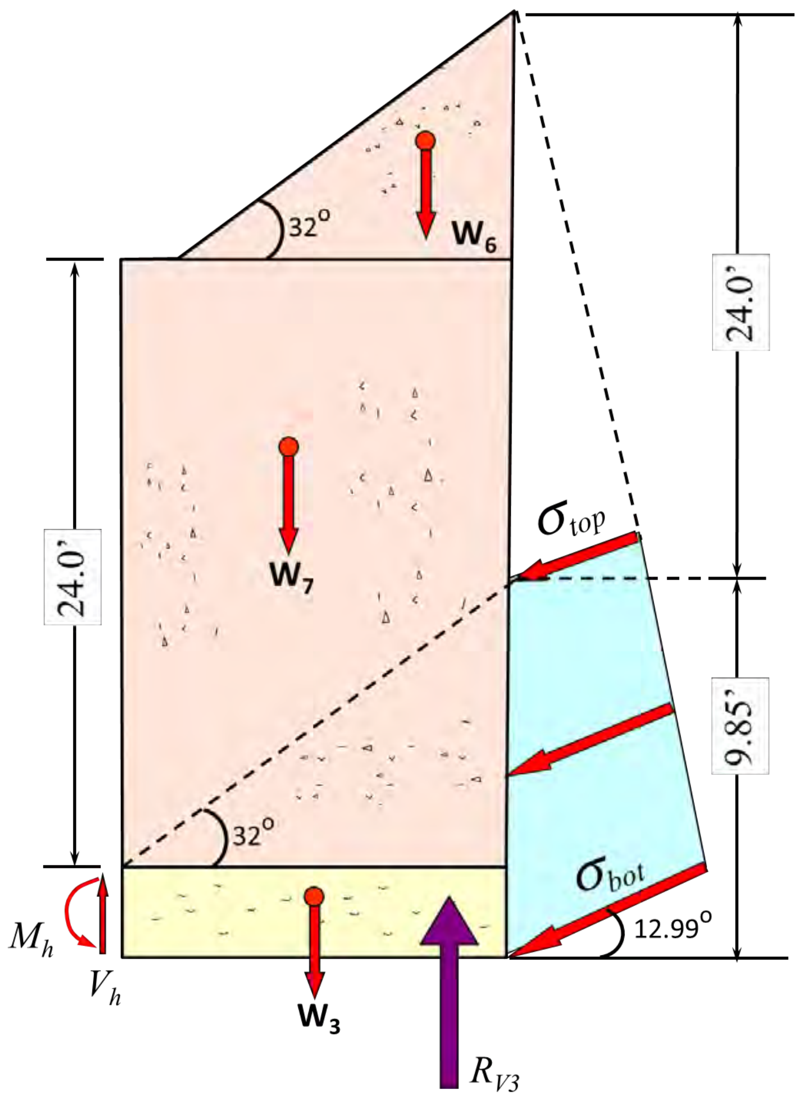


Figure 3-37: LRFD Heel Design for Static Loading Condition

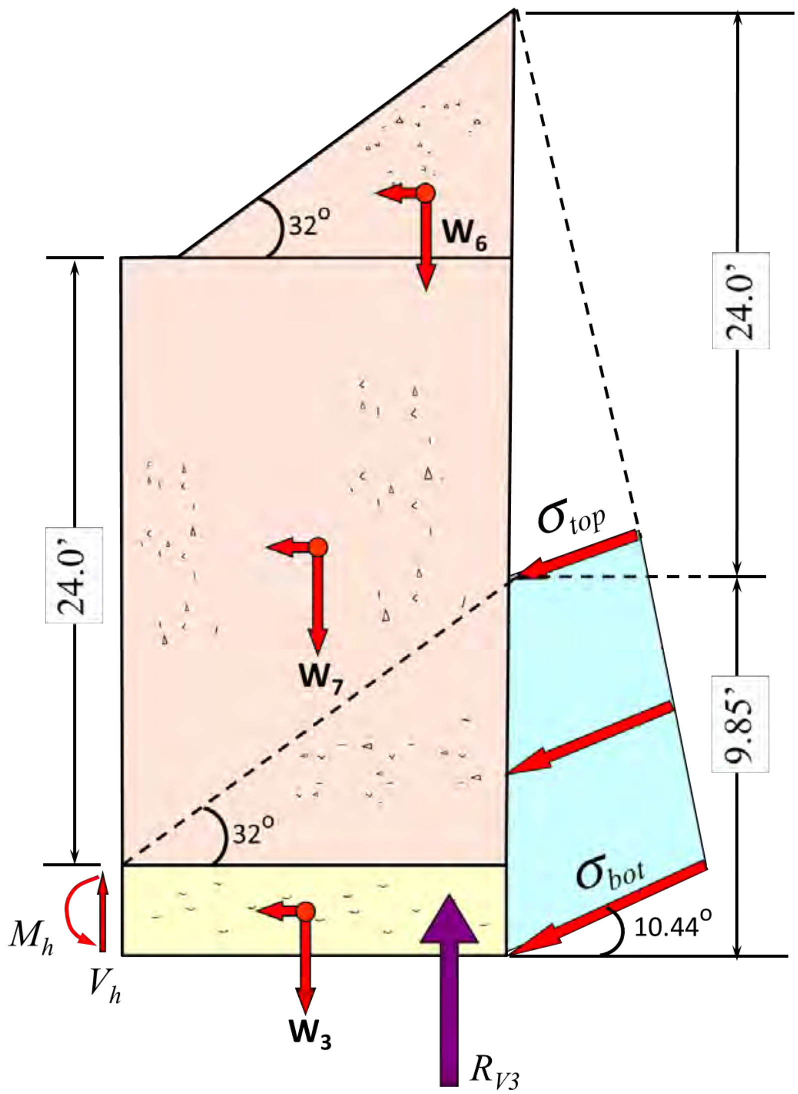


Figure 3-38: LRFD Heel Design for Seismic Loading Condition

(ii) Compute the shear force and bending moment at the critical section of heel.

The shear force and bending moment at the critical section of heel is determined by examining the equilibrium of the factored forces and moments acting on the heel. Only the trapezoidal portion of the active earth pressure as shown in Figure 3-37 and Figure 3-38 will impose shear force on the heel. The calculation for the Service Limit State is demonstrated below as an example.

Determine the Trapezoidal Stress Block:

The active earth pressure is assumed to be linearly distributed. The top and bottom values of the trapezoidal stress block is back-calculated from the total active earth-pressure force.

Static Case Calculations: $\sigma_{top} = 0.476$ ksf, $\sigma_{bottom} = 0.671$ ksf

Seismic Case Calculations: $\sigma_{top} = 0.813$ ksf, $\sigma_{bottom} = 1.146$ ksf

Factored forces and moments are applied to compute the design shear and design moment on heel, as demonstrated in Table 3-38 for Service Limit State and Table 3-39 for Extreme Event Limit State.

Static Case Calculations:

Table 3-38: Toe Design Calculations for Pile Connection Design

Force	Vertical Component (kips/ft)	Arm (ft)	Moment (kips–ft /ft)
W_6	0.5 (10.957) (6.85) (0.12) = 4.501	7.304	32.876
W_7	10.957 (24.0) (0.12) = 31.555	5.478	172.870
P_{AV}	$0.5(0.476+0.671)\ 9.85\cos(12.99°)\sin(12.99°)$ =1.237	10.957	13.556
W_3	10.957 (3.0) (0.15) = 4.931	5.478	27.011
$R_{v3}/4.5$	$56.538/4.5$ = −12.562	9.457	−118.791
P_{AH}	$0.5(0.476+0.671)\ 9.85\cos(12.99°)\cos(12.99°)$ = 5.363	3.144	−16.861

Compute the design shear and design moment:

$$V_u = \sum \text{Vertical Components} = 29.662 \text{ kips/ft}$$
$$M_u = \sum \text{Moments} = 110.660 \text{ kips-ft/ft}$$

Seismic Case Calculations:

Table 3-39: Toe Design Calculations for Pile Connection Design

Force	Vertical Component (kips/ft)	Arm (ft)	Moment (kips–ft /ft)
W_6	$0.5\,(10.957)\,(6.85)\,(0.12) = 4.501$	7.304	32.876
W_7	$10.957\,(24.0)\,(0.12) = 31.555$	5.478	172.870
P_{AEV}	$0.5(0.813+1.146)\;9.85\cos(10.44°)\sin(10.44°) = 1.718$	10.957	18.825
W_3	$(10.957)\,(3.0)\,(0.15) = 4.931$	5.478	27.011
$R_{v3}/4.5$	$54.289/4.5 = -12.064$	9.457	−114.088
P_{AEH}	$0.5(0.813+1.146)\;9.85\cos(10.44°)\cos(10.44°) = 9.325$	3.144	−29.316

Compute the design shear and design moment:

$$V_u = \sum \text{Vertical Components} = 30.640 \text{ kips/ft}$$

$$M_u = \sum \text{Moments} = 108.178 \text{ kips-ft/ft}$$

(iii) Obtain the design shear force and bending moment for all limit states.

Table 3-40: Heel Design Calculations for the Structural Element

Group	Design Shear Calculation						Design Moment Calculation				
	□W_S	W_3	P_{AV}	$R_{v3}/4.5$	V_u		□W_S	W_3	P_{AV}	$R_{v3}/4.5$	M_u
Service	36.056	4.931	1.237	−12.562	29.662		205.746	27.011	13.556	−118.79	110.660
Str. I (a)	36.056	4.438	1.856	−0.341	42.008		205.746	24.310	20.333	−3.228	221.869
Str. I (b)	48.678	6.163	1.856	−10.853	45.842		277.758	33.764	20.333	−102.63	203.931
Ext. Event	36.056	4.931	1.718	−12.064	30.640		205.746	27.011	18.825	−114.09	108.178

*Note: □$W_S = W_6 + W_7$

Pile Connection Design Calculations:

$$R_{V3} = 56.527 \quad \Rightarrow \quad w = 56.527/4.5 = 12.562 \text{ kips/ft}$$

$$V_u = 56.527/2 = 28.264 \text{ kips/ft}$$

$$M_u = -\frac{1}{10} w l_n^2 = -\frac{1}{10} 12.562 \times 4.5^2 = 25.438 \text{ kips/ft}$$

Table 3-41: Heel Design Calculations for Pile Connection Design

Group	R_{v3}	w	V_u	M_u
Service	56.527	12.562	28.264	25.438
Strength I (a)	1.536	0.341	0.768	0.695
Strength I (b)	48.838	10.853	28.269	24.419
Extreme Event	54.289	12.064	27.145	24.430

STRUCTURAL DESIGN:

Now that all of the calculations necessary to design the retaining wall have been made, these values will be compared and the retaining wall will be designed. Three separate designs will be analyzed: the stem, toe and heel. A table will be displayed summarizing the results obtained for each section and then the design will be determined.

STEM DESIGN:

Group	V_u	M_u
Service	11.698	131.517
Strength I (a)	19.116	218.884
Strength I (b)	**19.116**	**219.153**
Extreme Event	10.998	92.107

By observation, it can be seen that Strength I (b) Limit State controls the design of the stem.

Design the flexural and shear reinforcements:

Assuming #10 at 6 in.

Development length is $l_d = 42 \text{ in}$. (For the equations of basic development length of deformed bars and the modification factors associated with various conditions, please refer to AASHTO 5.11.2.)

$$A_s = 2.54 \text{ in}^2$$
$$h = 3.043 \text{ ft} = 36.520 \text{ in}$$
$$d = h - \text{clear cover} - d_b/2 = 36.520 - 2 - 1.270/2 = 33.885 \text{ in}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.54 \times 60{,}000}{0.85 \times 4{,}000 \times 12} = 3.735 \text{ in}$$

$$d_v = d - a/2 = 33.885 - 3.735/2 = 32.017 \text{ in}$$
$$d_v > 0.9 \times d = 0.9 \times 33.885 = 30.496 \text{ in (O.K.)}$$
$$d_v > 0.72 \times h = 0.72 \times 36.520 = 26.294 \text{ in (O.K.)}$$

In order to determine the required area of steel, the design moment, M_u, will be compared to the cracking moment (AASHTO 5.7.3.3.2):

$$M_{cr} = S_c f_r = \frac{2667.42 \times 0.74}{12} = 164.491 \text{ kips-ft/ft}$$

$$\text{where} \quad S_c \text{ (Section Modulus)} = \frac{1}{6}bh^2 = \frac{1}{6}1236.520^2 = 2667.42$$

$$f_r = 0.37\sqrt{f_c'} \text{ ksi} = 0.74 \text{ ksi} \text{ (AASHTO 5.4.2.6)}$$

The calculation for Strength I (b) is presented below as the example:

Check the flexural design: $M_u = 219.153 \text{ kips-ft/ft}$

(a) General requirement on factored flexural resistance (AASHTO 5.7.3.2)
The factored flexural resistance M_r shall be taken as: $M_r = \varphi M_n$

where: M_n = nominal flexural resistance (kip-in.)

φ = resistance factor as listed in Table 3-42 (AASHTO 5.5.4.2 and AASHTO 11.5.7).

Table 3-42: Resistance Factors for Tension-, Shear-, and Axial-Controlled RC Members

Limit State	φ_f (tension-controlled)	φ_v (shear)	φ_c (bearing on concrete)
Strength	0.9	0.9	0.7
Extreme Event	1.0	1.0	1.0

Check $M_r = \varphi M_n \geq M_u$:

$$\varphi M_n = \varphi A_s f_y (d - a/2) = \frac{1}{12} 0.9 \times 2.54 \times 60 \times (33.885 - 3.735/2)$$
$$= 365.958 \text{ kips-ft/f} > M_u = 219.153 \text{ kips-ft/ft (O.K.)}$$

(b) Minimum reinforcement (AASHTO 5.7.3.3.2)
The factored flexure resistance must be greater than or equal to the lesser of $1.2M_{cr}$ or $1.33M_u$.

$$1.2M_{cr} = 1.2 \times 164.491 = 197.389 \text{ kips-ft/ft}$$

$$1.33M_u = 219.153 \times 1.33 = 291.474 \text{ kips-ft/ft}$$

$$1.2M_{cr} < 1.33M_u \quad \Rightarrow \quad \text{check } \varphi M_n = 365.958 \geq 1.2M_{cr} \text{ (O.K.)}$$

(c) Additional requirement on longitudinal reinforcement (AASHTO 5.8.3.5)
At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_{ps} f_{ps} + A_s f_y = \frac{|M_u|}{d_v \varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left| \frac{V_u}{\varphi_v} - V_p \right| - 0.5V_s \right) \cot\theta \quad \text{(AASHTO Eq. 5.8.3.5-1)}$$

where θ = angle of crack (degrees)

$$\theta = 29 + 3500\varepsilon_s \quad \text{(AASHTO 5.8.3.4)}$$

$$\varepsilon_s = \frac{|M_u|/d_v + 0.5N_u + |V_u - V_p| - A_{ps} f_{po}}{\left(E_s A_s + E_p A_{ps}\right)} \qquad \text{(AASHTO Eq. 5.8.3.4.2-4)}$$

$M_u = 219.153$ kips-ft/ft
$d_v = 32.017$ in
$N_u = -\left(W_1 + W_2 + W_6 + P_{AV}\right) = -15.148$ kips/ft (compression is negative)
$V_u = 19.116$ kips/ft

$V_p = A_{ps} = 0$

$E_s = 29000$ ksi
$A_s = 2.54$ in^2

$$\varepsilon_s = \frac{\dfrac{219.153 \times 12}{32.017} - 0.5 \times 15.148 + 19.116}{29{,}000 \times 2.54} = 0.001272$$

$$\theta = 29 + 3500\,\varepsilon_s = 33.452°$$

$$A_{s,\min} = \frac{1}{f_y}\left[\frac{|M_u|}{d_v \varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left| \frac{V_u}{\varphi_v} - V_p \right| - 0.5V_s \right) \cot\theta \right]$$

$$= \frac{1}{60}\left[\frac{219.153 \times 12}{32.017 \times 0.9} + 0.5\frac{-15.148}{0.7} + \left(\left|\frac{19.116}{0.9} - 0\right| - 0\right)\cot(33.452^{o})\right] = 1.877$$

$$A_s = 2.54 \text{ in}^2 > A_{s,min} = 1.877 \text{ in}^2 \text{ (O.K.)}$$

Table 3-43: Checklist for the Flexural Design of Stem

Limit State	φ	φM_n	M_u	$1.2M_{cr}$	$1.33M_u$	A_s	$A_{s,min}$
Service	0.9	365.958	131.517	197.389	174.918	2.540	1.126
Strength I (a)	0.9	365.958	218.884	197.389	291.116	2.540	1.916
Strength I (b)	0.9	365.958	219.153	197.389	291.474	2.540	1.877
Extreme Event I	1.0	406.620	92.107	197.389	122.503	2.540	0.781

Check the shear design:

$$V_u = 19.116 \text{ kips/ft}$$

Per AASHTO 5.8.2.1, the factored shear resistance V_r shall be taken as:

$V_r = \varphi V_n$

where V_n = nominal shear resistance (kip)

φ = resistance factor as listed in Table 3-42 (AASHTO 5.5.4.2).

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3)

$$V_c = 0.0316\beta\sqrt{f'_c}\,b d_v$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$

where d_v = effective shear depth as determined in AASHTO 5.8.2.9 (in)

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

θ = angle of inclination of diagonal compressive stresses (degrees)

α = angle of inclination of transverse reinforcement to longitudinal axis (degrees)

A_v = area of shear reinforcement within a distance s (in^2)

s = spacing of transverse reinforcement (in)

β is determined by the following equations (AASHTO 5.8.3.4):

If minimum shear reinforcement is provided, then

$\beta = 4.8/1 + 750\varepsilon_s$

otherwise,

$$\beta = \frac{4.8}{1+750\varepsilon_s} \times \frac{51}{39+S_{xe}}$$

where $\quad S_{xe} = S_x\left(\frac{1.38}{a_g + 0.63}\right)$ is the crack spacing parameter (AASHTO Eq. 5.8.3.4.2-5)

$$12 \text{ in} \le S_{xe} \le 80 \text{ in}$$

$$S_x = d - a/2 = 33.885 - 3.735/2 = 32.017 \text{ in}$$

a_g (maximum aggregate size) = 1.5 in.

$$12 \le S_{xe} = 32.017\left(\frac{1.38}{1.5+0.63}\right) = 20.744 \le 80 \quad \text{(O.K.)}$$

$$\beta = \frac{4.8}{1+750\times 0.001272} \times \frac{51}{39+20.744} = 2.097$$

$$\Rightarrow \quad V_c = 0.0316 \times 2.097 \times \sqrt{4} \times 12 \times 32.017 = 50.923 \text{ kips/ft}$$

$$\text{A}_v = 0 \quad \Rightarrow \quad V_s = 0$$

Check $V_r = \varphi V_n \ge V_u$:

$$\varphi V_n = \varphi\left(V_c + V_s + V_p\right) = 0.9\left(50.923 + 0 + 0\right) = 45.831 \text{ kips/ft} \;>\; V_u = 19.116 \text{ kips/ft (O.K.)}$$

Table 3-44: Checklist for the Shear Design of Stem

Limit State	φ	N_u	ε_s	β	V_c	φV_n	V_u
Service	0.9	11.637	0.000749	2.624	63.707	57.337	11.698
Strength I (a)	0.9	11.512	0.001295	2.079	50.471	45.424	19.116
Strength I (b)	0.9	15.148	0.001272	2.097	50.923	45.831	19.116
Extreme Event I	1.0	12.124	0.000536	2.923	70.985	70.985	10.988

TOE DESIGN:

For the toe design, two designs will be developed: first, for the structural element and second, for the pile connection.

Structural Element Design:

Group	V_u	M_u
Service	20.425	74.638
Strength I (a)	30.549	109.847
Strength I (b)	**32.133**	**116.491**
Extreme Event	21.399	78.047

By observation, it can be seen that Strength I (b) controls the design of the toe.

Design the flexural and shear reinforcements:

Assuming #10 at 6”

$$A_s = 2.54 \text{ in}^2$$
$$h = 3 \text{ ft} = 36 \text{ in}$$

$$d = h - \text{clear cover} - \frac{d_b}{2} = 36 - 6 - \frac{1.270}{2} = 29.365 \text{ in}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.54(60{,}000)}{0.85(4{,}000)(12)} = 3.735 \text{ in}$$

$$d_v = d - \frac{a}{2} = 29.365 - \frac{3.735}{2} = 27.497 \text{ in}$$
$$d_v > 0.9 \times d = 0.9 \times 29.365 = 26.429 \text{ in , OK}$$
$$d_v > 0.72 \times h = 0.72 \times 36 = 25.920 \text{ in , OK}$$

In order to determine the required area of steel, the design moment, M_u, will be compared to the cracking moment (AASHTO 5.7.3.3.2).

$$M_{cr} = S_c f_r = \frac{2592(0.74)}{12} = 159.84 \text{ kips-ft/ft}$$

where $\quad S_c = \text{Section Modulus} = \frac{1}{6}bh^2 = \frac{1}{6}12(36)^2 = 2592$

$$f_r = 0.37\sqrt{f_c'} \text{ ksi} = 0.74 \text{ ksi} \quad \text{(AASHTO 5.4.2.6)}$$

The calculation for Strength I (b) is presented below as the example.

Check the flexural design:

$$M_u = 116.491 \text{ kips-ft/ft}$$

(a) General requirement on factored flexural resistance (AASHTO 5.7.3.2)
The factored flexural resistance M_r shall be taken as:

$M_r = \varphi M_n$

where: M_n = nominal flexural resistance (kip-in.)

φ = resistance factor as listed in Table 3-45 (AASHTO 5.5.4.2 and AASHTO 11.5.7).

Table 3-45: Resistance Factors for Tension-, Shear-, and Axial- Controlled RC Members

Limit State	φ_f (tension-controlled)	φ_v (shear)	φ_c (bearing on concrete)
Strength	0.9	0.9	0.7
Extreme Event	1.0	1.0	1.0

Check $M_r = \varphi M_n \geq M_u$:

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2}\right) = \frac{0.9(2.54)(60)\left(29.365 - \frac{3.735}{2}\right)}{12}$$

$$= 314.295 \text{ kips-ft/f} > M_u = 116.491 \text{ kips-ft/ft (O.K.)}$$

(b) Minimum reinforcement (AASHTO 5.7.3.3.2)

The factored flexure resistance must be greater than or equal to the lesser of $1.2M_{cr}$ or $1.33M_u$.

$$1.2M_{cr} = 1.2 \times 159.840 = 191.808 \text{ kips-ft/ft}$$

$$1.33M_u = 116.491 \times 1.33 = 154.934 \text{ kips-ft/ft}$$

$$1.2M_{cr} < 1.33M_u \Rightarrow \text{ check } \varphi M_n = 314.295 \geq 1.2M_{cr} \text{ (O.K.)}$$

(c) Additional requirement on longitudinal reinforcement (AASHTO 5.8.3.5)
At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_{ps}f_{ps} + A_s f_y = \frac{|M_u|}{d_v \varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta \text{ (AASHTO Eq. 5.8.3.5-1)}$$

where θ = angle of crack (degrees)

$$\theta = 29 + 3500\varepsilon_s \text{ (AASHTO 5.8.3.4)}$$

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5N_u + \left|V_u - V_p\right| - A_{ps}f_{po}}{\left(E_s A_s + E_p A_{ps}\right)} (AASHTO\, Eq. 5.8.3.4.2 - 4)$$

$$M_u = 116.491 \text{ kips-ft/ft}$$
$$d_v = 27.497 \text{ in}$$
$$N_u = -\left(W_1 + W_2 + W_6 + P_{AV}\right) = 0 \text{ kips/ft (Compression is negative.)}$$
$$V_u = 32.113 \text{ kips/ft}$$

$$V_p = A_{ps} = 0$$

$$E_s = 29000 \text{ ksi}$$
$$A_s = 2.54 \text{ in}^2$$

$$\varepsilon_s = \frac{\frac{116.491(12)}{27.497} - 0.5(0) + (32.133)}{(29000)(2.54)} = 0.001126$$

$$\theta = 29 + 3500\varepsilon_s = 32.941°$$

$$A_{s,\min} = \left[\frac{|M_u|}{d_v \varphi_f} + 0.5 \frac{N_u}{\varphi_c} + \left(\left| \frac{V_u}{\varphi_v} - V_p \right| - 0.5 V_s \right) \cot\theta \right] \Big/ f_y$$

$$= \left[\frac{116.491 \times 12}{27.497 \times 0.9} + 0.5 \frac{-0}{0.7} + \left(\left| \frac{32.133}{0.9} - 0 \right| - 0 \right) \cot(32.941^\circ) \right] \Big/ 60 = 1.860$$

$$A_s = 2.54 \text{ in}^2 > A_{s,\min} = 1.860 \text{ in}^2 \quad \text{(O.K.)}$$

Table 3-46: Checklist for the Flexural Design of Toe

Limit State	φ	φM_n	M_u	$1.2M_{cr}$	$1.33M_u$	A_s	$A_{s,min}$
Service	0.9	314.295	74.638	191.808	99.268	2.540	1.220
Strength I (a)	0.9	314.295	109.847	191.808	146.097	2.540	1.768
Strength I (b)	0.9	314.295	116.491	191.808	154.934	2.540	1.860
Extreme Event I	1.0	349.216	78.047	191.808	103.803	2.540	1.147

Check the shear design:

$V_u = 32.133$ kips/ft

Per AASHTO 5.8.2.1, the factored shear resistance V_r shall be taken as:

$V_r = \varphi V_n$

where: V_n = nominal shear resistance (kip)

φ = resistance factor as listed in Table 3-45 (AASHTO 5.5.4.2).

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3)

$$V_c = 0.0316 \beta \sqrt{f'_c}\, b d_v$$

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s}$$

where d_v = effective shear depth as determined in AASHTO 5.8.2.9 (in.)

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

θ = angle of inclination of diagonal compressive stresses (degrees)

α = angle of inclination of transverse reinforcement to longitudinal axis (degrees)

A_v = area of shear reinforcement within a distance s (in^2)

s = spacing of transverse reinforcement (in.)

β is determined by the following equations (AASHTO 5.8.3.4):

$\beta = \dfrac{4.8}{1+750\varepsilon_s}$, if minimum shear reinforcement is provided;

$\beta = \dfrac{4.8}{1+750\varepsilon_s} \cdot \dfrac{51}{39+S_{xe}}$, otherwise.

where $S_{xe} = S_x\left(\dfrac{1.38}{a_g+0.63}\right)$ is the crack spacing parameter (AASHTO Eq. 5.8.3.4.2-5)

$$12 \text{ in} \le S_{xe} \le 80 \text{ in}$$

$$S_x = d - a/2 = 29.365 - 3.735/2 = 27.497 \text{ in}$$

a_g (maximum aggregate size) = 1.5 in.

$$12 \le S_{xe} = 27.497\left(\frac{1.38}{1.5+0.63}\right) = 17.815 \le 80 \quad \text{(O.K.)}$$

$$\beta = \frac{4.8}{(1+750\times 0.001126)}\frac{51}{(39+17.815)} = 2.336$$

$$\Rightarrow \quad V_c = (0.0316)(2.336)\sqrt{4}(12)(27.497) = 48.707 \text{ kips/ft}$$

$$\text{A}_v = 0 \quad \Rightarrow \quad V_s = 0$$

Check $V_r = \varphi V_n \ge V_u$:

$$\varphi V_n = \varphi\left(V_c + V_s + V_p\right) = 0.9(48.707+0+0) = 43.836 \text{ kips/ft} \;>\; V_u = 32.133 \text{ kips/ft (O.K.)}$$

Table 3-47: Checklist for the Shear Design of Toe

Limit State	φ	N_u	ε_s	β	V_c	φV_n	V_u
Service	0.9	0	0.000719	2.799	58.361	52.525	20.425
Strength I (a)	0.9	0	0.001066	2.395	49.942	44.948	30.549
Strength I (b)	0.9	0	0.001126	2.336	48.707	43.836	32.133
Extreme Event I	1.0	0	0.000753	2.754	57.426	57.426	21.399

Pile Connection Design:

Group	V_u	M_u
Service	35.363	21.218
Strength I (a)	50.212	30.127
Strength I (b)	**54.245**	**32.544**
Extreme Event	35.363	21.218

By observation it can be seen that Strength I (b) controls the design of the pile connection at toe. The design procedure of the pile connection is identical to that of the structural element, except that the steel rebars are put parallel to the wall to prevent the structural failure of pile foundation in this direction. One can try 2 #6 at 12" for the pile connection design at toe.

HEEL DESIGN:

Structural Element Design

Group	V_u	M_u
Service	29.662	110.660
Strength I (a)	42.008	**221.869**
Strength I (b)	**45.842**	203.931
Extreme Event	30.640	108.178

By observation it can be seen that Strength I (b) and Strength I (a) controls the design of the shear and moment, respectively, for the heel. However, to achieve a more economical design, we do not pick the maximum shear and maximum moment directly as the design shear and design moment. Instead, we will check the capacity of the heel design against the demands under all four limit states independently.

Design the flexural and shear reinforcements:

Assuming #11 at 6"

$$A_s = 3.12\ in^2$$

$$h = 3\text{ ft} = 36\text{ in}$$

$$d = h - \text{clear cover} - \frac{d_b}{2} = 36 - 2 - \frac{1.410}{2} = 33.295\text{ in}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.12(60{,}000)}{0.85(4{,}000)(12)} = 4.588\text{ in}$$

$$d_v = d - \frac{a}{2} = 33.295 - \frac{4.588}{2} = 31.001\text{ in}$$

$$d_v > 0.9 \times d = 0.9 \times 33.295 = 29.966\text{ in (O.K.)}$$

$$d_v > 0.72 \times h = 0.72 \times 36 = 25.920\text{ in (O.K.)}$$

In order to determine the required area of steel, the design moment, M_u, will be compared to the cracking moment (AASHTO 5.7.3.3.2).

$$M_{cr} = S_c f_r = \frac{2592(0.74)}{12} = 159.84\text{ kips-ft/ft}$$

$$\text{where}\quad S_c = \text{Section Modulus} = \frac{1}{6}bh^2 = \frac{1}{6}12(36)^2 = 2592$$

$$f_r = 0.37\sqrt{f_c'}\text{ ksi} = 0.74\text{ ksi (AASHTO 5.4.2.6)}$$

The calculation for Strength I (a) is presented below as the example.

Check the flexural design:

$M_u = 221.869$ **kips-ft/ft**

(a) General requirement on factored flexural resistance (AASHTO 5.7.3.2)
The factored flexural resistance M_r shall be taken as:

$M_r = \varphi M_n$

where: M_n = nominal flexural resistance (kip-in.)

φ = resistance factor as listed in Table 3-48 (AASHTO 5.5.4.2 and AASHTO 11.5.7).

Table 3-48: Resistance Factors for Tension-, Shear-, and Axial- Controlled RC Members

Limit State	φ_f (tension-controlled)	φ_v (shear)	φ_c (bearing on concrete)
Strength	0.9	0.9	0.7
Extreme Event	1.0	1.0	1.0

Check $M_r = \varphi M_n \geq M_u$:

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right) = \frac{0.9(3.12)(60)\left(33.295 - \frac{4.588}{2}\right)}{12}$$

$$= 435.252 \text{ kips-ft/f} > M_u = 221.869 \text{ kips-ft/ft (O.K.)}$$

(b) Minimum reinforcement (AASHTO 5.7.3.3.2)
The factored flexure resistance must be greater than or equal to the lesser of $1.2M_{cr}$ or $1.33M_u$.

$$1.2M_{cr} = 1.2(159.840) = 191.808 \text{ kips-ft/ft}$$

$$1.33M_u = 221.869(1.33) = 295.086 \text{ kips-ft/ft}$$

$$1.2M_{cr} < 1.33M_u \Rightarrow \text{check } \varphi M_n = 435.252 \geq 1.2M_{cr} \text{ (O.K.)}$$

(c) Additional requirement on longitudinal reinforcement (AASHTO 5.8.3.5)
At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_{ps}f_{ps} + A_s f_y = \frac{|M_u|}{d_v \varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left| \frac{V_u}{\varphi_v} - V_p \right| - 0.5V_s \right)\cot\theta \text{ (AASHTO Eq. 5.8.3.5-1)}$$

where θ = angle of crack (degrees)

$$\theta = 29 + 3500\varepsilon_s \text{ (AASHTO 5.8.3.4)}$$

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{(E_sA_s + E_pA_{ps})} \text{(AASHTO Eq.5.8.3.4.2 – 4)}$$

$M_u = 221.869$ kips-ft/ft

$d_v = 31.001$ in

$N_u = -(W_1 + W_2 + W_6 + P_{AV}) = 0$ kips/ft (Compression is negative.)

$V_u = 42.008$ kips/ft

$V_p = A_{ps} = 0$

$E_s = 29000$ ksi

$A_s = 3.12 \text{ in}^2$

$$\varepsilon_s = \frac{\frac{221.869(12)}{31.001} - 0.5(0) + (42.008)}{(29000)(3.12)} = 0.001413$$

$$\theta = 29 + 3500\varepsilon_s = 33.946°$$

$$A_{s,\min} = \left[\frac{|M_u|}{d_v\varphi_f} + 0.5\frac{N_u}{\varphi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta\right] \Big/ f_y$$

$$= \left[\frac{221.869 \times 12}{31.001 \times 0.9} + 0.5\frac{-0}{0.7} + \left(\left|\frac{42.008}{0.9} - 0\right| - 0\right)\cot(33.946°)\right] \Big/ 60 = 2.746$$

$A_s = 3.12 \text{ in}^2 > A_{s,\min} = 2.746 \text{ in}^2$ (O.K.)

Table 3-49: Checklist for the Flexural Design of Heel

Limit State	φ	φM_n	M_u	$1.2M_{cr}$	$1.33M_u$	A_s	$A_{s,min}$
Service	0.9	435.252	110.660	191.808	147.178	3.12	1.679
Strength I (a)	0.9	435.252	221.869	191.808	295.086	3.12	2.746
Strength I (b)	0.9	435.252	203.931	191.808	271.228	3.12	2.729
Extreme Event I	1.0	483.614	108.178	191.808	143.877	3.12	1.521

Check the shear design:

$V_u = 42.008$ kips/ft

Per AASHTO 5.8.2.1, the factored shear resistance V_r shall be taken as:

$V_r = \varphi V_n$

where: V_n = nominal shear resistance (kip)

φ = resistance factor as listed in Table 3-48 (AASHTO 5.5.4.2).

$V_n = V_c + V_s + V_p$ (AASHTO 5.8.3.3)

$$V_c = 0.0316\beta\sqrt{f'_c}\,bd_v$$

$$V_s = \frac{A_v f_y d_v\left(\cot\theta + \cot\alpha\right)\sin\alpha}{s}$$

where d_v = effective shear depth as determined in AASHTO 5.8.2.9 (in.)

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

θ = angle of inclination of diagonal compressive stresses (degrees)

α = angle of inclination of transverse reinforcement to longitudinal axis (degrees)

A_v = area of shear reinforcement within a distance s (in^2)

s = spacing of transverse reinforcement (in.)

β is determined by the following equations (AASHTO 5.8.3.4):

$\beta = \dfrac{4.8}{1+750\varepsilon_s}$, if minimum shear reinforcement is provided;

$\beta = \dfrac{4.8}{1+750\varepsilon_s}\cdot\dfrac{51}{39+S_{xe}}$, otherwise.

where $S_{xe} = S_x\left(\dfrac{1.38}{a_g+0.63}\right)$ is the crack spacing parameter (AASHTO Eq. 5.8.3.4.2-5)

$$12\text{ in} \le S_{xe} \le 80\text{ in}$$

$$S_x = d - a/2 = 33.295 - 4.588/2 = 31.001 \text{ in}$$

a_g (maximum aggregate size) = 1.5 in.

$$12 \le S_{xe} = 31.001\left(\frac{1.38}{1.5+0.63}\right) = 20.085 \le 80 \quad \text{(O.K.)}$$

$$\beta = \frac{4.8}{(1+750\times 0.001413)}\frac{51}{(39+20.085)} = 2.011$$

$$\Rightarrow \quad V_c = (0.0316)(2.011)\sqrt{4}(12)(31.001) = 47.284 \text{ kips/ft}$$

$$\text{A}_v = 0 \quad \Rightarrow \quad V_s = 0$$

Check $V_r = \varphi V_n \ge V_u$:

$$\varphi V_n = \varphi\left(V_c + V_s + V_p\right) = 0.9(47.284+0+0) = 42.556 \text{ kips/ft} \;>\; V_u = 42.008 \text{ kips/ft (O.K.)}$$

Table 3-50: Checklist for the Shear Design of Heel

Limit State	φ	N_u	ε_s	β	V_c	φV_n	V_u
Service	0.9	0	0.000801	2.588	60.846	54.761	29.662
Strength I (a)	0.9	0	0.001413	2.011	47.284	42.556	42.008
Strength I (b)	0.9	0	0.001379	2.037	47.884	43.095	45.842
Extreme Event I	1.0	0	0.000801	2.588	60.840	60.840	30.640

Pile Connection Design

Group	V_u	M_u
Service	28.264	**25.438**
Strength I (a)	0.768	0.695
Strength I (b)	**28.269**	24.419
Extreme Event	27.145	24.430

The design procedure of the pile connection is identical to that of the structural element, except that the steel rebars are put parallel to the wall to prevent the structural failure of pile foundation in this direction. One can try 2 #6 at 12" for the pile connection design at heel.

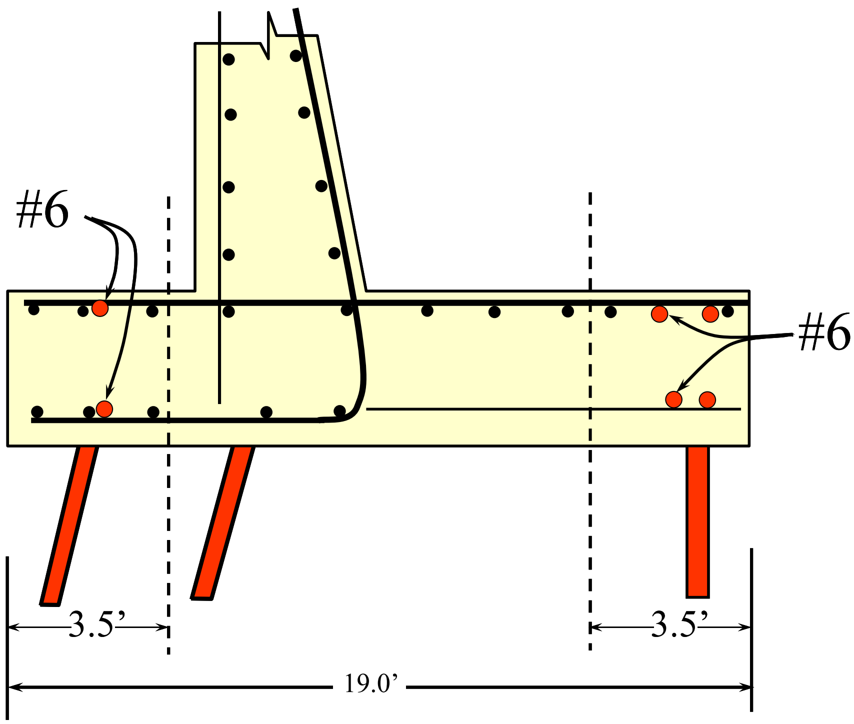


Figure 3-39: Final Cross-section for Pile Supported Retaining Wall

CHAPTER 4 **NONGRAVITY EARTH RETAINING STRUCTURES**

Non-gravity earth retaining systems are constructed of vertical structural members consisting of above-ground height (*H*) and partially embedded soldier piles or continuous sheet piles into the ground with and embedment depth (*D*) as shown in Figure 4-1.

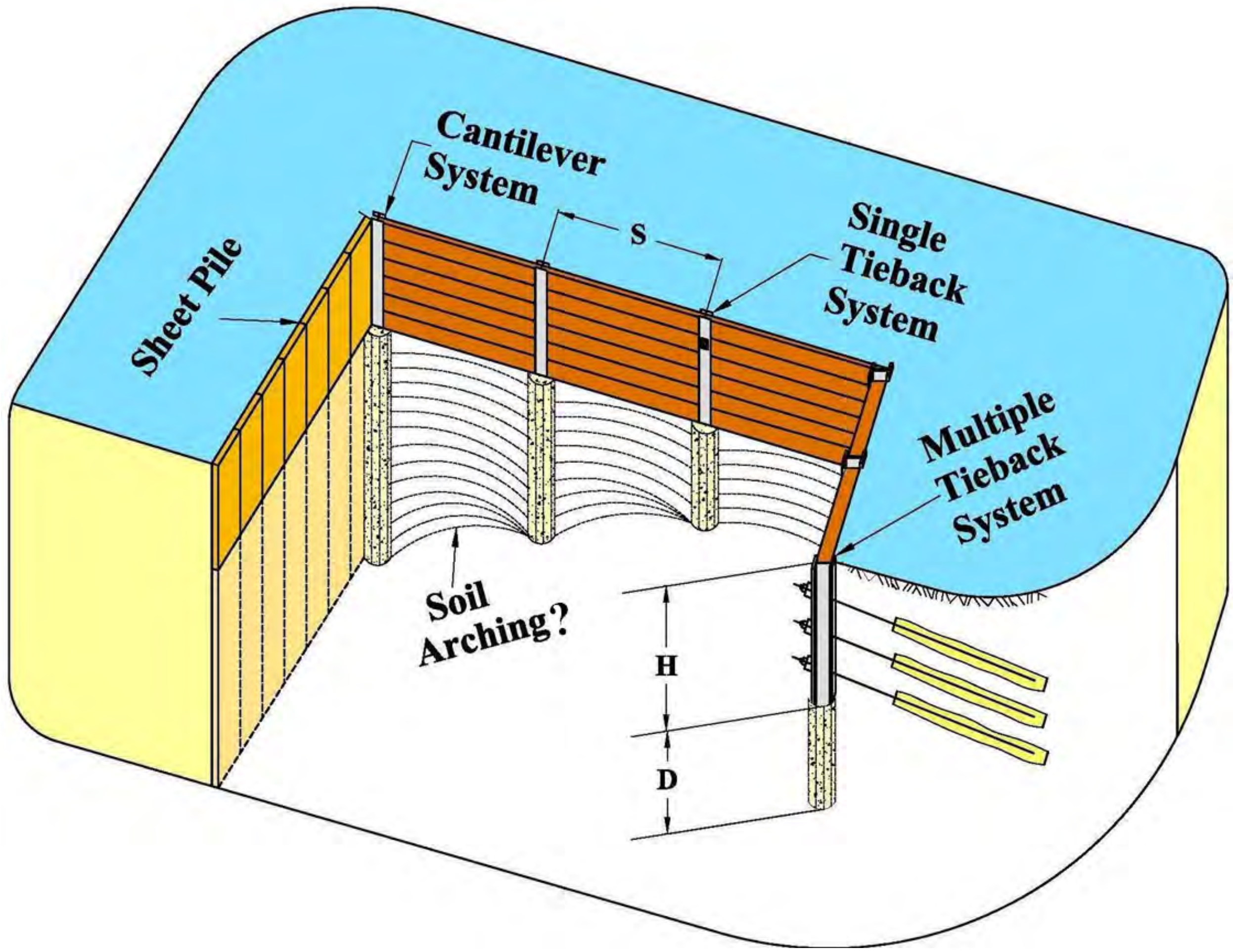


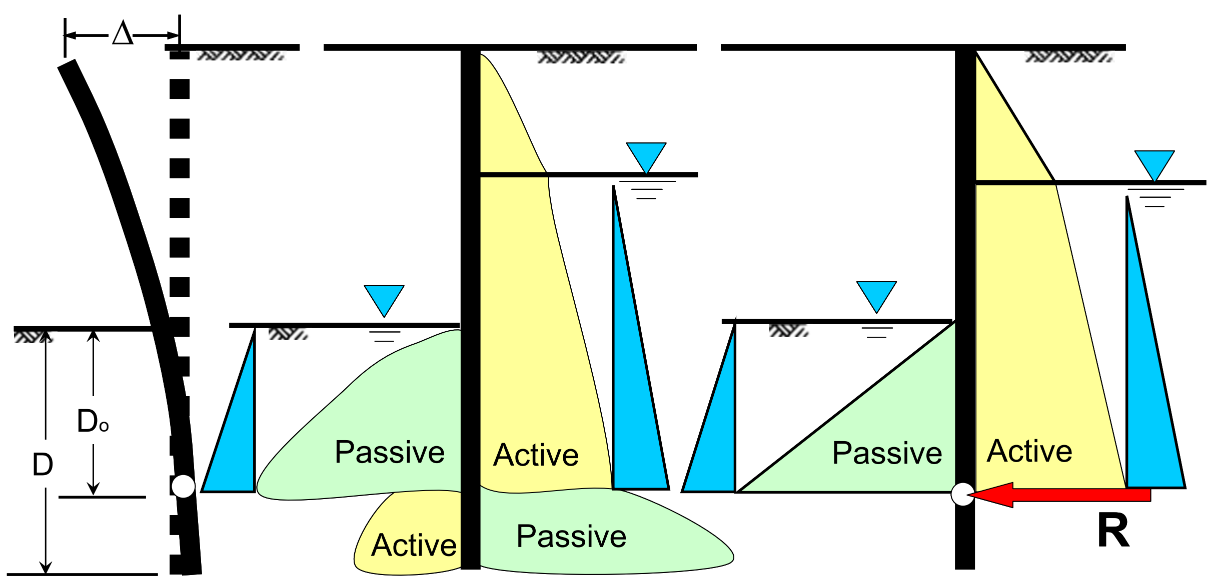
Figure 4-1: Non-Gravity Earth Retaining Systems

These types of walls are either cantilever or anchored walls. The magnitude of load distribution against the wall for the cantilever varies linearly with depth. In contrast, the magnitude of load for the anchored wall is distributed uniformly with depth.

4.1. NON-GRAVITY CANTILEVERED EARTH RETAINING SYSTEM

Non-gravity cantilevered walls are constructed of vertical structural members consisting of partially embedded soldier piles or continuous sheet piles. Soldier piles may be constructed with driven steel piles, treated timber, precast concrete or steel piles placed in drilled holes and backfilled with concrete or cast-in-place reinforced concrete. Continuous sheet piles may be constructed with driven precast pre-stressed concrete sheet piles or steel sheet piles. Soldier piles are faced with either treated-timber, reinforced shotcrete, reinforced cast-in-place concrete, precast concrete or metal elements. This type of wall depends on passive resistance of the foundation material and the moment resisting capacity of the vertical structural members for stability. Therefore, its maximum height is limited by the competence of the foundation material and the moment resisting capacity of the vertical structural members. The economical height of this type of wall is generally limited to a maximum height of 18 feet or less.

Non-gravity cantilever retaining walls are analyzed by assuming that the vertical structural member rotates at point, O, at the distance, D_O, below the excavation line, as shown in Figure 4-2 (a). The realistic load distribution is shown in Figure 4-2 (b). As a result, the mobilized active pressure develops above point O in the back of the wall and below point O in the front of the wall. The mobilized passive pressure develops in front of the wall above point O and at the back of the wall below point O. The simplified load distribution is shown in Figure 4-2 (c). Force R is assumed at point O to compensate the resultant net active and passive pressure below point of rotation O. D_O is increased by 20% to approximate the total embedment depth of the vertical wall element ($D = 1.2D_O$, AASHTO 3.11.5.6). Load distributions for typical non-gravity cantilever earth retaining systems are discussed in §4.5.3.1 of this manual.



(a) Deformed Wall (b) Realistic Load Distribution (c) Simplified Load Distribution

Figure 4-2: Non-Gravity Cantilever Retaining Walls

4.2. SHEET PILE WALLS

Cantilever sheet pile wall is a common type of temporary shoring system made of individual sheet piles driven side by side into the ground and, thus, forming a continuous vertical wall. Due to the large deflections that may develop, cantilever sheet pile walls are mainly used for temporary excavations less than about 18 feet. Cantilever sheet pile walls with adequate structural capacity and embedment depth can be used for permanent retaining walls. Figure 4-3 shows a typical cantilever sheet pile wall supporting bridge abutment (FHWA, NHI 2007).

Figure 4-3: Sheet Pile Wall (FHWA, NHI 2007)

4.3. SOLDIER PILE WALLS

Soldier piles are steel "I Beams" installed vertically into drilled holes and encased in concrete. The typical drilled-hole diameter for soldier beams is between 18 and 48 inches, and the beams are usually placed on spacing of 6 to 10 feet. Soldier piles can provide support by acting as a cantilever or by being braced by either tiebacks or internal struts. For wall heights below 15 feet, most soldier beam walls are installed without tiebacks. For wall heights in excess of 15 feet, it is usually more economical to employ tiebacks rather than cantilevers with larger steel I beams. Tiebacks are steel tendons grouted into a drilled hole that is inclined through the retained soil and anchored into competent material. Tiebacks are then post-tensioned and "locked-off" at a pre-determined load to minimize wall deflection. Lagging, typically consisting of timber, reinforced shotcrete, or pre-cast concrete panels, is installed next. The lagging spans the distance between the soldier piles to prevent soil movement between the piles.

The effective width of a soldier pile is, generally, considered to be the element width *b* (i.e., dimension of the soldier pile taken parallel to the line of the wall for driven piles or drilled piles backfilled with material other then concrete). The effective width of the soldier piles may be taken as the diameter of the drilled-hole when concrete is used. A phenomenon known as soil arching, as is shown in Figure 4-4, however, can greatly increase the effective width described above. Arching action of the soil between soldier piles can increase the effective width of a soldier pile up to 3 times the diameter of the hole or the width of the vertical element (AASHTO, 3.11.5.6).

If the element is embedded in soft clay having a stability number less than 3, soil arching will not occur and the actual element width shall be used as the effective width for passive resistance. Where a vertical element is embedded in rock (AASHTO, Figure 3.11.5.6-2) the passive resistance of the rock is assumed to develop through the shear failure of a rock wedge equal in width to the vertical element (*b*) and defined by a plane extending upward from the base of the element at an angle of 45°. For the active zone behind the wall and below the mudline or ground line in front of the wall, the active pressure is assumed to act over one vertical element width (*b*) in all cases.

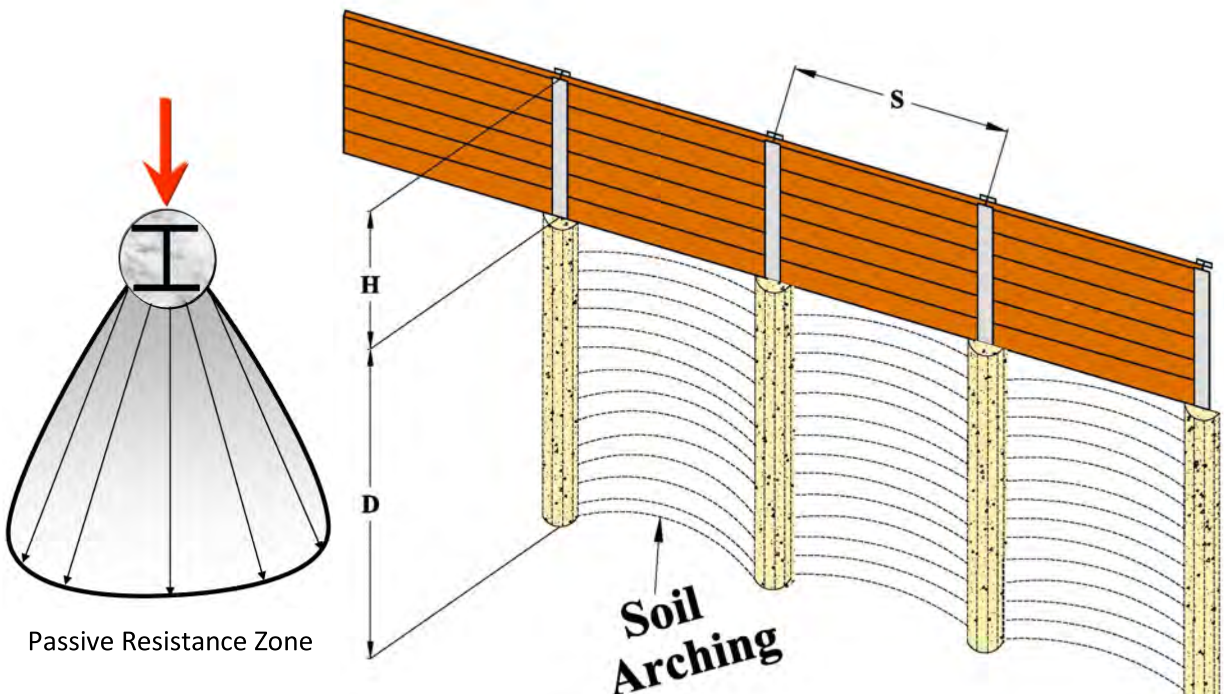


Figure 4-4: Soldier Piles with Arching

4.4. GROUND ANCHOR RETAINING WALLS

An anchored wall includes an exposed design height (*h*) over which soil is retained. Also, an embedded depth (*D*) may provide vertical and lateral support in addition to either structural anchors or ground anchors, as shown in Figure 4-5. In developing the lateral earth pressure for braced or anchored walls, consideration must be given to the wall displacement that may affect adjacent structures and/or underground utilities.

Depending on the soil type, the lateral earth pressure acting on the wall may be determined using an appropriate earth pressure theory. Generally, the earth pressure increases with depth against a wall. However, for braced or tieback walls, this is not the case. A trapezoidal shaped apparent earth pressure distribution needs to be developed for this type of wall design. The load distributions for the single and multiple anchor and/or braced earth retaining systems are described in §4.5.3.2 of this manual.

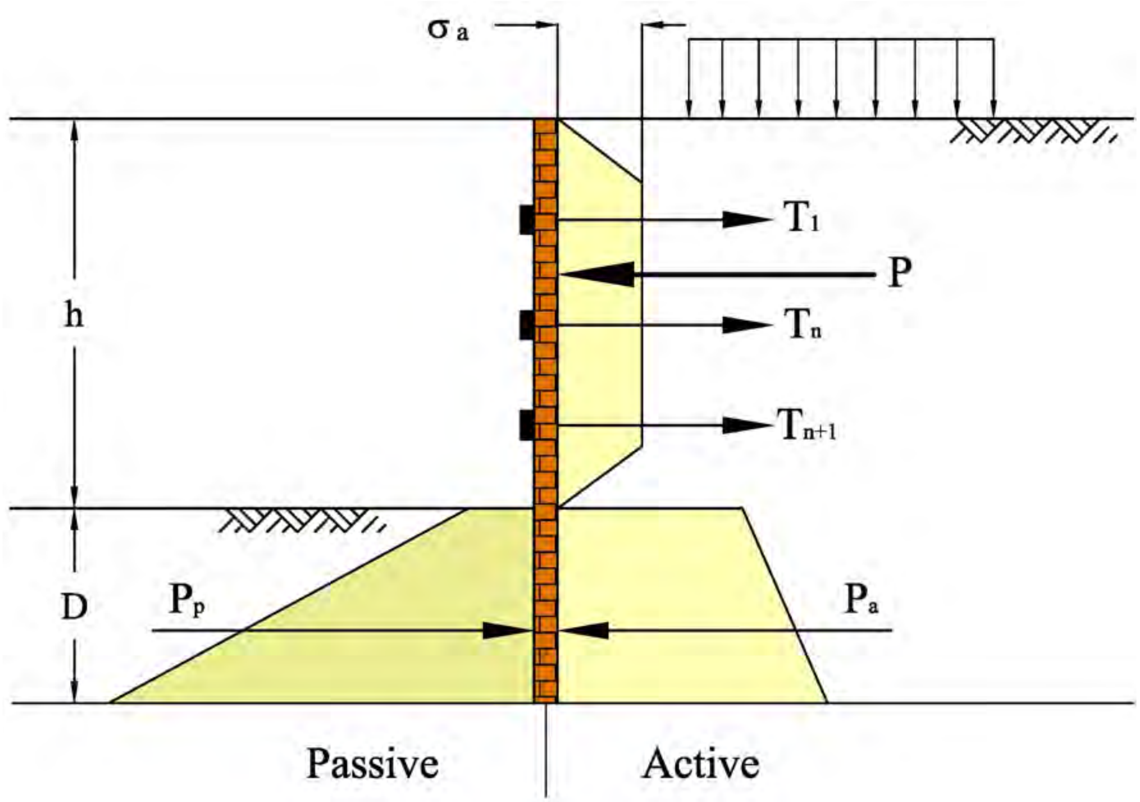


Figure 4-5: Lateral Earth Pressure for Anchored/Braced Walls

4.5. PERFORMANCE BASED DESIGN

Performance of the earth retaining systems is largely affected by method of wall construction. It is impossible to perform stage construction using the classical limit equilibrium to calculate stresses and/or wall deformation. Either a beam-column-spring model—i.e., the so-called "*p-y*" approach—or continuum a finite element approach shall be used to evaluate the wall performance for important structures.

The *p-y* model considers soil-wall interaction using a generalized beam-column model. The soil is represented by nonlinear discrete springs attached to the nodal points at the beam interface. Beam-column spring model calculates the shear, moment and deflection of the beam as a function of applied active thrust load above the excavation line, non-linear springs below the excavation line, beam-column stiffnesses, and the specified anchor post-tension loading.

The continuum Finite Element Method considers the complete solution of the earth retaining system, including the computation of stresses and deformation in both the wall and the adjacent soil. The finite element method is very useful in special situations to address issues that cannot be readily resolved by the limit equilibrium or the *p-y* approach, such as staged construction processes, and the prediction of lateral and vertical displacements around and below the wall.

4.5.1 The *p-y* Approach

The classical earth pressure theory is used to develop the active earth pressure above the excavation line behind the wall, as shown in Figure 4-6. The wall is modeled as a linear or nonlinear beam-column element and the *p-y* approach analyzes the behavior of a flexible retaining wall or solider pile wall with or without tiebacks. The active and passive earth pressure below the excavation is modeled using distributed nonlinear *p-y* springs. The following are the parameters that may be taken into consideration when designing cantilever or tieback system.

- Soil properties
- Soil sub-grade modulus parameter ($E_s = k_x$)
- Flexural Stiffness of the vertical wall element

The subgrade modulus (k) is used to calculate the soil's reaction (P) as a function of wall deflection (x).

Table 4-1: Typical Values of Sub-Grade Modulus *k* for Different Sand Properties

Relative Density	Loose	Medium	Dense
Submerged Sand	20 lb/in^3	60 lb/in^3	125 lb/in^3
Sand above WT	25 lb/in^3	90 lb/in^3	225 lb/in^3

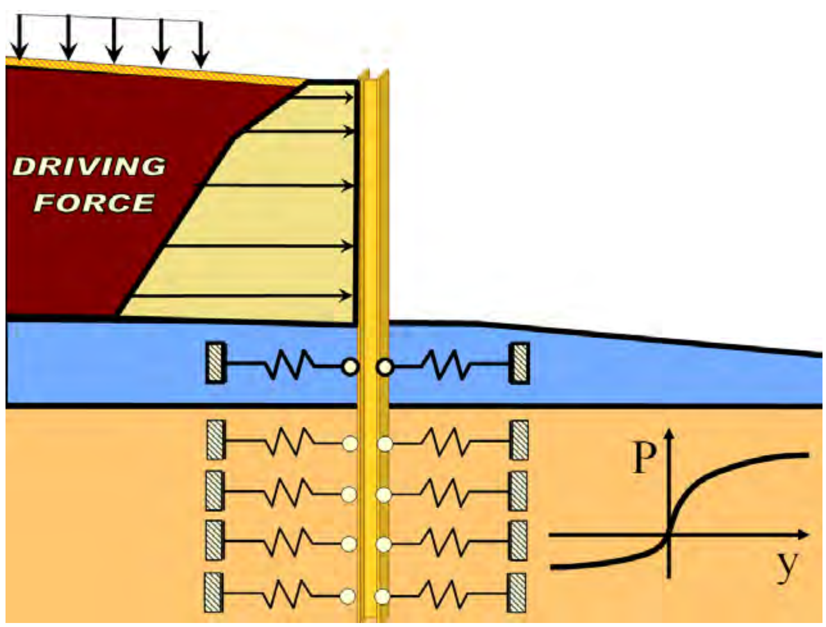


Figure 4-6: Conceptual *p-y* Approach for Cantilever Systems

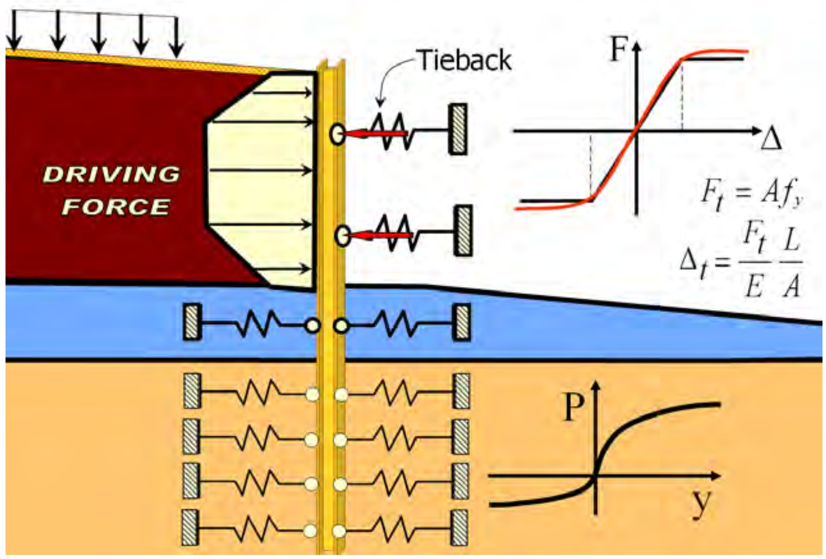


Figure 4-7: Conceptual *p-y* Approach for Tieback Systems

The *p-y* model of a tieback system is shown in Figure 4-7. For a tieback system, the classical earth pressure theory is used to develop the triangular and trapezoidal distributions above the excavation line. The active and passive earth pressure below the excavation line is modeled using a non-linear *p-y* spring. The tieback forces are analyzed using non-linear and/or bi-linear springs, meaning the force varies with wall displacement, as shown in Figure 4-8. The calculation for the tie back elongation is shown below.

$$\Delta = PL/EA_a \tag{109}$$

where Δ= Tieback elongation at the specified load test

L = Un-bonded tieback length

E = Modulus of elasticity of the tieback

A_a = Cross sectional area of strands

The cross sectional area and the ultimate capacity of a 0.6 in diameter single ASTM A-416 anchor is shown in Table 4-2.

Table 4-2: Properties of 0.6 in diameter Pre-stressing Steel Strands (ASTM A416, Grade 270)

Number of Strands	A_a (in^2)	Ultimate Strength F_t (kips)
1	0.217	58.6

The limiting tension force is given by

$$F_t = A_a f_y \tag{110}$$

The limiting force in compression F_c depends both on the manner in which tiebacks and/or tie rods are connected to the vertical element and on the axial load capacity of the tiebacks or tie rods which may vary from zero to the yield value as the limiting tension force given in the above equation.

The displacements of Δ_{yt} and Δ_{yc} on the tension and compression side, respectively, are expressed in the following two equations:

$$\Delta y_t = F_t L / E A_a \quad (111)$$

$$\Delta y_c = F_c L / E A_a \quad (112)$$

The *p-y* approach may be used to determine the deformation of the non-gravity earth retaining systems for a service load design.

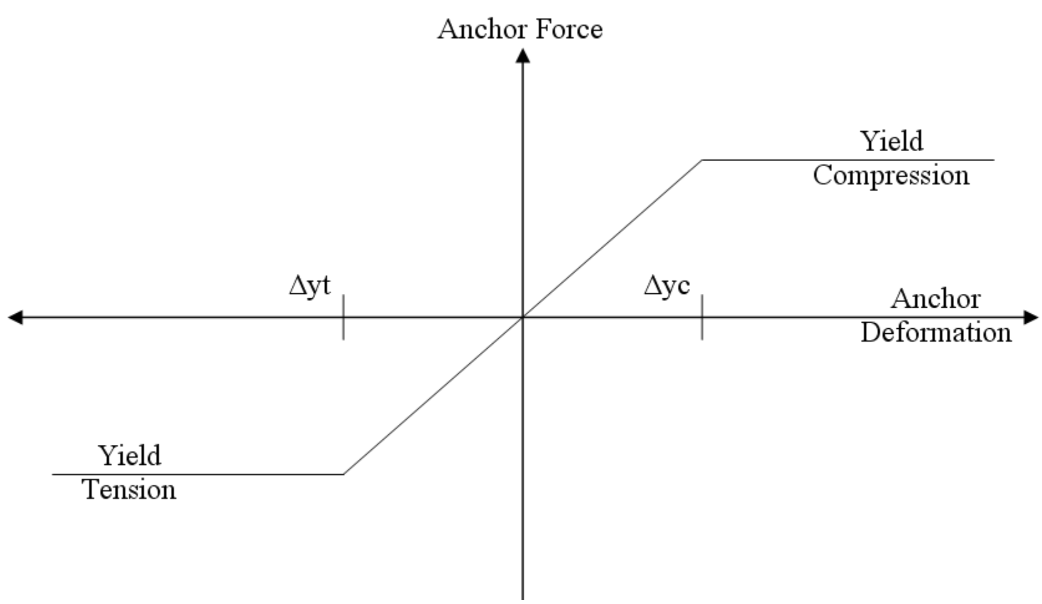
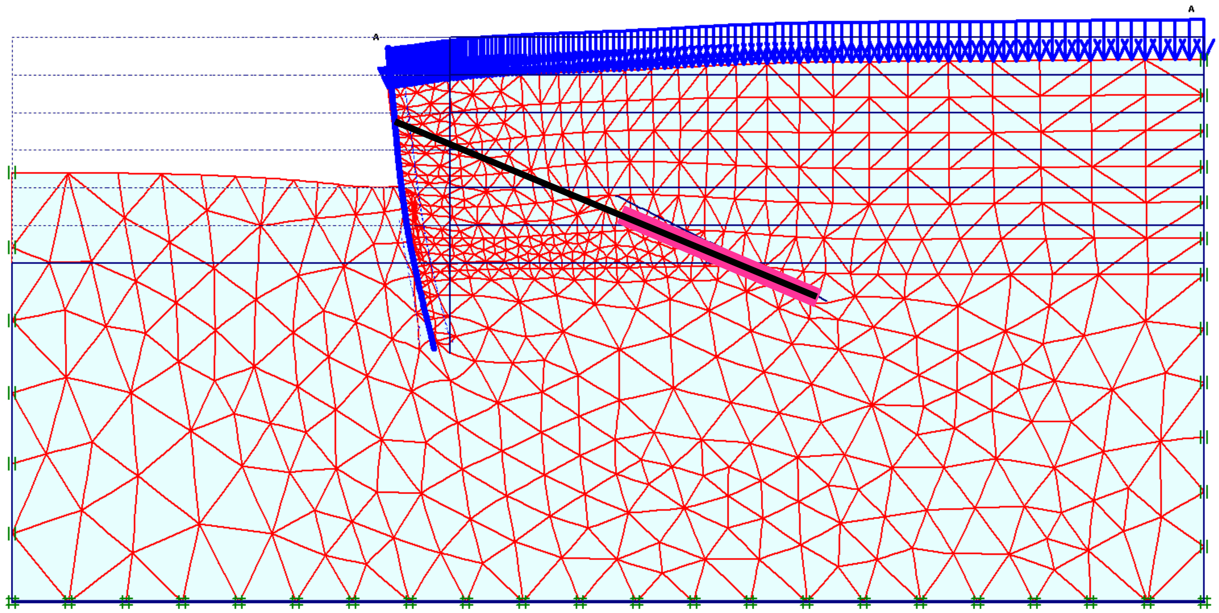


Figure 4-8: Tieback Modeled as an Anchor Spring

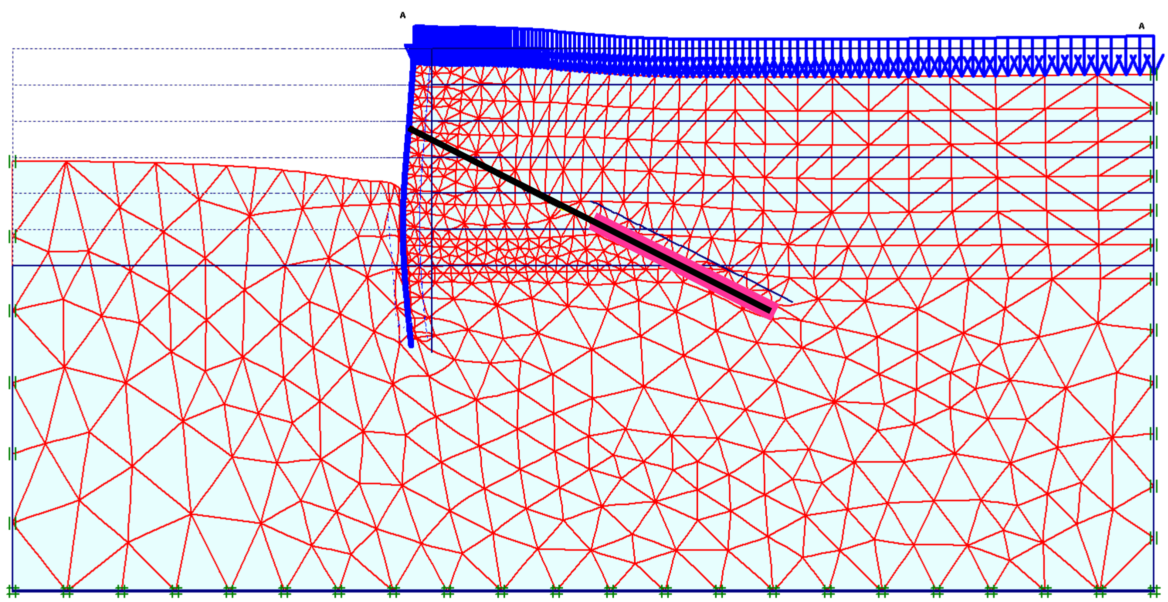
4.5.2 The Finite Element Approach

Performance of the earth retaining system in particularly flexible walls such as MSE walls, soil nail walls, solder pile and sheet pile walls, depends on many factors, in particular, successive stages of construction. The finite element method, which is well accepted in design practice today, can be used for modeling complex soil-wall interaction problems. The contrast to the *p-y* approach of the soil is modeled as a nonlinear continuum and the structural elements are modeled as a beam element.

When using the finite element model, it is important to create a model with a realistic geometric representation of the project. A geometry model should include a representative division of the subsoil into distinct soil layers, structural objects, loading conditions and construction stages. The model must be sufficiently large so that the boundaries do not influence the results of the studied problem. A typical finite element continuum model of a single tieback wall before and after the pre-stress tieback load is applied is shown in Figure 4-9 (a) and (b).



(a) Before pre-stressing



(b) After pre-stressing

Figure 4-9: Finite Element Models

4.5.3 Load Resistance Factor Design (LRFD) for Earth Retaining Systems Design

Per AASHTO LRFD Specifications (2010), the following three limit states should be considered for the design of earth retaining systems: (1) Service I Limit State, (2) Strength I Limit State, and (3) Extreme Event I Limit State. Earth retaining systems shall be designed to satisfy all three states.

4.5.3.1 *Cantilever Wall*

Depending on the site soil profile, the un-factored simplified lateral earth pressure distribution, shown in Figure 4-10 through Figure 4-13, may be used for the design of cantilever earth retaining systems. The LRFD loads and resistance factors listed in Table 4-3 should be applied to the load distributions shown in these figures, in order to calculate various load combinations for the design of the wall.

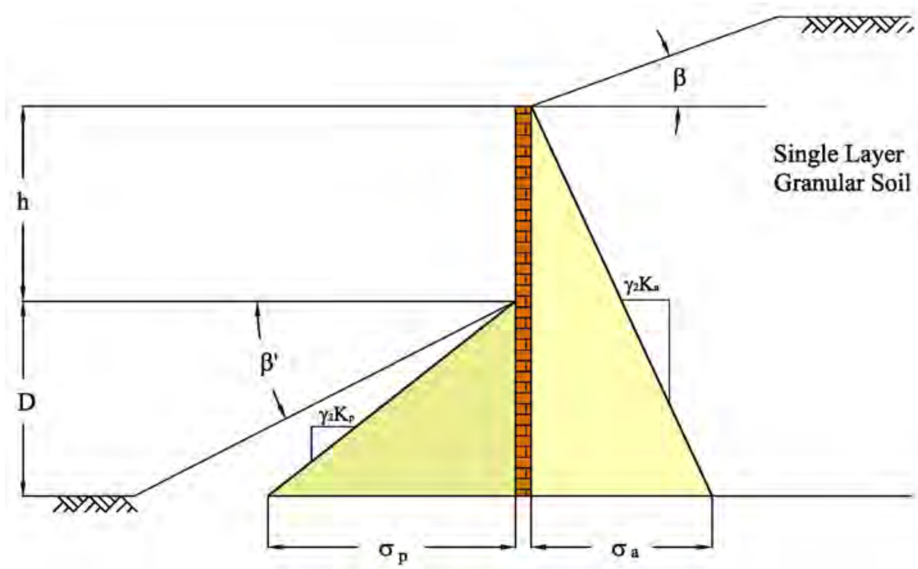


Figure 4-10: Loading Diagram for Single Layer Soil

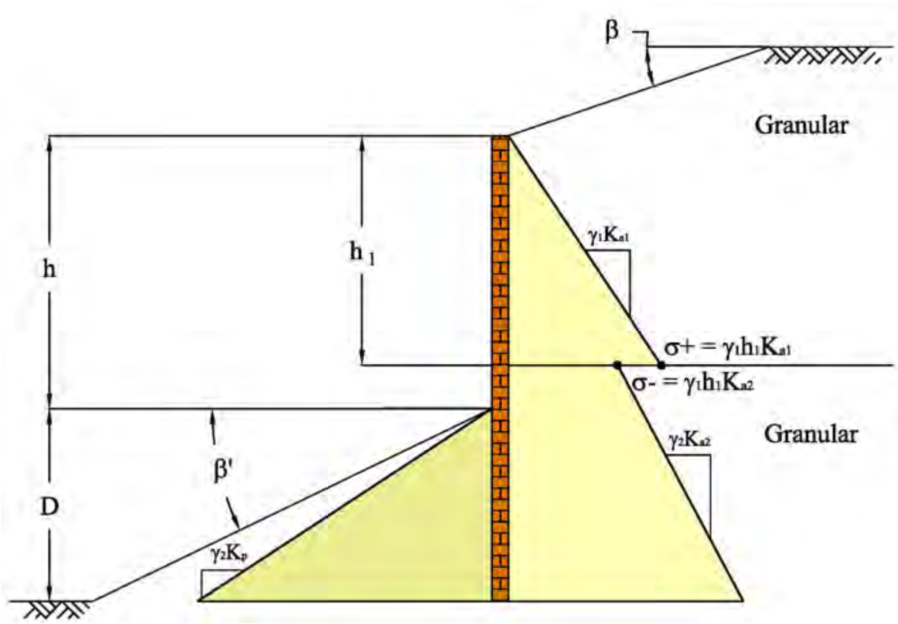


Figure 4-11: Loading Diagram for Multi-Layer Soil (Granular Soil on Granular Soil)

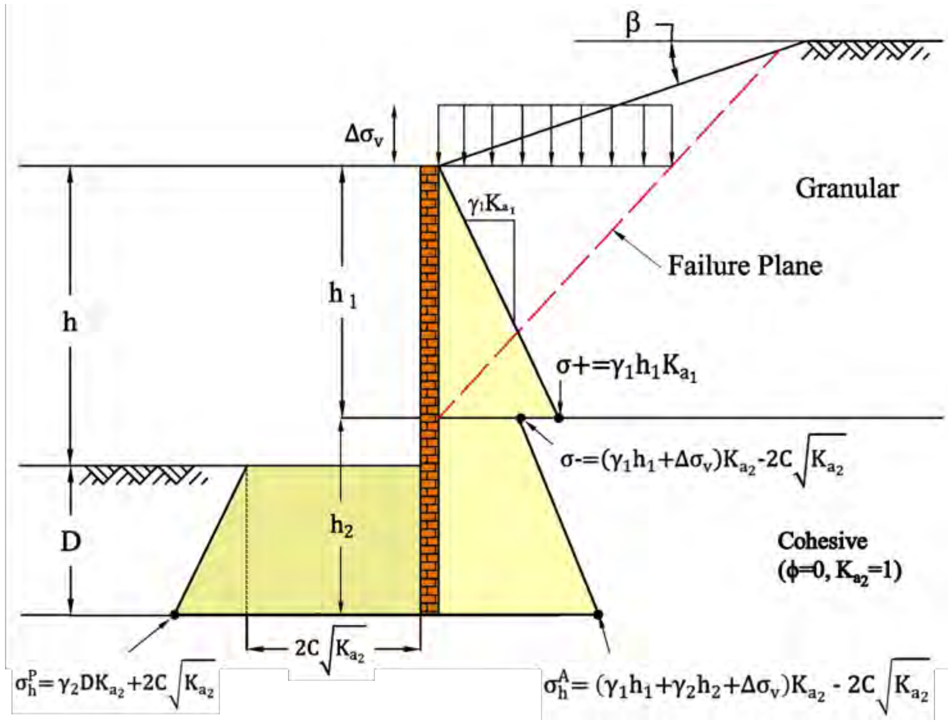


Figure 4-12: Loading Diagram for Multi-Layer Soil (Granular Soil on Purely Cohesive Soil)

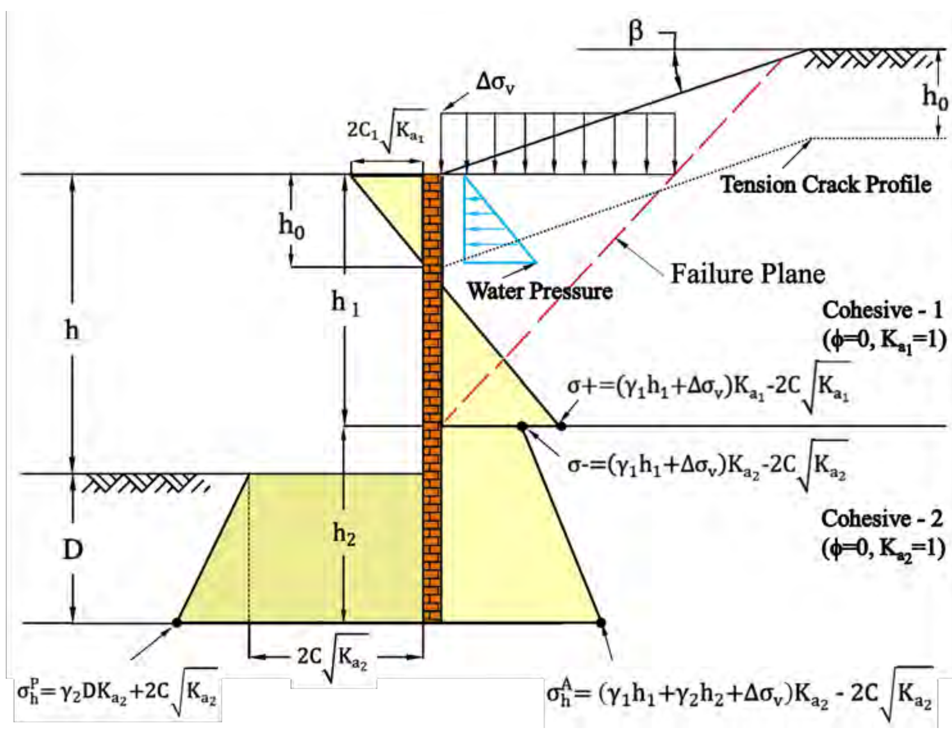


Figure 4-13: Loading Diagram for Multi-Layer Soil (Purely Cohesive Soil on Purely Cohesive Soil)

To determine the active lateral earth pressure on the embedded wall element shown in Figure 4-13, the sloping backfill above the top of the wall within the active failure wedge is treated as an additional surcharge ($\Delta\sigma_v$). The portion of the negative loading at the top of the wall due to cohesion is ignored and any hydrostatic pressure in the tension crack needs to be considered.

In addition, the following two points must be satisfied:

- The ratio of total overburden pressure to un-drained shear strength (*NS*) must be < 3 at the design grade in front of wall.
- The active lateral earth pressure acting over the wall height (*H*) should not be less than 0.25 times the effective overburden pressure at any depth, or 0.035 ksf/ft of wall height—which ever is greater.

Table 4-3: LRFD Factors for Cantilever Retaining Walls

Limit State	Active Pressure	Passive Earth Pressure	Live Load Surcharge	Seismic Addition
Service I	1.0	1.0	1.0	0
Strength I	1.5	1.0	1.75	0
Extreme Event I	1.0	1.0	0.0	1.0

Design Steps for a Non-Gravity Cantilever Wall

The following procedure is used for the design of a non-gravity cantilever wall:

1. Calculate Active/Passive Earth Pressure to arbitrary point O at the distance, D_o, below the excavation line.
2. Apply appropriate LRFD Factors in Table 4-3.
3. Take a moment about Point O to eliminate force R and determine embedment depth D_o.
4. Increase D_o by 20 percent ($D = 1.2D_o$)
5. Calculate R by summation of force in horizontal direction ($R \leq 0$, if R is larger than zero, increase D)
6. Calculate Maximum Bending Moment (M_{max}) and Maximum Shear Force (V_{max}) to design the vertical structural member and lagging.
7. Calculate the wall deformation for the service limit state

4.5.3.2 ***Anchored Wall***

The design of the anchored wall involves many of the same considerations as the non-gravity cantilever walls. However, presence of one or more anchors to the vertical elements of the wall introduces trapezoidal active loads behind the wall above the excavation line.

During the seismic loading, the anchored wall develops additional driving loads behind the wall. The additional seismic load should be resisted through the reaction of the anchors and the passive resistance of the soil bellow the excavation depth.

Cohesionless Soils

The lateral earth pressure distribution for the design of braced or anchored walls constructed in cohesion-less soils for single braced/tieback walls and multiple braced/tieback walls are demonstrated in Figure 4-17 and Figure 4-18, respectively (AASHTO Figure 3.11.5.7.1-1). The maximum ordinate (σ_a) of the pressure diagram is determined as given in Eqns. 113 and 114.

For walls with a single level of anchors or braces (see Figure 4-17):

$$\sigma_a = \frac{3P}{2h} \qquad (113)$$

For the multiple tieback walls (see Figure 4-18):

$$\sigma_a = \frac{P}{\left[h - \frac{1}{3}\left(h_1 + h_{n+1}\right)\right]} \qquad (114)$$

where the total active earth pressure is:

$$P = 1.3\ P_{aH} \qquad (115)$$

Cohesive Soils

The lateral earth pressure distribution for cohesive soils is related to the stability number (N_s), which is defined as:

$$N_s = \frac{\gamma_s h}{C} \qquad (116)$$

- The ratio of total overburden pressure to undrained shear strength, Ns (see AASHTO 3.11.5.7.2), should be < 3 at the wall base.
- The active earth pressure shall not be less than 0.25 times the effective overburden pressure at any depth, or 5.5×10–6 MPa of wall height—whichever is greater.

(i) **Stiff to Hard Cohesive Soils**

For braced or anchored walls in stiff to hard cohesive soils with the stability number (N_s) less than or equal to 4, the lateral earth pressure may be determined using Figure 4-19 with the maximum ordinate (σ_a) of the pressure diagram determined as:

$$\sigma_a = 0.2\ \gamma_s h \ \sim \ 0.4\ \gamma_s h \tag{117}$$

(ii) **Soft to Medium Stiff Cohesive Soils**

The lateral earth pressure on a restrained shoring system in soft to medium stiff cohesive soils with the stability number equal to or larger than 6 may be determined using Figure 4-19 in which the maximum ordinate (σ_a) of the pressure diagram is determined as:

$$\sigma_a = K_a\ \gamma_s\ h \tag{118}$$

The coefficient of active lateral earth pressure (K_a) may be determined using Eqn. 119.

$$K_a = 1 - \frac{4\ S_u}{\gamma_s\ h} + 2\sqrt{2}\,\frac{D}{h}\left(\frac{1 - 5.14\ S_{ub}}{\gamma_s\ h}\right) \geq 0.22 \tag{119}$$

where S_u = undrained strength of retained soil (ksf)
S_{ub} = undrained strength of soil below excavation base (ksf)

D = depth of potential base failure surface below base of excavation (ft)

The value of d is taken as the thickness of soft to medium stiff cohesive soil below the excavation base up to a maximum value of $B_e/\sqrt{2}$, where B_e is the excavation width.

For soils with $4<N_s<6$, use the larger σ_a from Eqns. 117 and 118.

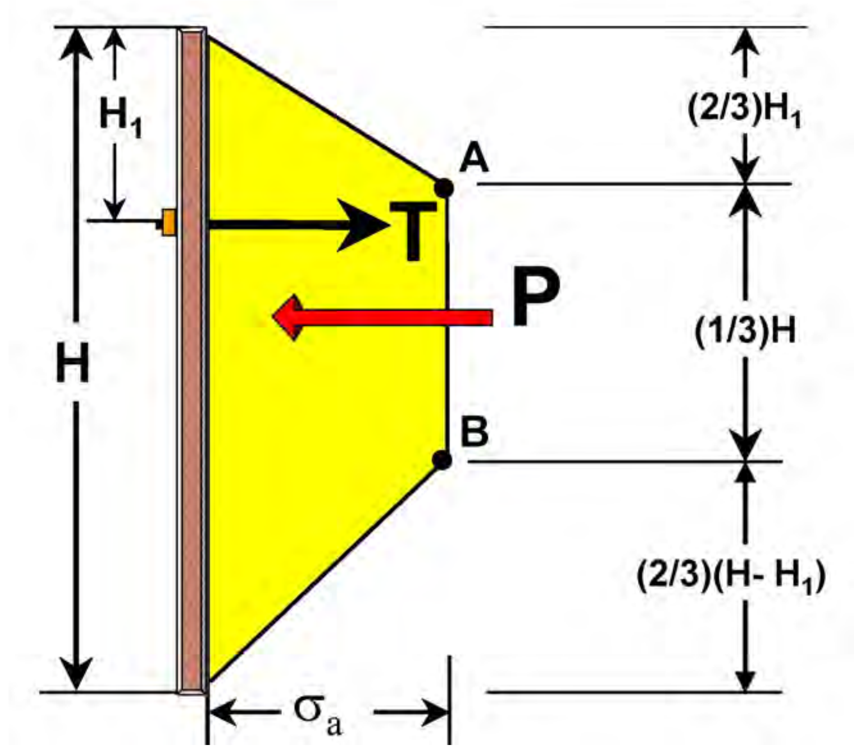


Figure 4-14: Pressure Diagram for Single Tieback Wall in Granular Soil

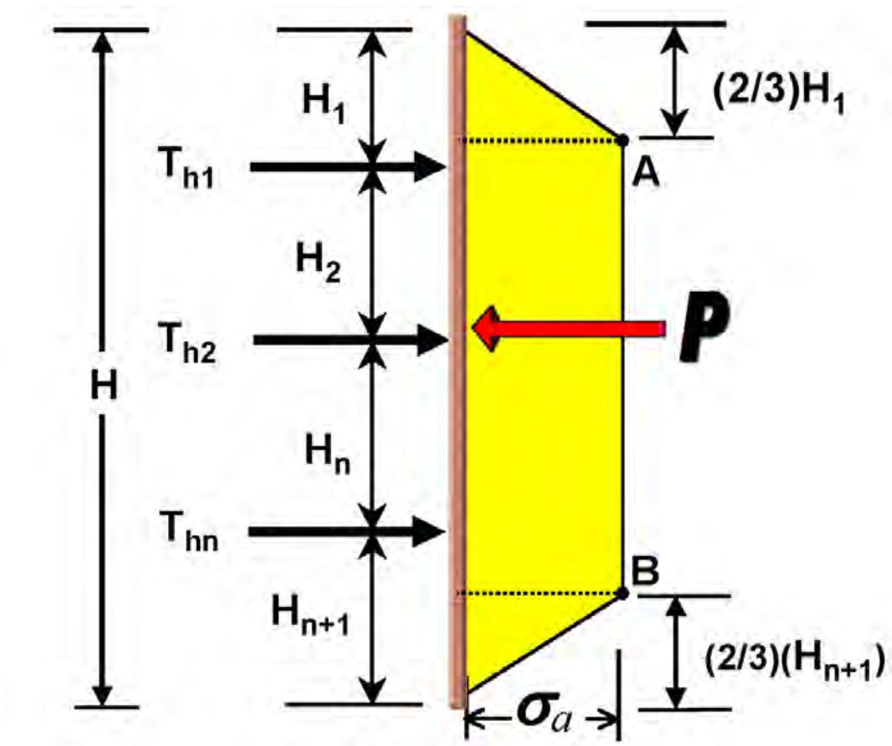


Figure 4-15: Pressure Diagram for Multiple Tieback Wall in Granular Soil

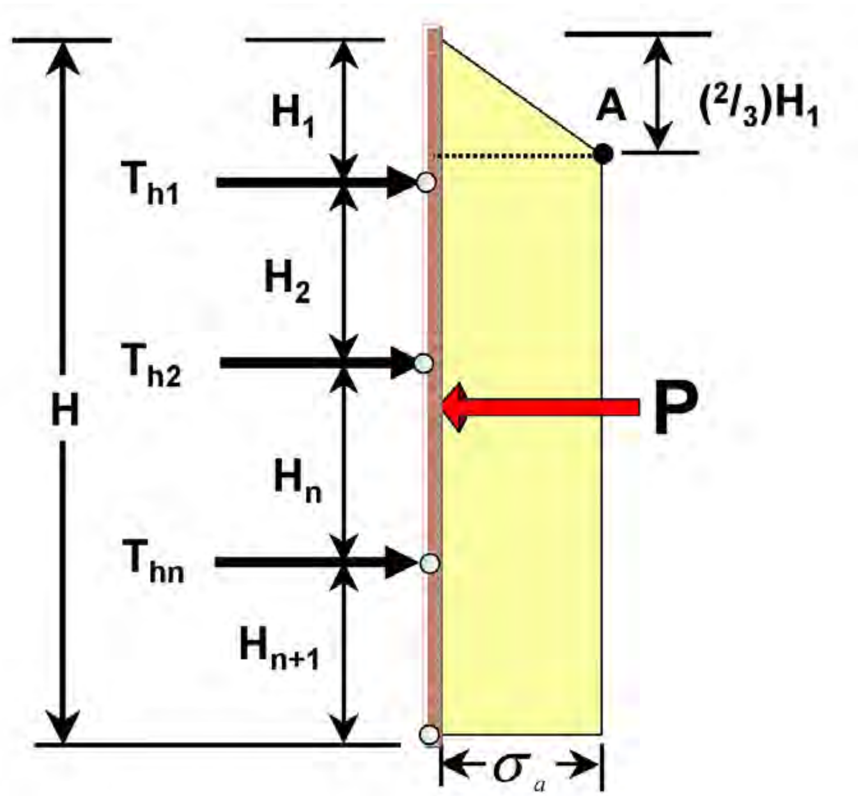


Figure 4-16: Pressure Diagram for Multiple Tieback Wall in Purely Cohesive Soil

LRFD Factors for non-cohesive soil Tieback Retaining Walls

Table 4-4: LRFD Factors for Tieback Retaining Walls

Limit State	Active Earth Pressure	Passive Earth Pressure	Live Load Surcharge	Seismic Addition
Service I	1.0	1.0	1.0	0
Strength I	1.5	1.0	1.75	0
Extreme Event I	1.0	1.0	0.0	1.0

Calculation Procedure for Single Tieback/Brace System

The following procedure is used for the design of a Single Tieback/Brace System wall:

1. Determine the Earth Pressure Coefficients using the classical Earth Pressure Theories described in Chapter 2.
2. Apply appropriate LRFD Factors listed in Table 4-4 to the active and passive earth pressure.
3. Convert the active earth pressure above the excavation line to a trapezoidal earth pressure.
4. Take a moment about the tieback to calculate embedment depth, D.
5. Set summation of forces equal to zero in horizontal direction to calculate tieback/brace force T.
6. Calculate Maximum Bending Moment (M_{max}) and Maximum Shear Force (V_{max}) to design the vertical structural member and lagging.
7. Calculate wall deformation for the service limit state.

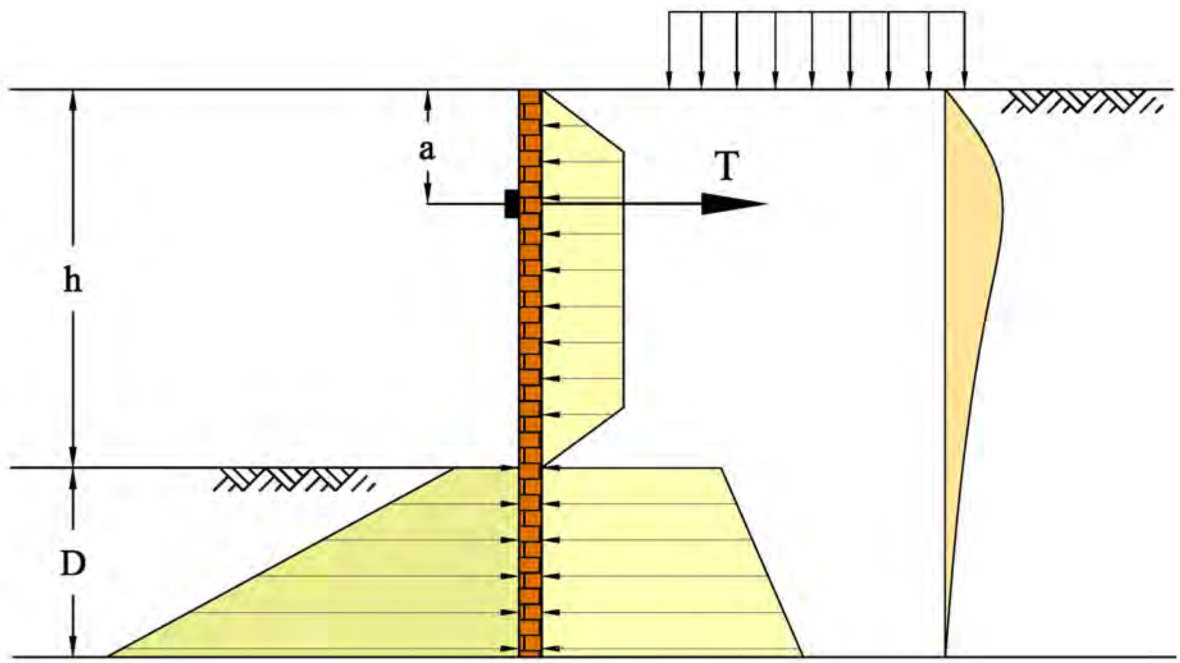


Figure 4-17: Single Tieback System

Calculation Procedure for Multiple Tieback System

Depending on the backfill properties, the trapezoidal pressure diagrams in soil as shown in Figure 4-15 and Figure 4-16 are used for the analysis and design of multiple tieback systems. Figure 4-18 shows a simple trapezoidal pressure diagram for a multiple tieback system. The wall is divided into three types of spans:

- Starting Cantilever Span, S_1
- Interior Spans, S_n
- Embedment Span, S_D

The Hinge method, shown in Figure 4-19, is used to solve multiple Tieback/Brace system.

1. Take a moment M_1 about the upper level tieback due to cantilever action of the soil pressure above the upper tieback.
2. Use combination of the moment, M_1, and tributary area to calculate the remaining tieback loads except the last tieback load.
3. Take a moment about the last tieback to calculate embedment depth, D using a factor of safety of 1.0.
4. Set summation of forces equal to zero in horizontal direction to calculate the last tieback force, $T_{n+1.}$
5. Calculate Maximum Bending Moment (M_{max}) and Maximum Shear Force (V_{max}) to design the vertical structural member and lagging.
6. Calculate wall deformation for the service limit state.

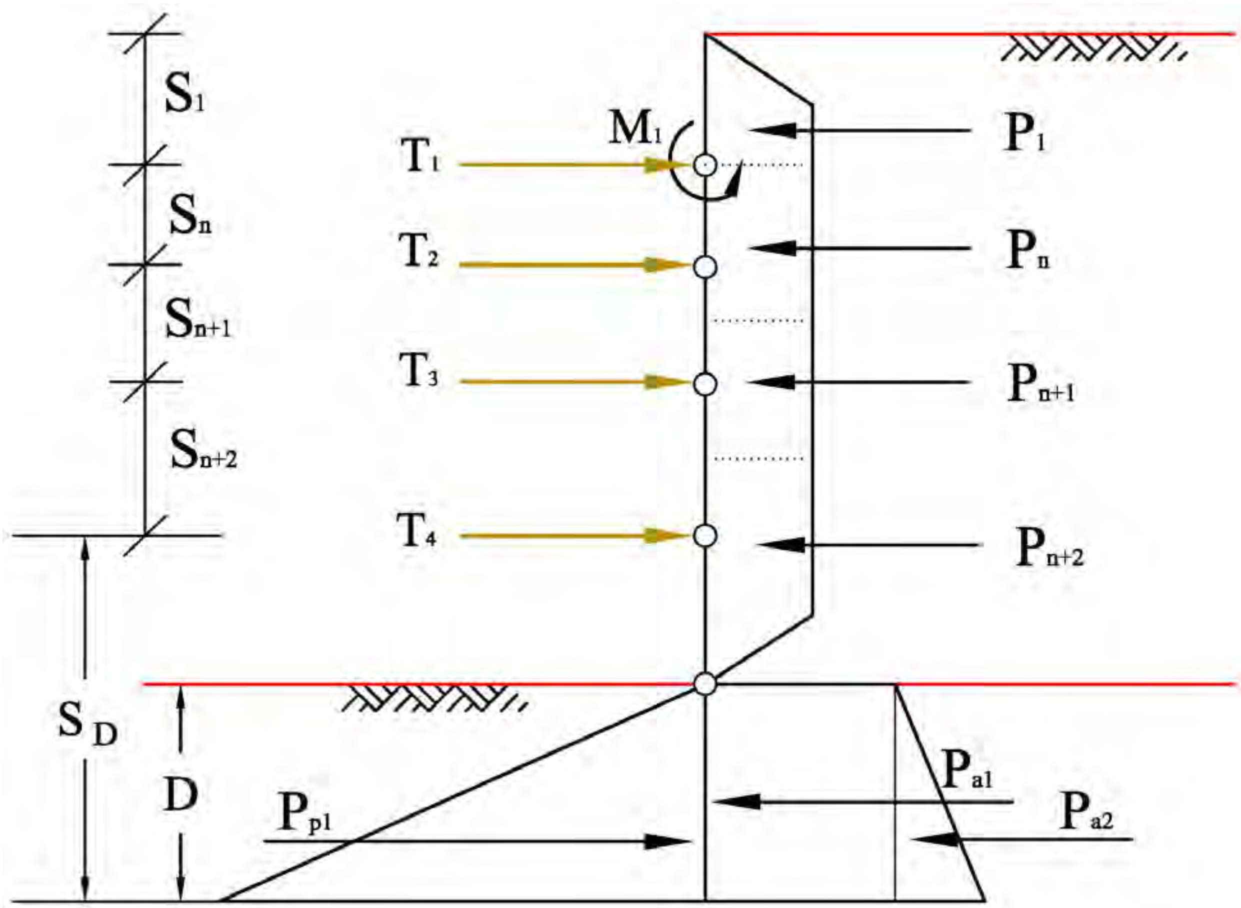


Figure 4-18: Multiple Tieback System

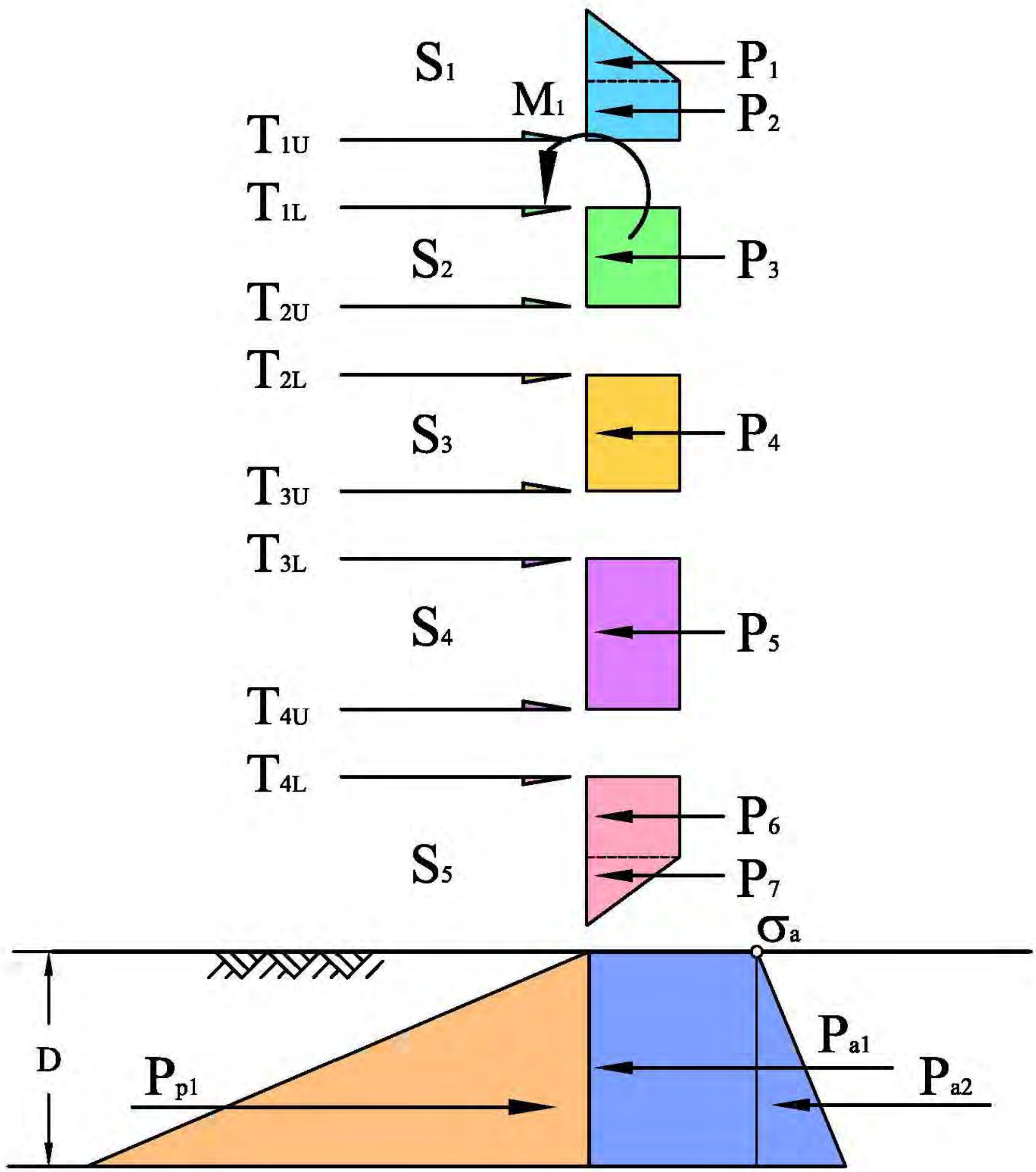
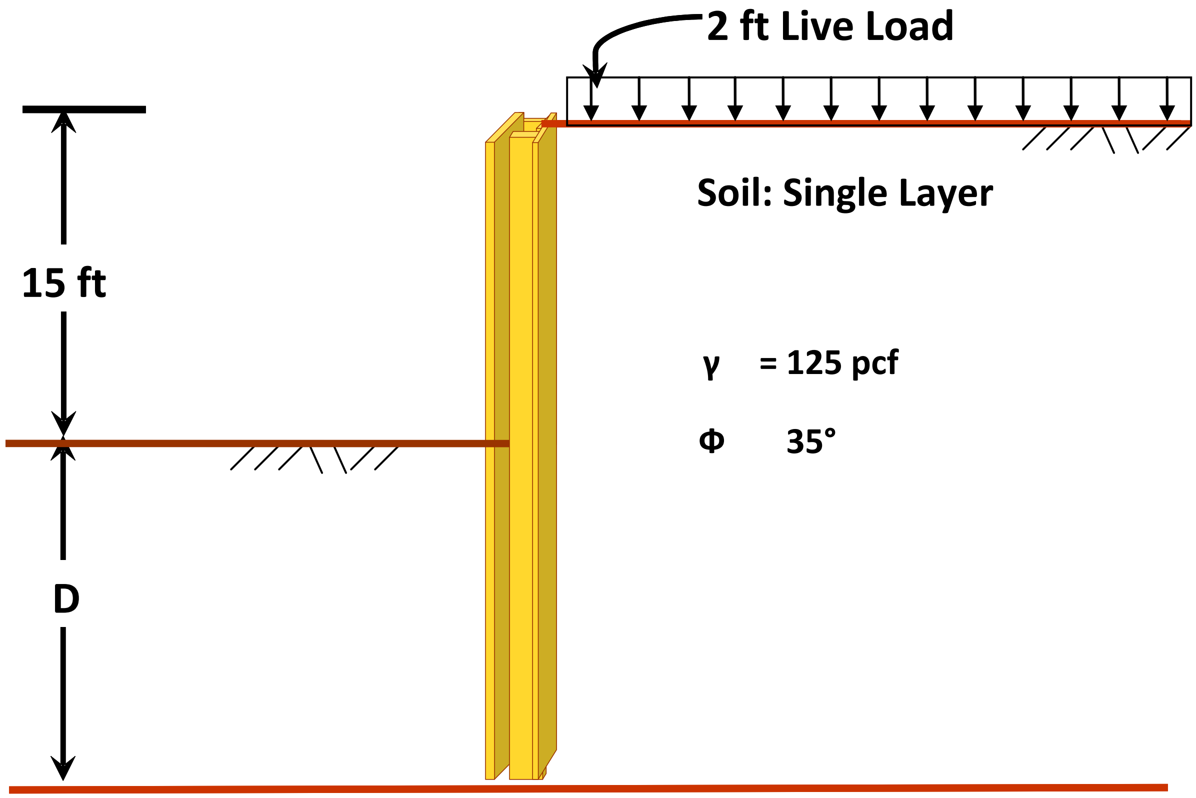


Figure 4-19: Detail Hinge Method for Tieback Design

4.6. DESIGN EXAMPLES OF NON-GRAVITY CANTILEVER WALLS

4.6.1 Example 4-1: Cantilever Sheet Pile Wall

Design a cantilevered sheet pile wall with single soil layer.



Determine:

1. Active & Passive Earth Pressures
2. Pile Embedment *D*
3. Maximum Shear
4. Maximum Moment

PART A: SERVICE I CALCULATIONS

Determine Active and Passive Earth Pressures:

- Calculate active and passive earth pressure coefficients: Since the wall friction (δ) is zero, use Rankine Earth Pressure Theory to calculate the active and passive earth pressure coefficients.

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45^o - \frac{35^o}{2}\right) = 0.271$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = \tan^2\left(45^o + \frac{35^o}{2}\right) = 3.690$$

Note: Rankine Theory tends to underestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to compute the passive earth force.

- Calculate earth pressure distribution

Lateral load due to surcharge above the excavation line only (LRFD Factor – 1.0):

$$s_{sur} = 0.125 \times 2 \times 0.271 \times 1.0 = 0.068 \text{ ksf}$$

Lateral load distribution at excavation level (LRFD Factor – 1.0):

$$\sigma = 0.125 \times 15 \times 0.271 \times 1.0 = 0.508 \text{ ksf}$$

Lateral load distribution for the second layer at depth D_o (LRFD Factor – 1.0):

$$\sigma_D = 0.508 + 0.125 \times 0.271 \times 1.0 D_o = \left(0.508 + 0.0339\, D_o\right) \text{ksf}$$

Passive Lateral load distribution for the second layer in the front at depth D_o (LRFD Factor – 1.0):

$$\sigma_{pD} = 0.125 \times 3.69 \times 1.0 D_o = 0.461\, D_o \text{ ksf}$$

- Calculate resultant earth forces

Calculate active earth pressure due to surcharge P_{AS}:

$$P_{As} = 15 \times 0.068 = 1.016 \text{ klf}$$

Calculate active earth pressure for the first soil layer P_{A1}:

$$P_{A1} = \frac{15}{2} \times 0.508 = 3.811 \text{ klf}$$

Calculate active earth pressure for the second soil layer P_{A2}:

$$P_{A21} = 0.508\, D_o \text{ klf}$$

$$P_{A22} = 0.0339 \times D_o \left(\frac{D_o}{2}\right) = 0.0169\, D_o^2 \text{ klf}$$

Calculate passive earth pressure for the second soil layer P_P:

$$P_p = 0.462\, D_o \left(\frac{D_o}{2}\right) = 0.231\, D_o^2 \text{ klf}$$

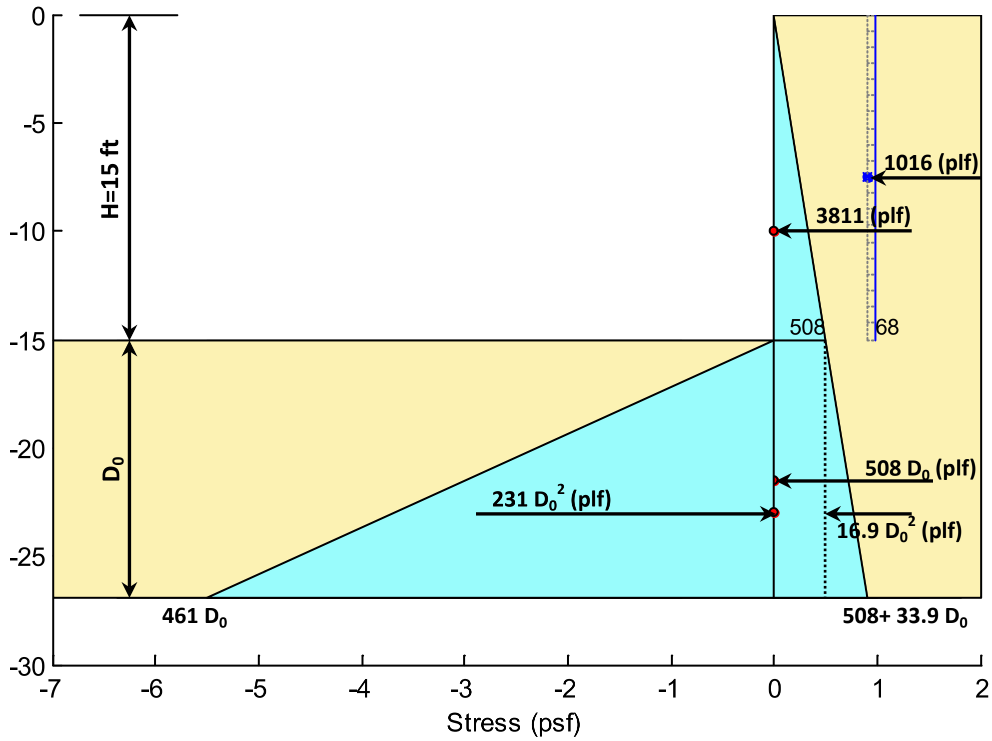


Figure 4-20: Pressure Diagram for Service I Loading Condition

*Note: For Simplicity of the graph, all numbers are rounded to whole numbers. However, in the calculation of D_o all numbers are rounded to two digits after the decimal point.

- Calculate driving and resisting moments:

Driving Force (klf)	Arm (ft)	Driving Moment M_{RS}
1.016	$7.5+D_o$	$7.622 + 1.016\ D_o$
3.811	$5+ D_o$	$19.055 + 3.811\ D_o$
$0.508\ D_o$	$^{Do}/_2$	$0.254\ D_o^2$
$0.0169\ D_o^2$	$^{Do}/_3$	$0.00565\ D_o^3$
Resisting Force (klf)	Arm (ft)	Resisting Moment M_{RS}
$0.231\ D_o^2$	$^{Do}/_3$	$0.0769\ D_o^3$

$M_{DR} = 0.005657 D_o^3 + 0.254\ D_o^2 + 3.811\ D_o + 19.055 + 1.016\ D_o + 7.622$

$M_{RS} = 0.0769\ D_o^3$

- Calculate embedment depth:

$M_{RS} = M_{DR}$: $-0.0712 D_o^3 + 0.254 D_o^2 + 4.827 D_o + 26.677 = 0$

$\Rightarrow \quad D_o^3 - 3.57 D_o^2 - 67.76 D_o - 374.49 = 0 \quad \Rightarrow \quad D_o = 11.903 \text{ ft} \quad \Rightarrow \quad D = 1.2 D_o = 14.284 \text{ ft}$

- Calculate Maximum Moment:

The maximum moment is located at distance *Y* below the excavation line where the shear is equal to zero. Therefore the summation of horizontal forces at the distance *Y* must be set to equal zero.

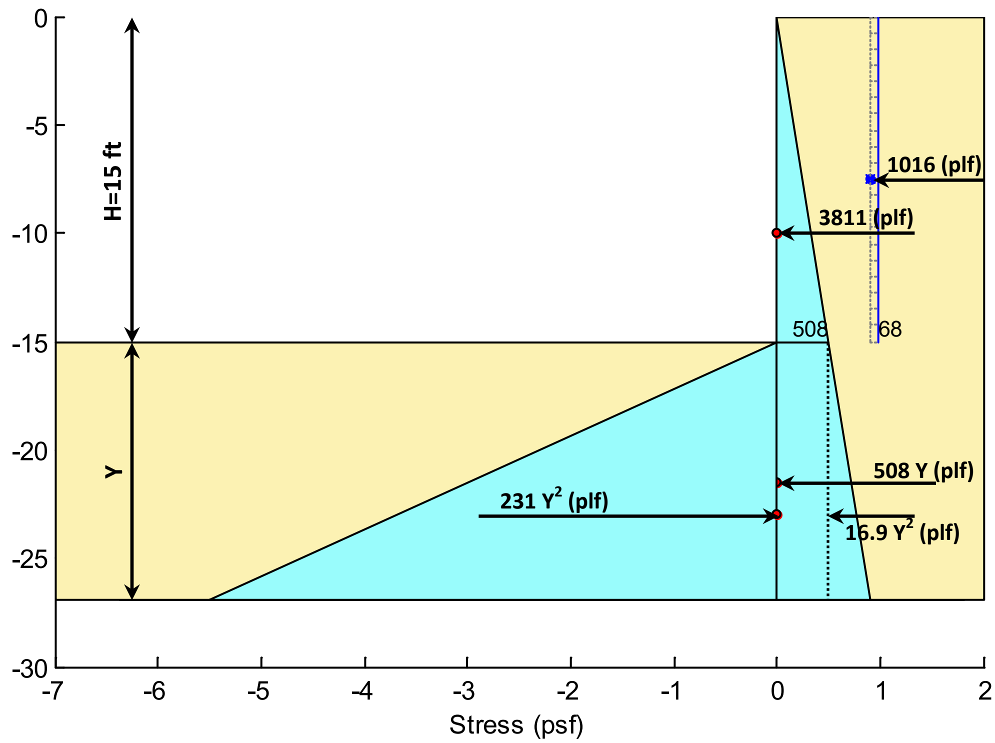


Figure 4-21: Location of Zero and Maximum Moment for Service I Loading Condition

$$\sum F_x = 0$$

$$0.231Y^2 - 0.0169Y^2 - 0.508Y - 3.811 - 1.016 = 0$$

$$\Rightarrow \quad Y^2 - 2.378Y - 22.590 = 0 \quad \Rightarrow \quad Y = 6.088 \text{ ft (below the dredge line)}$$

$$M_{\max} = \left\{ \begin{aligned} &1.016(7.5+6.088) + 3.811(5+6.088) + 0.508(6.088)\left(\frac{6.088}{2}\right) \\ &+ 0.0169(6.088)^2\left(\frac{6.088}{3}\right) - 0.231(6.088)^2\left(\frac{6.088}{3}\right) \end{aligned} \right\}$$

$$M_{max} = 49.409 \text{ kips-ft/ft}$$

The figure below displays the Shear and Moment Diagram for the Strength I Loading Condition:

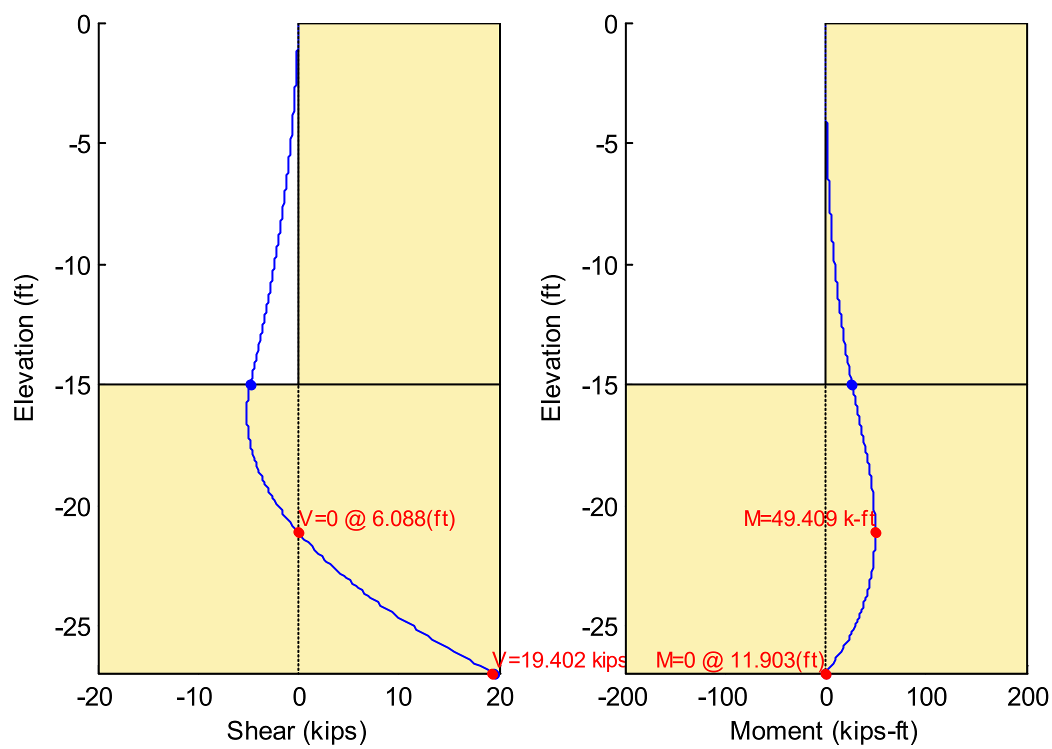


Figure 4-22: Shear and Moment Diagram for Service I Loading Condition

- Design Sheet Pile for the above Shear and Moment:
  (Note: Assuming Grade 55 Steel will be used)

  Check Flexural:

$$\sigma_{allowable} = 1.0 f_y = 55 \text{ ksi}$$

$$S_m = S_x = \frac{M_{max}}{\sigma_{allowable}} = \frac{49.409 \text{ kips} - ft\left(12 \text{ in/ft}\right)}{55 \; ksi} = 10.780 \text{ in}^3$$

Try Sheet Pile Type: $PZ27 \quad \Rightarrow \quad S_m = 30.2 \text{ in}^3 > 10.780 \text{ in}^3$

  Check Shear:

$$q_{v_{allowable}} = 1.0 f_y = 55 \text{ ksi}$$

$$q_v = \frac{V_{max}}{A} = \frac{19.402 \text{ kips}}{7.9 \text{ in}^2} = 2.456 \text{ ksi}$$

*Note: A is based on Sheet Pile Properties

$$q_{v_{allowable}} > q_v \quad \Rightarrow \quad \textbf{Assumed sheet pile O.K. for design}$$

PART B: STRENGTH I CALCULATIONS

Determine Active and Passive Earth Pressures:

- Calculate active and passive earth pressure coefficients: Since the wall friction (δ) is zero, use Rankine Earth Pressure Theory to calculate the active and passive earth pressure coefficients.

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45^o - \frac{35^o}{2}\right) = 0.271$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = \tan^2\left(45^o + \frac{35^o}{2}\right) = 3.690$$

Note: Rankine Theory tends to underestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to compute the passive earth force.

- Calculate earth pressure distribution

Lateral load due to surcharge above the excavation line only (LRFD Factor – 1.75):

$$s_{sur} = 0.125 \times 2 \times 0.271 \times 1.75 = 0.119 \text{ ksf}$$

Lateral load distribution at excavation level (LRFD Factor – 1.5):

$$\sigma = 0.125 \times 15 \times 0.271 \times 1.5 = 0.762 \text{ ksf}$$

Lateral load distribution for the second layer at depth D_o (LRFD Factor – 1.5):

$$\sigma_D = 0.762 + 0.125 \times 0.271 \times 1.5 D_o = \left(0.762 + 0.0508\, D_o\right) \text{ksf}$$

Passive Lateral load distribution for the second layer in the front at depth D_o (LRFD Factor – 1.0):

$$\sigma_{pD} = 0.125 \times 3.69 \times 1.0 D_o = 0.461\, D_o \text{ ksf}$$

- Calculate resultant earth forces

Calculate active earth pressure due to surcharge P_{AS}:

$$P_{As} = 15 \times 0.119 = 1.778 \text{ klf}$$

Calculate active earth pressure for the first soil layer P_{A1}:

$$P_{A1} = \frac{15}{2} \times 0.762 = 5.716 \text{ klf}$$

Calculate active earth pressure for the second soil layer P_{A2}:

$$P_{A21} = 0.762\, D_o \text{ klf}$$

$$P_{A22} = 0.0508 \times D_o \left(\frac{D_o}{2}\right) = 0.0254\, D_o^2 \text{ klf}$$

Calculate passive earth pressure for the second soil layer P_P:

$$P_p = 0.462\, D_o \left(\frac{D_o}{2}\right) = 0.231\, D_o^2 \text{ klf}$$

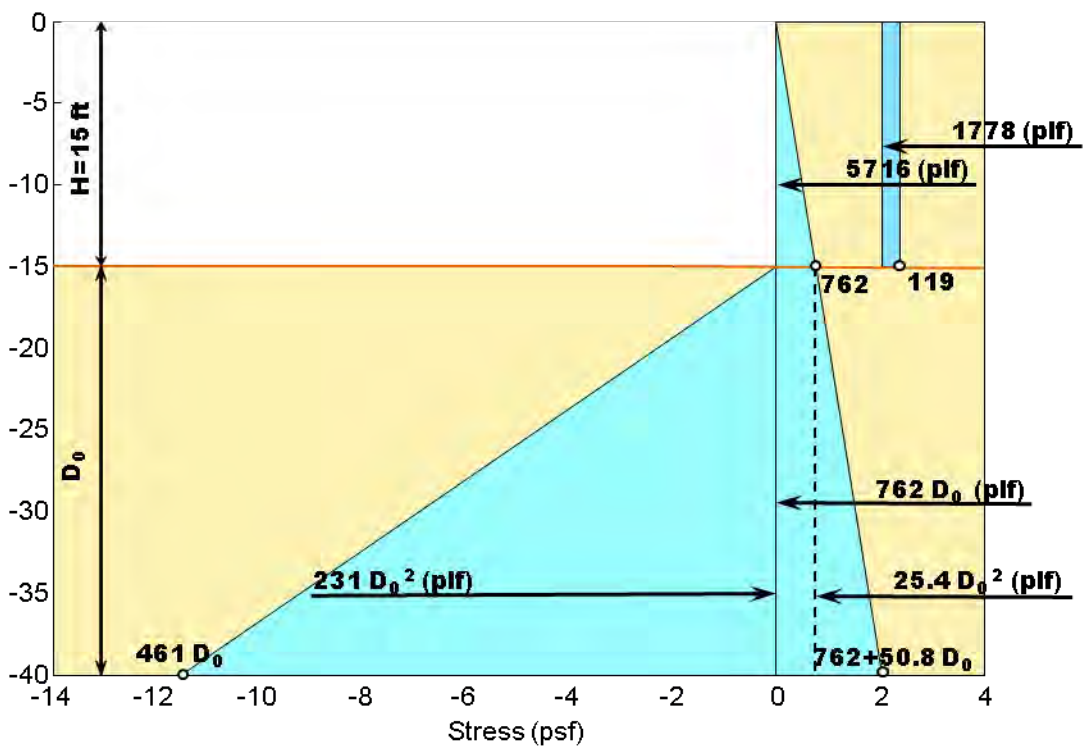


Figure 4-23: Pressure Diagram for Strength I Loading Condition

*Note: For Simplicity of the graph, all numbers are rounded to whole numbers. However, in the calculation of D_o all numbers are rounded to two digits after the decimal point.

- Calculate driving and resisting moments:

Driving Force (klf)	Arm (ft)	Driving Moment M_{RS}
1.778	$7.5+D_o$	$13.338 + 1.778\ D_o$
5.716	$5+ D_o$	$28.582 + 5.716\ D_o$
$0.762\ D_o$	$^{Do}/_2$	$0.381\ D_o^2$
$0.0254\ D_o^{\ 2}$	$^{Do}/_3$	$0.00847\ D_o^3$
Resisting Force (klf)	Arm (ft)	Resisting Moment M_{RS}
$0.231\ D_o^2$	$^{Do}/_3$	$0.0769\ D_o^3$

$M_{DR} = 0.00847 D_o^3 + 0.381\ D_o^2 + 5.716\ D_o + 28.582 + 1.778\ D_o + 13.338$

$M_{RS} = 0.0769\ D_o^3$

- Calculate embedment depth:

$M_{RS} = M_{DR}$

$$-0.0684 D_o^3 + 0.381 D_o^2 + 7.495 D_o + 41.920 = 0$$

$$D_o^3 - 5.57 D_o^2 - 109.55 D_o - 612.76 = 0 \quad \Rightarrow \quad D_o = 15.328 \text{ ft} \quad \Rightarrow \quad D = 1.2 D_o = 18.393 \text{ ft}$$

- Calculate Maximum Moment:

  The maximum moment is located at distance Y below the excavation line where the shear is equal to zero. Therefore the summation of horizontal forces at the distance Y must be set to equal zero.

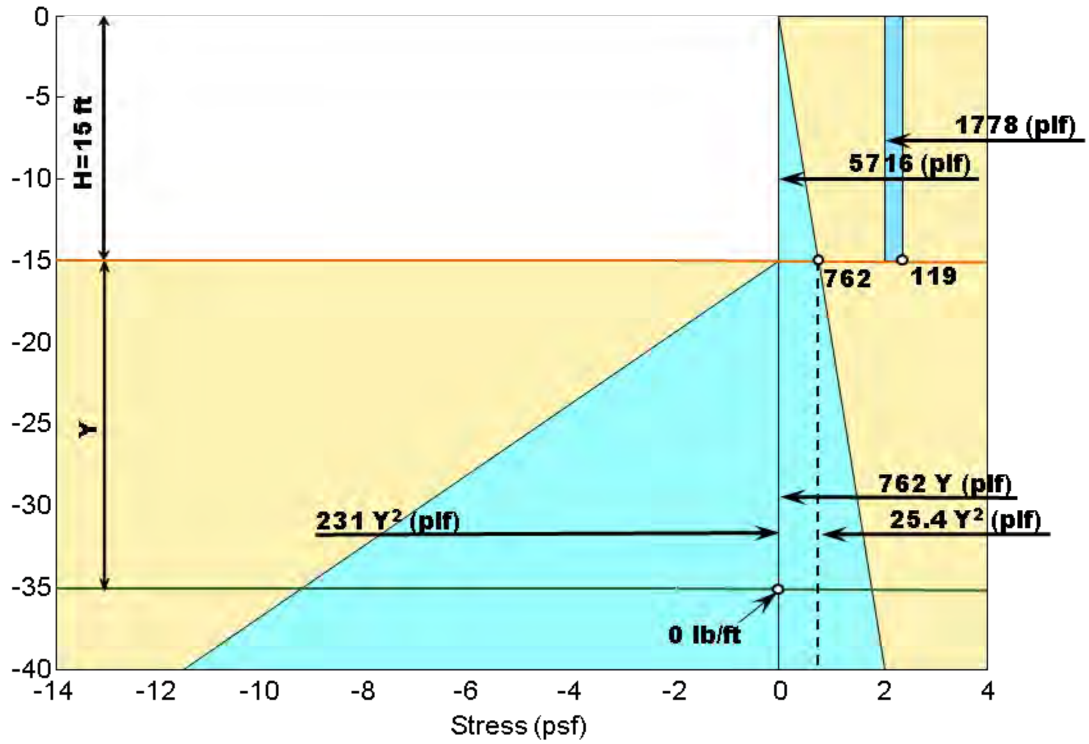


Figure 4-24: Location of Zero and Maximum Moment for Strength I Loading Condition

$$\sum F_x = 0$$

$$0.231Y^2 - 0.0254Y^2 - 0.762Y - 5.716 - 1.778 = Y^2 - 3.714Y - 36.521 = 0$$

$$\Rightarrow \quad Y = 8.179 \text{ ft(below the dredge line)}$$

$$M_{max} = \left\{ \begin{array}{l} 1.778(7.5+8.179) + 5.716(5+8.179) + 0.762(8.179)\left(\frac{8.179}{2}\right) \\ + \ 0.0254(8.179)^2\left(\frac{8.179}{3}\right) - 0.231(8.179)^2\left(\frac{8.179}{3}\right) \end{array} \right\}$$

$$M_{max} = 91.286 \text{ kips-ft/ft}$$

The figure below displays the Shear and Moment Diagram for the Strength I Loading Condition:

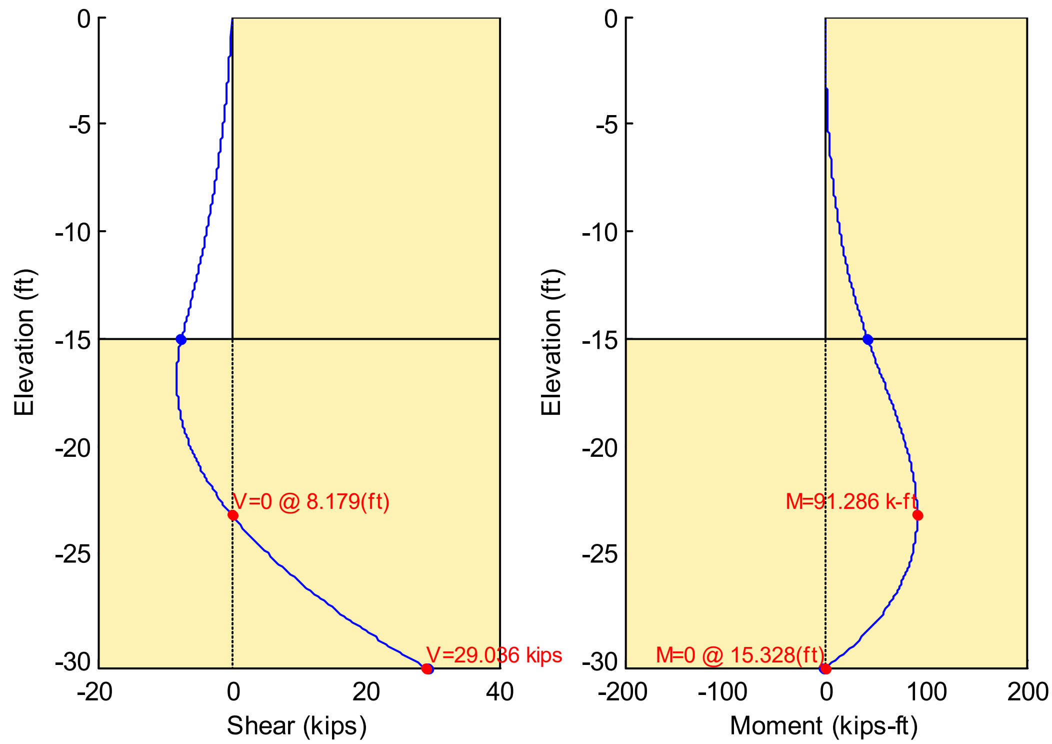


Figure 4-25: Shear and Moment Diagram for Strength I Loading Condition

- Design Sheet Pile for the above Shear and Moment:
  (Note: Assuming Grade 55 Steel will be used)

  Check Flexural:

$$\sigma_{allowable} = 1.0 f_y = 55 \text{ ksi}$$

$$S_m = S_x = \frac{M_{max}}{\sigma_{allowable}} = \frac{91.286 \text{ kips} - ft\left(12 \text{ in/ft}\right)}{55 \; ksi} = 19.917 \text{ in}^3$$

$$\text{Try Sheet Pile Type: } PZ27 \quad \Rightarrow \quad S_m = 30.2 \text{ in}^3 > 19.917 \text{ in}^3$$

  Check Shear:

$$q_{v_{allowable}} = 1.0 f_y = 55 \text{ ksi}$$

$$q_v = \frac{V_{max}}{A} = \frac{29.036 \text{ kips}}{7.9 \text{ in}^2} = 3.675 \text{ ksi}$$

***Note: A is based on Sheet Pile Properties**

$$q_{v_{allowable}} > q_v \quad \Rightarrow \quad \text{Assumed sheet pile O.K. for design}$$

PART C: EXTREME EVENT CALCULATIONS (for k_h = 0.35)

- Calculate active and passive earth pressure coefficients: For seismic condition, normally the Trial Wedge method or the Mononobe-Okabe equation is applied to calculate the active and passive earth pressure coefficients.

$$K_{ae} = 0.526, \quad K_{pe} = 2.945$$

Note: Trial Wedge method and Mononobe-Okabe equations tend to overestimate the passive earth pressure. To achieve a conservative design, it is recommended to use the Log-Spiral-Rankine model to compute the passive earth force.

- Calculate earth pressure distribution

Lateral pressure due to live load surcharge is not considered in the Extreme Event Limit State:

$$s_{sur} = 0$$

Total Seismic lateral load distribution at the excavation level:

$$\sigma^{-} = 0.125 \times 15 \times 0.526 = 0.986 \text{ ksf}$$

Lateral load distribution for the second layer at depth D_0:

$$\sigma_D = 0.986 + 0.125 \times 0.526\ D_o = \left(0.986 + 0.0658\ D_o\right) \text{ksf}$$

Passive Lateral load distribution for the second layer in the front at depth D_0:

$$\sigma_{pD} = 0.125 \times 2.945 D_o = 0.368\ D_o \text{ ksf}$$

- Calculate resultant earth forces

Calculate active earth pressure for the first soil layer P_{A1}:

$$P_{A1} = 0.986\left(\frac{15}{2}\right) = 7.397\ \text{klf}$$

Calculate active earth pressure for the second soil layer P_{A2}:

$$P_{A21} = 0.986 D_o\ \text{klf}$$

$$P_{A22} = 0.0658\ D_o\left(\frac{D_o}{2}\right) = 0.0329\ D_o^2\ \text{klf}$$

Calculate passive earth pressure for the second soil layer P_P:

$$P_p = 0.368 D_o\left(\frac{D_o}{2}\right) = 0.184\ D_o^2\ \text{lbs/ft}$$

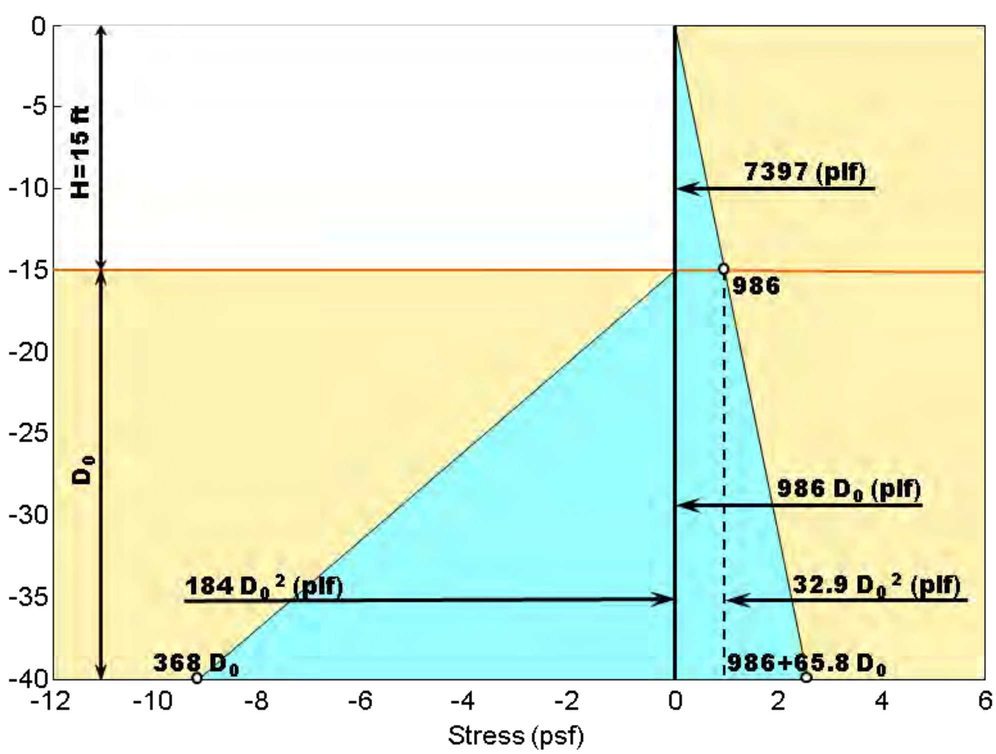


Figure 4-26: Pressure Diagram for Extreme Event Loading Condition

- Calculate driving and resisting moments

Driving Force (klf)	Arm (ft)	Driving Moment M_{RS}
7.397	$5+D_o$	$36.984 + 7.397\ D_o$
$0.986\ D_o$	$^{Do}/_2$	$0.493\ D_o^{\ 2}$
$0.0329\ D_o^{\ 2}$	$^{Do}/_3$	$0.0110\ D_o^3$
Resisting Force (plf)	Arm (ft)	Resisting Moment M_{RS}
$0.184\ D_o^{\ 2}$	$^{Do}/_3$	$0.0614\ D_o^3$

$M_{DR} = 0.0110\ D^3 + 0.493\ D^2 + 7.397D + 36.984$

$M_{RS} = 0.0614\ D^3$

- Calculate embedment depth:

  $M_{RS} = M_{DR}$

$$-0.0504D_o^3 + 0.492D_o^2 + 7.397D_o + 36.984 = 0$$
$$\Rightarrow D_o^3 + 9.785D_o^2 - 146.776D_o + 733.878 = 0$$

$$\Rightarrow \quad \boldsymbol{D_o = 19.338 \text{ ft}} \quad \Rightarrow \quad \boldsymbol{D = 1.2D_o = 23.205 \text{ ft}}$$

- Calculate maximum moment:

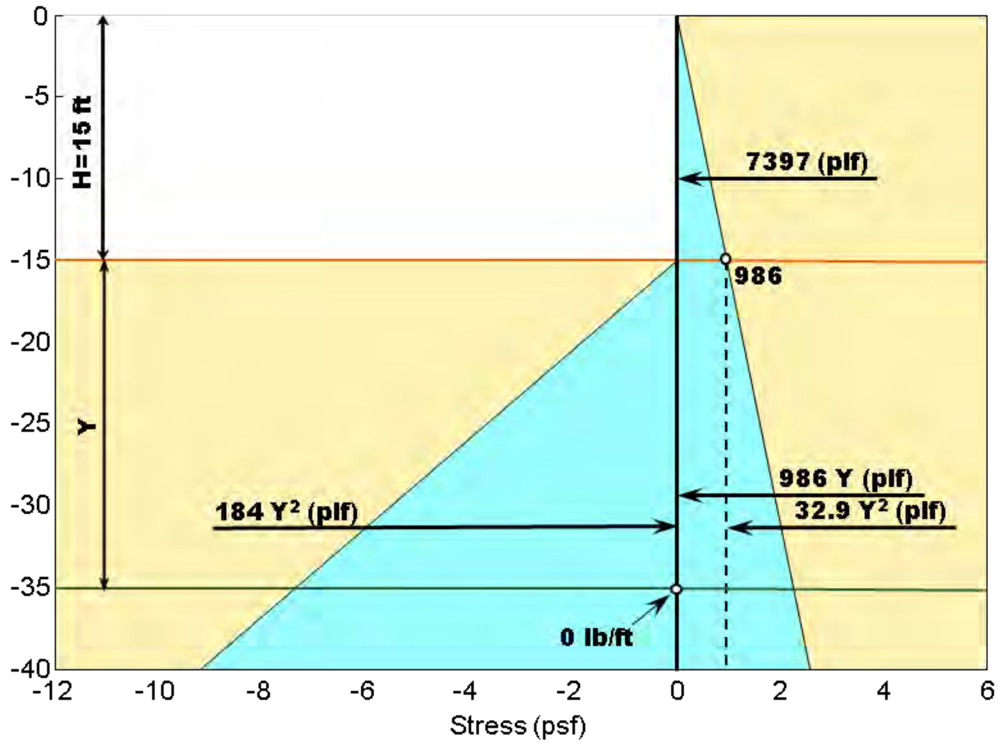


Figure 4-27: Location of Zero Shear and Maximum Moment for Extreme Event Loading Condition

$$\sum F_x = 0$$

$$0.184Y^2 - 0.0329Y^2 - 0.986Y - 7.397 = 0.151Y^2 - 0.986Y - 7.397 = 0$$

$$Y^2 - 6.523Y - 48.925 = 0$$

$$\Rightarrow \quad Y = 10.979 \text{ ft(below the dredge line)}$$

$$M_{max} = \begin{Bmatrix} 7.397(5+10.979)+0.986(10.979)\left(\dfrac{10.979}{2}\right) \\ +0.0329(10.979)^2\left(\dfrac{10.979}{3}\right)-0.184(10.979)^2\left(\dfrac{10.979}{3}\right) \end{Bmatrix}$$

$$M_{max} = 110.942 \text{ kips-ft/ft}$$

The figure below shows the Shear and Moment Diagram for the Extreme Event Loading Condition:

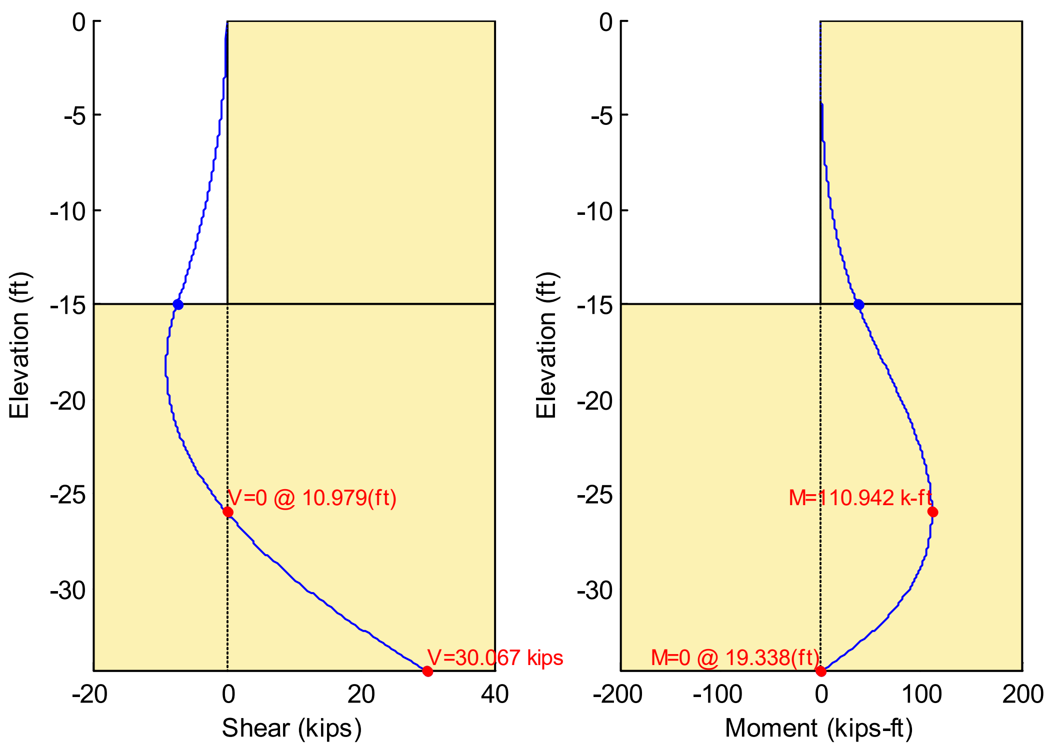


Figure 4-28: Shear and Moment Diagram for Extreme Event Loading Condition

Design Sheet Pile for the above Shear and Moment:

*Note: Assuming Grade 55 Steel will be used

Check Flexural:

$$\sigma_{allowable} = 1.0 f_y = 55 \text{ ksi}$$

$$S_m = S_x = \frac{M_{\max}}{\sigma_{\text{allowable}}} = \frac{110.942\ K-\text{ft}(12 \text{ in/ft})}{55 \text{ ksi}} = 24.206 \text{ in}^3$$

$$\text{Try Sheet Pile Type: } PZ27 \quad \Rightarrow \quad S_m = 30.2\ in^3 > 24.206\ \text{in}^3,\ \text{OK}$$

Check Shear:

$$q_{v_{allowable}} = 1.0 f_y = 55\ \text{ksi}$$

$$q_v = \frac{V_{max}}{A} = \frac{30.067\ K}{7.9\ in^2} = 3.806\ \text{ksi}$$

***Note: A is based on Sheet Pile Properties**

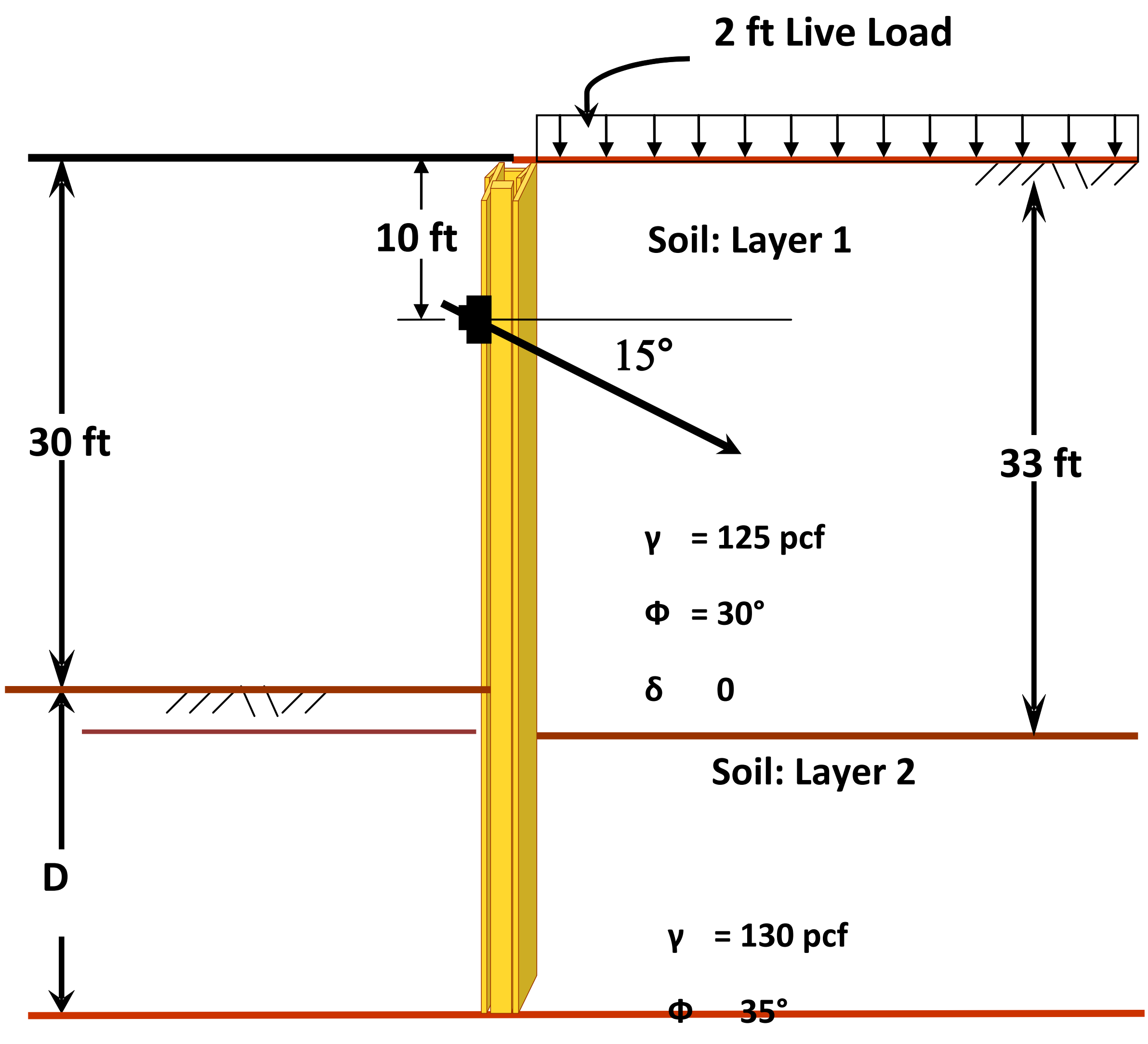
$$q_{v_{allowable}} > q_v \quad \Rightarrow \quad \text{Assumed Sheet Pile OK for Design}$$

Based on the above calculation, it is seen that when k_h=0.35, the Extreme Event Limit State controls the design. However, when k_h=0.25, the Strength I Limit State controls, as summarized in the table below.

Limit State	D_0 (ft)	1.2 D_0 (ft)	V @ M=0 (kips)	Y (ft)	M_{max} (kips-ft)
Service	11.903	14.284	19.402	6.088	49.409
Strength I	15.328	18.393	29.036	8.179	91.286
Extreme Event I (k_h=0.25)	15.958	19.149	24.492	8.813	77.083
Extreme Event I (k_h=0.35)	19.338	23.205	30.067	10.979	110.942

4.6.2 Example 4-2: Single Tieback Sheet Pile Wall

Design a tieback sheet pile wall with two soil layers and 2 feet of uniform surcharge that should extend to a depth of 30 feet. The tieback spacing is 8 feet.



Determine:

1. Active & Passive Earth Pressures
2. Pile Embedment *D*
3. Maximum Shear
4. Maximum Moment
5. Maximum Deflection

PART A: SERVICE I LOAD CALCULATIONS

- Active and Passive Earth Pressures

  Active Earth Pressures: Used Rankine to solve for K_{a1} and K_{a2}

  $$K_{a_1} = \tan^2\left(45^o - \frac{\varphi}{2}\right) = \tan^2\left(45^o - \frac{30^o}{2}\right) = 0.333$$

  $$K_{a_2} = \tan^2\left(45^o - \frac{\varphi}{2}\right) = \tan^2\left(45^o - \frac{35^o}{2}\right) = 0.271$$

  Passive Earth Pressures: Used Rankine to solve for K_{p1} and K_{p2}

  $$K_{p_1} = \tan^2\left(45^o + \frac{\varphi}{2}\right) = \tan^2\left(45^o + \frac{30^o}{2}\right) = 3.000$$

  $$K_{p_2} = \tan^2\left(45^o + \frac{\varphi}{2}\right) = \tan^2\left(45^o + \frac{35^o}{2}\right) = 3.690$$

Note: Rankine Theory tends to underestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to compute the passive earth force.

- Develop Active and Passive Earth Pressure Diagram

Lateral load due to surcharge above the excavation line only (LRFD Factor – 1.0):

$$s_{sur} = (0.125)(2)(0.333)(1.0) = 0.083 \text{ ksf}$$

Lateral load distribution at excavation line (LRFD Factor – 1.0):

$$\sigma_A = \gamma_1(h = 30)K_{a1}(1.0) = (0.125)(30)(0.333)(1.0) = 1.249 \text{ ksf}$$

Lateral load distribution at Δh = 3ft below excavation line (LRFD Factor – 1.0):

$$\sigma_B^+ = \sigma_A + \gamma_1(\Delta h)K_{a1}(1.35) = 1.249 + (0.125)(3)(0.333)(1.0) = 1.374 \text{ ksf}$$

$$\sigma_B^- = \gamma_1(h + \Delta h)K_{a2}(1.0) = (0.125)(33)(0.271)(1.0) = 1.118 \text{ ksf}$$

Lateral load distribution for the second layer at depth D_o (LRFD Factor – 1.0):

$$\sigma_D = \sigma_B^- + \gamma_2(D_o - 3)K_{a2}(1.0) = 1.118 + (0.130)(D_o - 3)(0.271)(1.0)$$
$$= (1.012 + 0.0352\, D_o) \text{ ksf}$$

Calculate passive earth pressure at Δh = 3ft below excavation line (LRFD Factor – 1.0):

$$\sigma_p^+ = \gamma_1(\Delta h)K_{p1}(1.0) = (0.125)(3)(3.00)(1.0) = 1.125\text{ ksf}$$

$$\sigma_p^- = \gamma_1(\Delta h)K_{p2}(1.0) = (0.125)(3)(3.69)(1.0) = 1.384\text{ ksf}$$

Calculate passive earth pressure for the second layer at depth D_o (LRFD Factor – 1.0):

$$\sigma_{pD} = \sigma_p^- + \gamma_2(D_o - 3)K_{p2}(1.0) = 1.384 + (0.130)(D_o - 3)(3.69)(1.0)$$
$$= (0.480\,D_o - 0.0551)\text{ ksf}$$

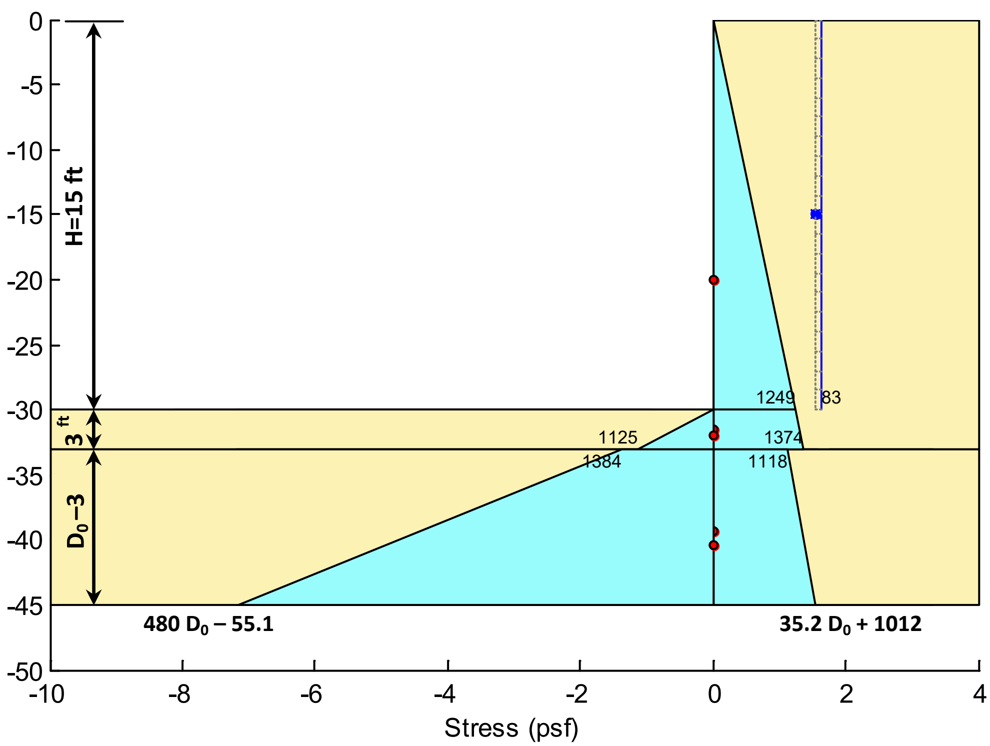


Figure 4-29: Triangular Earth Pressure Diagram for Service I Loading Condition

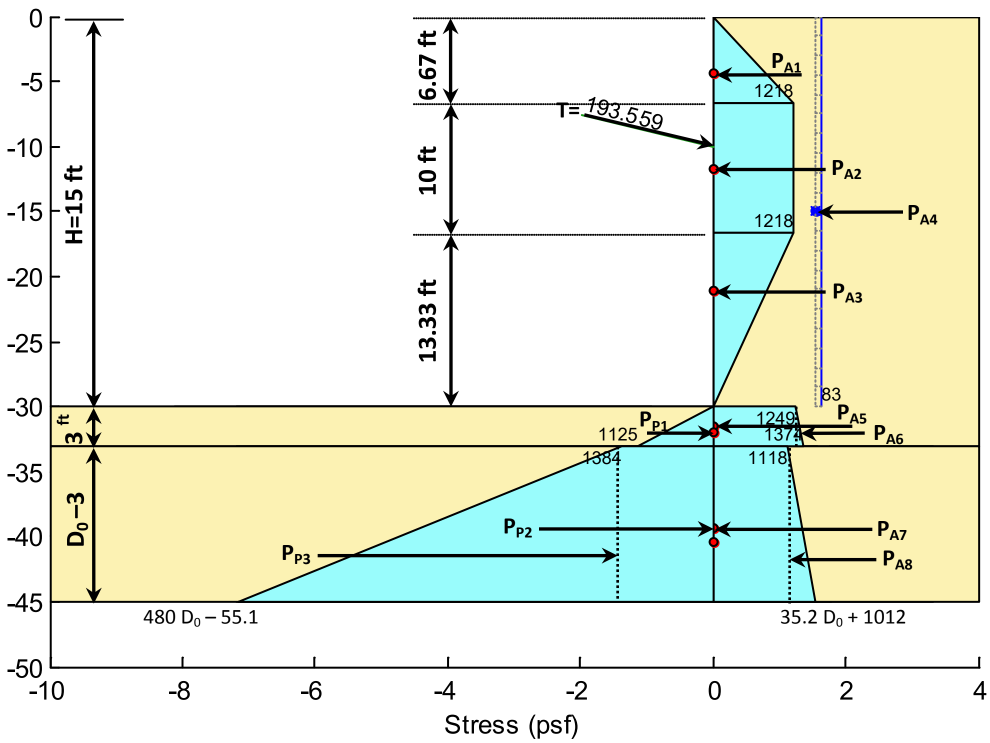


Figure 4-30: Trapezoidal Earth Pressure Diagram for Service I Loading Condition

Develop Trapezoidal loading (LRFD Factor – 1.0):

$$\sigma_{Trapezoid} = \frac{1.3P}{\frac{2}{3}h}(1.0) \quad \text{where} \quad P = \frac{1}{2}\gamma_1 (h = 30)^2 K_{a1}$$

$$P = \frac{1}{2}(0.125)(30)^2(0.333) = 18.731\,\text{klf}$$

$$\sigma_{Trapezoid} = \frac{1.3(18.731)}{\frac{2}{3}(30)}(1.0) = 1.2175\,\text{ksf}$$

- Calculating Driving and Resisting Moments taken about the Tieback Force

Driving Force (klf)	**Arm (ft)**	**Driving Moment M_{DR} (k)**
P_{A1}= (½)(6.667)(1.2175) = 4.058	$^{6.667}/_3$ + 3.33 = 5.556	−22.547
P_{A2} = (1.2175)(10) = 12.175	(6.667+5) – 10 = 1.667	20.292
P_{A3}= $^1/_2$(13.333)(1.2175) = 8.117	20 – $^2/_3$(13.333)= 11.111	90.188
P_{A4} = (0.083)(30) = 2.498	5	12.488
P_{A5} = (3)(1.249) = 3.747	1.5+20 = 21.5	80.548
P_{A6} = $^3/_2$(1.374 – 1.249) = 0.187	2+20 = 22	4.118
P_{A7}= 1.118 (D_o– 3) = 1.118 D_o – 3.354	20 + 3 + $^{(Do-3)}/_2$ = 21.5 + $^{Do}/_2$	0.559D_o^2 +22.358 D_o – 72.103
P_{A8} = $^1/_2$ (-0.106 + 0.0352 D_o)(D_o– 3) = 0.0176D_o^2- 0.106 D_o + 0.159	20 + 3 + $^2/_3$ (D_o – 3) = 21 + $^2/_3$ D_o	0.0117D_o^3 + 0.299D_o^2 –2.114 D_o +3.329
Resisting Force (klf)	**Arm (ft)**	**Resisting Moment M_{RS} (k)**
P_{P1} = $^1/_2$(3)(1.125) = 1.688	20 + $^2/_3$(3) = 22	37.125
P_{P2} = 1.384(D_o– 3) = 1.384 D_o – 4.151	20 + 3 + $^{(Do–3)}/_2$ = 21.5 + $^{Do}/_2$	0.692D_o^2 +27.675 D_o – 89.252
P_{P3} = $^1/_2$(0.480 D_o – 1.439) (D_o– 3) = 0.240D_o^2– 1.439 D_o + 2.159	20 + 3 + $^2/_3$ (D_o– 3) = 21 + $^2/_3$ D_o	0.160D_o^3 + 4.077D_o^2 –28.782 D_o + 45.332

M_{DR}= 0.0117D_o^3 + 0.299D_o^2 –2.114 D_o + 3.329 + 0.559D_o^2 + 22.358 D_o – 72.103 + 4.118 + 80.548 + 12.488 + 90.188 + 20.292 – 22.547

M_{RS} = 0.160D_o^3 + 4.077D_o^2 –28.782 D_o + 45.332 + 0.692D_o^2 +27.675 D_o – 89.252 + 37.125

Calculate Embedment Depth:

$M_{RS} = M_{DR}$

$$0.148D_o^3 + 3.911D_o^2 - 21.351D_o - 123.107 = 0$$

$$D_o^3 + 26.397D_o^2 - 144.109D_o - 830.925 = 0$$

$D_o = 7.512$ ft

Calculate Tieback Force:

$$\sum F_x = 0$$

$$\begin{Bmatrix} T_H + 1.688 + 1.384(7.512) \\ -4.151 + 0.240(7.512)^2 \\ -1.439(7.512) + 2.159 \end{Bmatrix} = \begin{Bmatrix} 4.058 + 12.175 + 8.117 + 2.498 \\ +3.747 + 0.187 + 1.118(7.512) - 3.354 \\ +0.0176(7.512)^2 - 0.106(7.512) + 0.159 \end{Bmatrix}$$

$T_H = 23.371\,\text{klf}$ in horizontal direction

Multiply by 8 ft for spacing

$$T = \frac{23.371(8)}{\cos(15°)} = 193.559\,\text{kips} \quad \text{(along the 15 degree angle)}$$

The Shear and Moment Diagrams are displayed below:

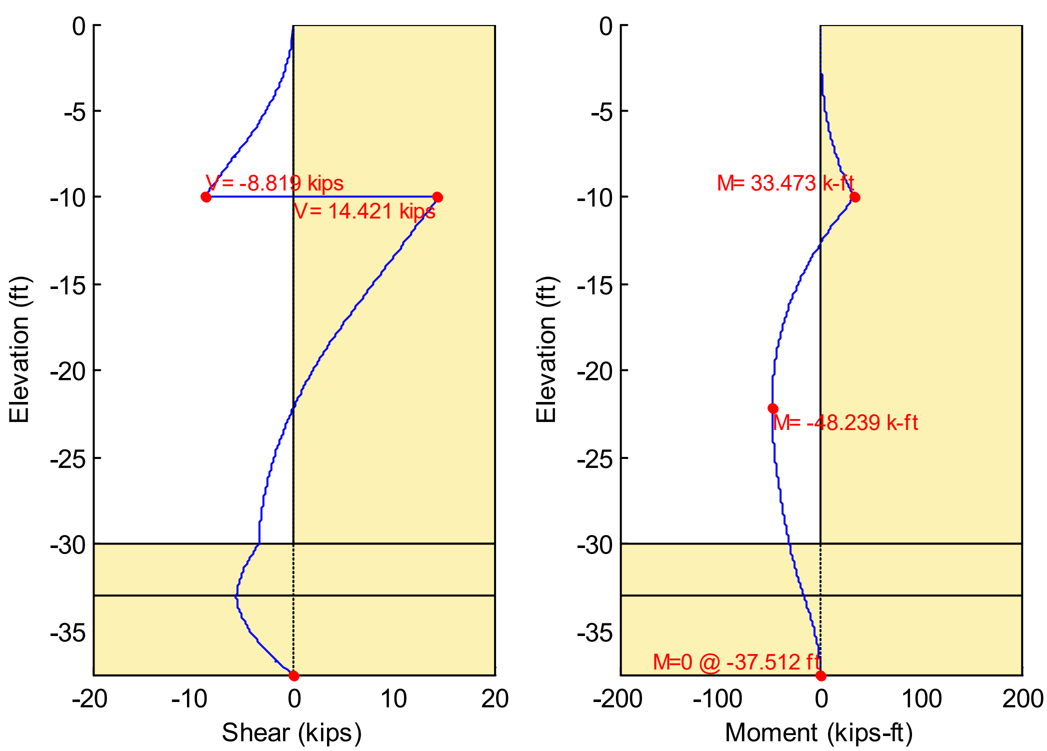


Figure 4-31: Shear and Moment Diagrams for Service I Loading Condition

PART B: STRENGTH I LOAD CALCULATIONS

- Active and Passive Earth Pressures

  Active Earth Pressures: Used Rankine to solve for K_{a1} and K_{a2}

$$K_{a_1} = \tan^2\left(45^o - \frac{\varphi}{2}\right) = \tan^2\left(45^o - \frac{30^o}{2}\right) = 0.333$$

$$K_{a_2} = \tan^2\left(45^o - \frac{\varphi}{2}\right) = \tan^2\left(45^o - \frac{35^o}{2}\right) = 0.271$$

  Passive Earth Pressures: Used Rankine to solve for K_{p1} and K_{p2}

$$K_{p_1} = \tan^2\left(45^o + \frac{\varphi}{2}\right) = \tan^2\left(45^o + \frac{30^o}{2}\right) = 3.000$$

$$K_{p_2} = \tan^2\left(45^o + \frac{\varphi}{2}\right) = \tan^2\left(45^o + \frac{35^o}{2}\right) = 3.690$$

Note: Rankine Theory tends to underestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to compute the passive earth force.

- Develop Active and Passive Earth Pressure Diagram

Lateral load due to surcharge above the excavation line only (LRFD Factor – 1.75):

$$s_{sur} = (0.125)(2)(0.333)(1.75) = 0.146 \text{ ksf}$$

Lateral load distribution at excavation line (LRFD Factor – 1.35):

$$\sigma_A = \gamma_1 (h = 30) K_{a1} (1.35) = (0.125)(30)(0.333)(1.35) = 1.686 \text{ ksf}$$

Lateral load distribution at Δh = 3ft below excavation line (LRFD Factor – 1.35):

$$\sigma_B^+ = \sigma_A + \gamma_1 (\Delta h) K_{a1} (1.35) = 1.686 + (0.125)(3)(0.333)(1.35) = 1.854 \text{ ksf}$$

$$\sigma_B^- = \gamma_1 (h + \Delta h) K_{a2} (1.35) = (0.125)(33)(0.271)(1.35) = 1.509 \text{ ksf}$$

Lateral load distribution for the second layer at depth D_o (LRFD Factor – 1.35):

$$\sigma_D = \sigma_B^- + \gamma_2 (D_o - 3) K_{a2} (1.35) = 1.509 + (0.130)(D_o - 3)(0.271)(1.35)$$
$$= (1.652 - 0.0476 D_o) \text{ ksf}$$

Calculate passive earth pressure at Δh = 3ft below excavation line (LRFD Factor – 1.0):

$$\sigma_p^+ = \gamma_1(\Delta h)K_{p1}(1.0) = (0.125)(3)(3.00)(1.0) = 1.125 \text{ ksf}$$

$$\sigma_p^- = \gamma_1(\Delta h)K_{p2}(1.0) = (0.125)(3)(3.69)(1.0) = 1.384 \text{ ksf}$$

Calculate passive earth pressure for the second layer at depth D_o (LRFD Factor – 1.0):

$$\begin{aligned}\sigma_{pD} &= \sigma_p^- + \gamma_2(D_o - 3)K_{p2}(1.0) = 1.384 + (0.130)(D_o - 3)(3.69)(1.0) \\ &= (0.480\,D_o - 0.0551)\text{ ksf}\end{aligned}$$

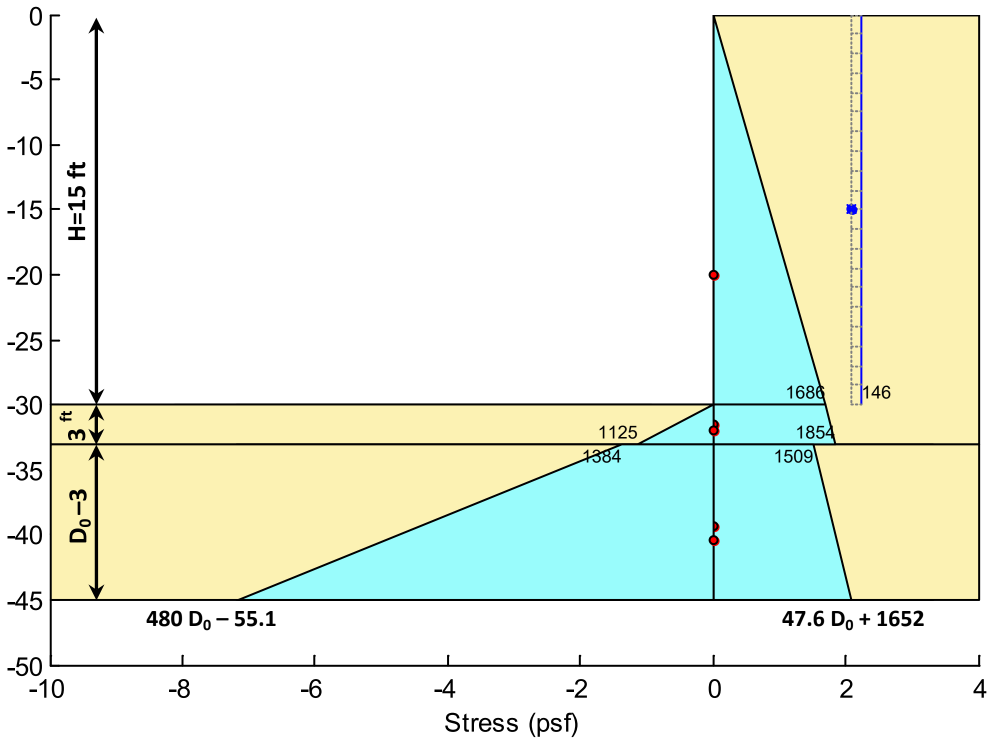


Figure 4-32: Triangular Earth Pressure Diagram for Strength I Loading Condition

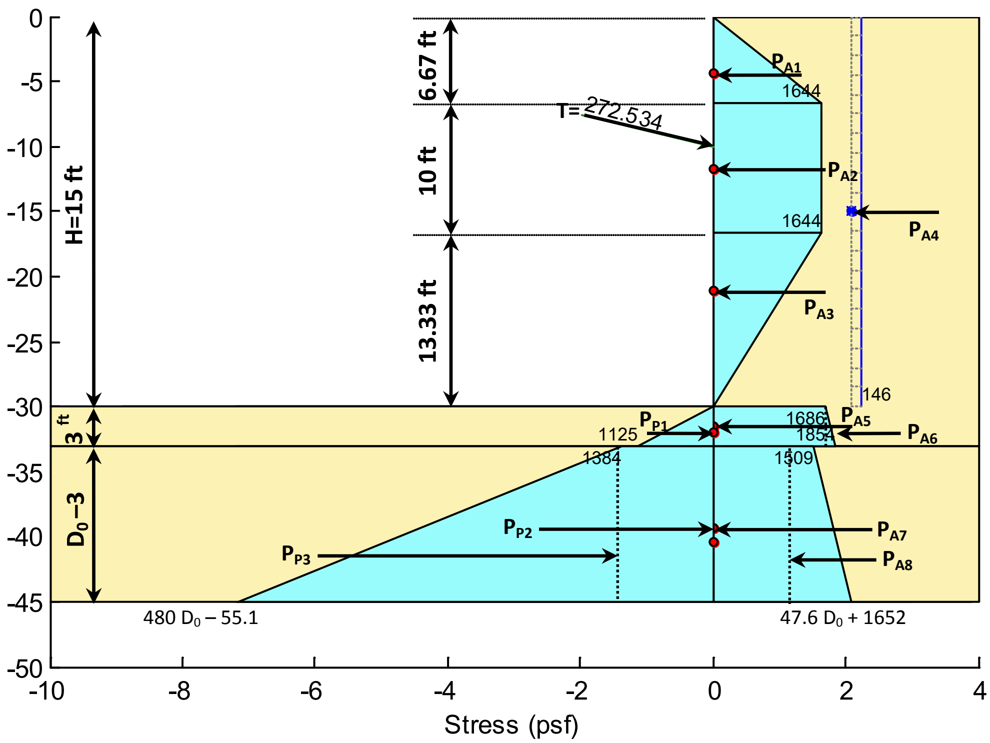


Figure 4-33: Trapezoidal Earth Pressure Diagram for Strength I Loading Condition

Develop Trapezoidal loading (LRFD Factor – 1.35):

$$\sigma_{Trapezoid} = \frac{1.3P}{\frac{2}{3}h}(1.35) \quad \text{where} \quad P = \frac{1}{2}\gamma_1 (h = 30)^2 K_{a1}$$

$$P = \frac{1}{2}(0.125)(30)^2(0.333) = 18.731\,\text{klf}$$

$$\sigma_{Trapezoid} = \frac{1.3(18.731)}{\frac{2}{3}(30)}(1.35) = 1.644\,\text{ksf}$$

- Calculating Driving and Resisting Moments taken about the Tieback Force

Driving Force (klf)	**Arm (ft)**	**Driving Moment M_{DR} (k)**
P_{A1}= (½)(6.667)(1.644) = 5.480	$^{6.667}/_{3}$ + 3.33 = 5.556	−30.438
P_{A2} = (1.644)(10) = 16.44	(6.667+5) – 10 = 1.667	27.395
P_{A3}= $^{1}/_{2}$(13.333)(1.644) = 10.958	20 – $^{2}/_{3}$(13.333)= 11.111	121.753
P_{A4} = (0.146)(30) = 4.371	5	21.853
P_{A5} = (3)(1.686) = 5.057	1.5+20 = 21.5	108.734
P_{A6} = $^{3}/_{2}$(1.854 – 1.686) = 0.253	2+20 = 22	5.564
P_{A7}= 1.509 (D_o– 3) = 1.509D_o – 4.527	20 + 3 + $^{(Do-3)}/_{2}$ = 21.5 + $^{Do}/_{2}$	0.755D_o^2 +30.183 D_o –97.339
P_{A8} = $^{1}/_{2}$ (-0.143 + 0.0476 D_o)(D_o– 3) = 0.0238D_o^2- 0.143 D_o + 0.214	20 + 3 + $^{2}/_{3}$ (D_o – 3) = 21 + $^{2}/_{3}$ D_o	0.0159D_o^3 + 0.404D_o^2 –2.854 D_o +4.494
Resisting Force (klf)	**Arm (ft)**	**Resisting Moment M_{RS} (k)**
P_{P1} = $^{1}/_{2}$(3)(1.125) = 1.688	20 + $^{2}/_{3}$(3) = 22	37.125
P_{P2} = 1.384(D_o– 3) = 1.384 D_o – 4.151	20 + 3 + $^{(Do–3)}/_{2}$ = 21.5 + $^{Do}/_{2}$	0.692D_o^2 +27.675 D_o – 89.252
P_{P3} = $^{1}/_{2}$(0.480 D_o – 1.439) (D_o– 3) = 0.240D_o^2– 1.439 D_o + 2.159	20 + 3 + $^{2}/_{3}$ (D_o– 3) = 21 + $^{2}/_{3}$ D_o	0.160D_o^3 + 4.077D_o^2 –28.782 D_o + 45.332

M_{DR}= 0.0159D_o^3 + 0.404D_o^2 –2.854 D_o + 4.494 + 0.755D_o^2 + 30.183 D_o – 97.337 + 5.564 + 108.734 + 21.853 + 121.753 + 27.395 – 30.438

M_{RS} = 0.160D_o^3 + 4.077D_o^2 –28.782 D_o + 45.332 + 0.692D_o^2 +27.675 D_o – 89.252 + 37.125

Calculate embedment depth:

$M_{RS} = M_{DR}$

$$0.144D_o^3 + 3.611D_o^2 - 28.436D_o - 168.811 = 0$$

$$D_o^3 + 25.065D_o^2 - 197.408D_o - 1171.92 = 0$$

$D_o = 9.367$ ft

Calculate Tieback Force:

$$\sum F_x = 0$$

$$\begin{Bmatrix} T_H + 1.688 + 1.384(9.367) \\ -4.151 + 0.240(9.367)^2 \\ -1.439(9.367) + 2.159 \end{Bmatrix} = \begin{Bmatrix} 5.480 + 16.44 + 10.958 + 4.371 \\ +5.057 + 0.253 + 1.509(9.367) - 4.527 \\ +0.0238(9.367)^2 - 0.143(9.367) + 0.214 \end{Bmatrix}$$

$T_H = 32.906$ klf in horizontal direction

Multiply by 8 ft for spacing

$$T = \frac{32.906(8)}{\cos(15°)} = 272.534 \text{ kips} \quad \text{(along the 15 degree angle)}$$

The Shear and Moment Diagrams are displayed below:

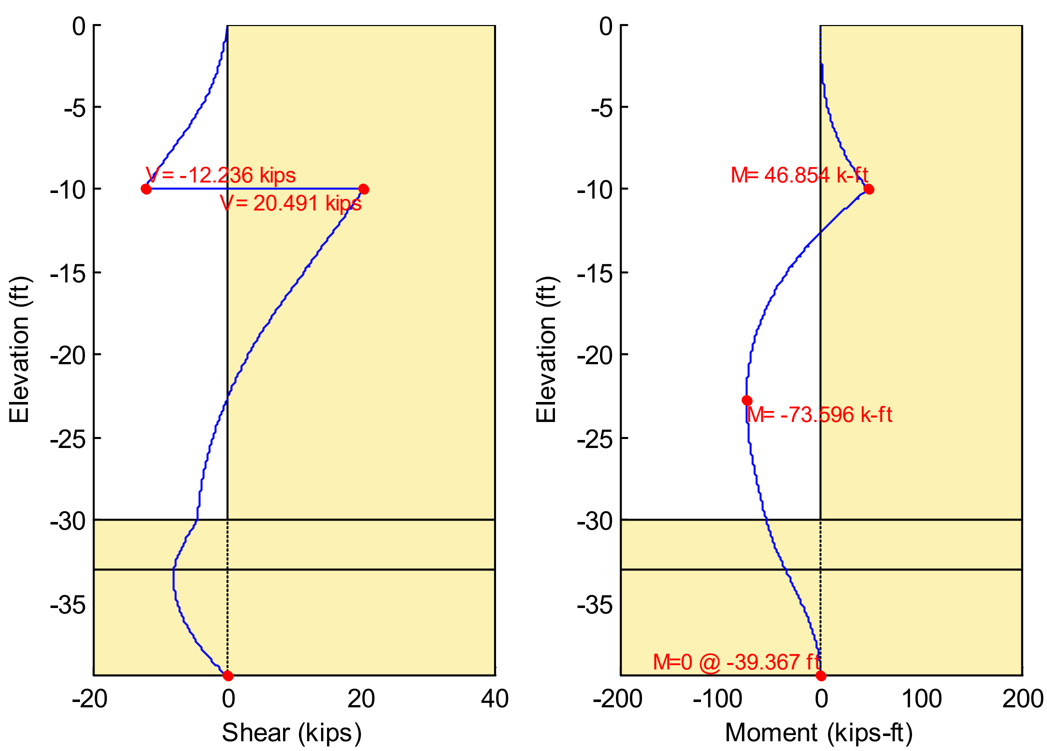


Figure 4-34: Shear and Moment Diagrams for Strength I Loading Condition

PART C: EXTREME EVENT CALCULATIONS (k_h = 0.35)

- *Calculate active and passive earth pressure coefficients*:

For seismic condition, normally the Trial Wedge method or the Mononobe-Okabe equation is applied to calculate the active and passive earth pressure coefficients.

$$K_{ae_1} = 0.628 \qquad K_{pe_1} = 2.301$$

$$K_{ae_2} = 0.526 \qquad K_{pe_2} = 2.945$$

Note: Trial Wedge method and Mononobe-Okabe equations tend to overestimate the passive earth pressure. To achieve a conservative design, it is recommended to use the Log-Spiral-Rankine model to compute the passive earth force.

- *Calculate Earth Pressure Distribution:*

Lateral pressure due to live load surcharge is not considered in the Extreme Event Limit State:

$$s_{sur} = 0$$

Total Seismic lateral load distribution at the excavation level:

$$\sigma_A^- = \gamma_1 (h = 30) K_{ae_1} = (0.125)(30)(0.628) = 2.355 \text{ ksf}$$

Lateral load distribution at Δh = 3ft below excavation line:

$$\sigma_B^+ = \sigma_A^- + \gamma_1 (\Delta h) K_{ae_1} = 2.355 + (0.125)(3)(0.628) = 2.591 \text{ ksf}$$

$$\sigma_B^- = \gamma_1 (h + \Delta h) K_{ae_2} = (0.125)(30+3)(0.526) = 2.170 \text{ ksf}$$

Lateral load distribution at a depth D_o below excavation line:

$$\sigma_D = \sigma_B^- + \gamma_2 (D_o - 3) K_{ae_2} = 2.170 + (0.130)(D_o - 3)(0.526) = (1.965 + 0.0684\, D_o) \text{ ksf}$$

Calculate passive earth pressure at Δh = 3ft below excavation line:

$$\sigma_p^+ = \gamma_1 (\Delta h) K_{pe_1} = (0.125)(3)(2.301) = 0.863 \text{ ksf}$$

$$\sigma_p^- = \gamma_1 (\Delta h) K_{pe_1} = (0.125)(3)(2.945) = 1.104 \text{ ksf}$$

Calculate passive earth pressure at a depth D_o below excavation line:

$$\sigma_{pD} = \sigma_p^- + \gamma_2\left(D_o - 3\right)K_{pe_2} = 1.104 + \left(0.130\right)\left(D_o - 3\right)\left(2.945\right) = \left(0.383D_o - 0.0446\right)\text{ksf}$$

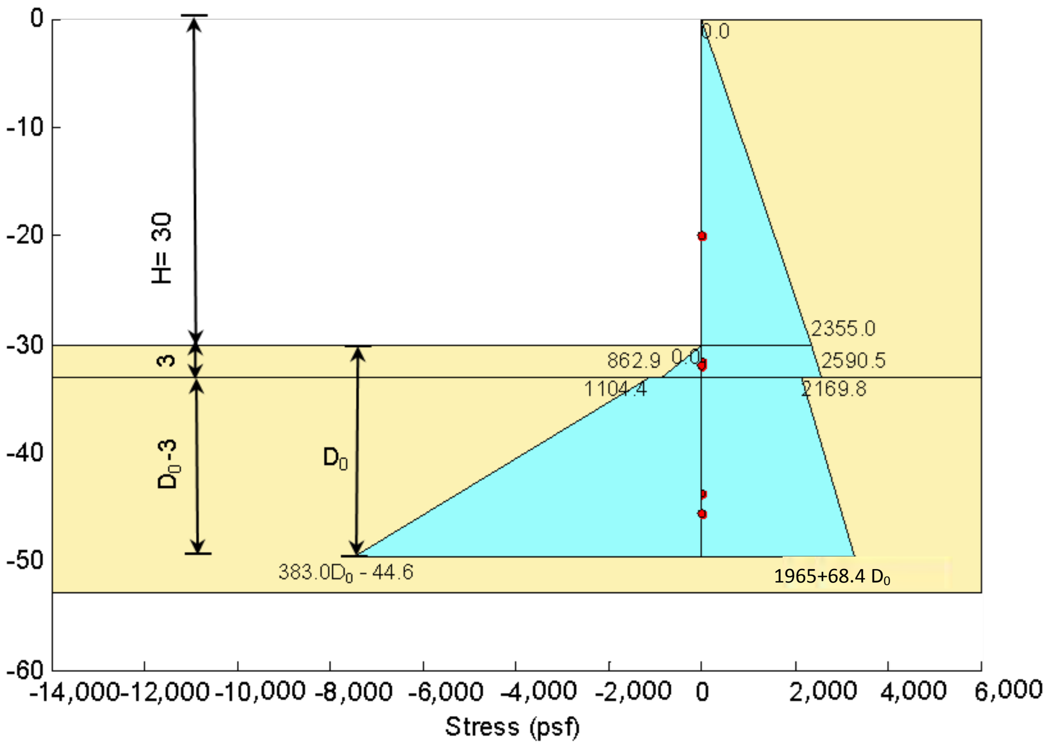


Figure 4-35: Triangular Earth Pressure Diagram for Extreme Event Loading Case

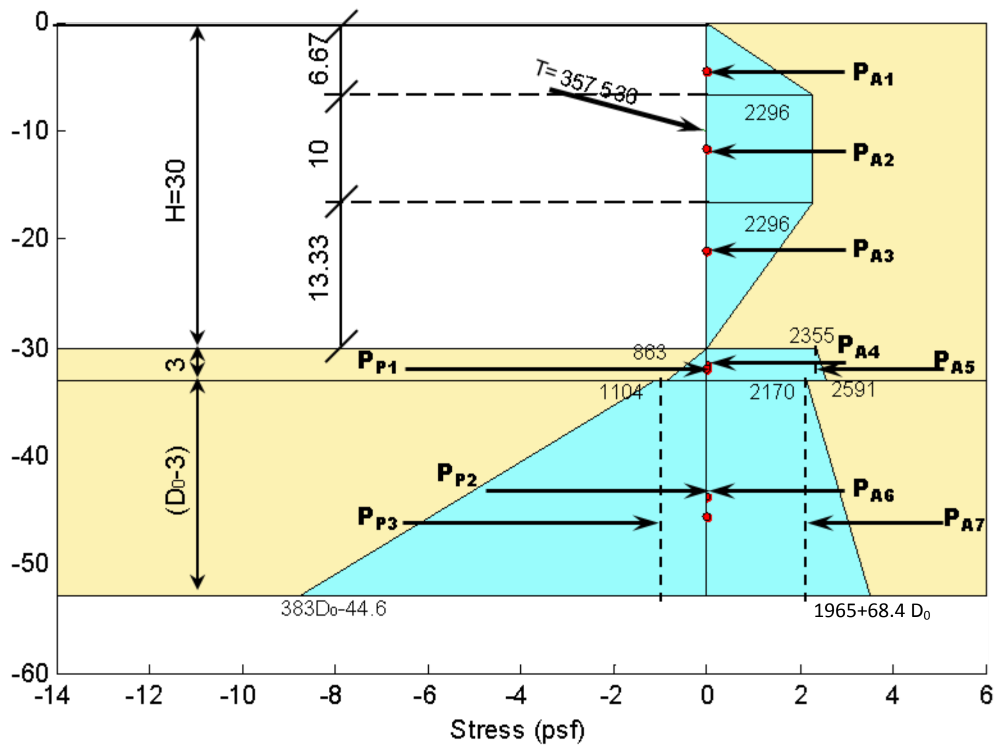


Figure 4-36: Load Distributions for Extreme Event Loading Case

Develop Trapezoidal loading:

$$\sigma_{Trapezoid} = \frac{1.3P}{2/_{3}h} \quad \text{where} \quad P = \frac{1}{2}\gamma_1 (h = 30)^2 K_{ae1}$$

$$P = \frac{1}{2}(0.125)(30)^2(0.628) = 35.325 \text{ klf}$$

$$\sigma_{Trapezoid} = \frac{1.3(35.325)}{2/_{3}(30)} = 2.296 \text{ ksf}$$

Calculating Driving and Resisting Moments taken about the Tieback Force

Driving Force (klf)	Arm (ft)	Driving Moment M_{RS} (kip)
P_{A1}= ½(6.667)(2.296) =7.654	6.667/3 + 3.333 = 5.556	−42.521
P_{A2} = (2.296)(10) = 22.961	(6.667+5) – 10 = 1.667	38.269
P_{A3}= 1/2(13.333)(2.296) = 15.308	20 − 2/3(13.333)= 11.111	170.083
P_{A4} = (3)(2.355) = 7.065	1.5+20 = 21.5	151.898
P_{A5} = 3/2(2.591-2.355) = 0.353	2+20 = 22	7.772
P_{A6}= 2.170 (Do– 3) = 2.170 Do – 6.509	20 + 3 + (D_o −3)/2 = 21.5 + D_o /2	1.085 D_o^2 + 43.395 D_o – 139.949
P_{A7} =1/2 (−0.205 + 0.0684 D_o)(D_o – 3) = 0.0342 D_o^2 – 0.205 D_o + 0.308	20 + 3 + 2/3 (D_o – 3) = 21 + 2/3 D_o	0.0228 D_o^3 + 0.581 D_o^2 – 4.103 D_o + 6.462
Resisting Force (klf)	**Arm (ft)**	**Resisting Moment M_{RS}**
P_{P1} = 1/2(3)(0.863) = 1.294	20 + 2/3(3) = 22	28.475
P_{P2} = 1.104 (D_o – 3) = 1.104 D_o – 3.313	20 + 3 + (D_o −3)/2 = 21.5 + D_o /2	0.552 D_o^2 + 22.086 D_o – 71.232
P_{P3} = 1/2(0.383 D_o – 1.149) (D_o – 3) = 0.191 D_o^2– 1.149 D_o + 1.723	20 + 3 + 2/3 (D_o – 3) = 21 + 2/3 D_o	0.128 D_o^3 + 3.254 D_o^2 – 22.971 D_o + 36.179

$M_{DR} = 0.0228D_o^3 + 0.581D_o^2 – 4.103\ D_o + 6.462+1.085\ D_o^2 + 43.395\ D_o – 139.949 + 7.772 + 151.898 + 170.083 + 38.269 − 42.521$

$M_{RS} = 0.128\ D_o^3 + 3.254\ D_o^2 – 22.971\ D_o + 36.179 + 0.552\ D_o^2 + 22.086\ D_o – 71.232 + 28.475$

Calculate embedment depth:

$M_{RS} = M_{DR}$

$$0.105D_o^3 + 2.140D_o^2 - 40.176D_o - 198.591 = 0$$

$$D_o^3 + 20.418D_o^2 - 383.271D_o - 1894.53 = 0$$

D_o = 14.630 ft

Calculate Tieback Force

$$M_{RS} = M_{DR}$$

$$\sum F_x = 0$$

$$\begin{Bmatrix} T_H + 1.294 + 1.104(14.630) \\ -3.313 + 0.191(14.630)^2 \\ -1.149(14.630) + 1.723 \end{Bmatrix} = \begin{Bmatrix} 7.654 + 22.961 + 15.308 + 7.065 \\ +0.353 + 2.170(14.630) - 6.509 \\ +0.0342(14.630)^2 - 0.205(14.630) + 0.308 \end{Bmatrix}$$

$T_H = 43.169$ klf in the horizontal direction

Multiply by 8 ft for spacing

$$T = \frac{43.169(8)}{\cos(15^\circ)} = 357.536 \text{ kips (along the 15 degree angle)}$$

The Shear and Moment Diagrams are displayed below:

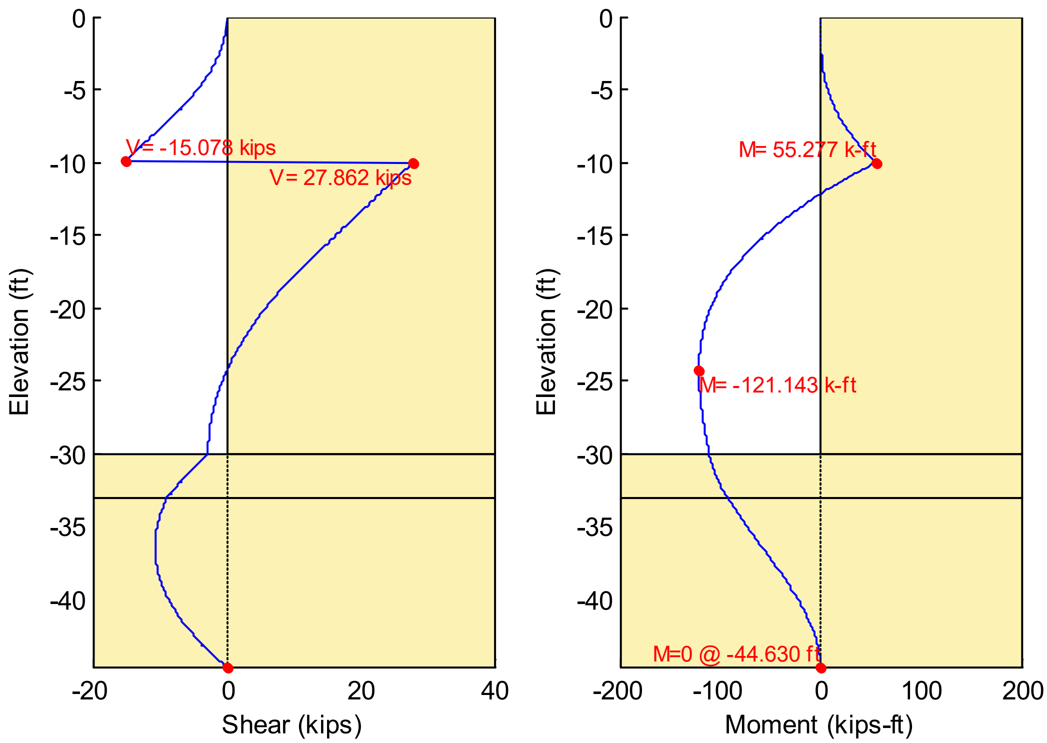


Figure 4-37: Shear and Moment Diagrams for Extreme Event Loading Case

When k_h=0.35, the Extreme Event Limit State controls the design. However, if k_h<0.22, the Strength I Limit State may control, as summarized in the table below.

Limit State	D_0 (ft)	$V_{\max}$ (kips)	$M_{\max}$ (kips-ft)
Service I	7.512	14.421	48.239
Strength I	9.367	20.491	73.596
Extreme Event I (k_h=0.22)	10.46	19.704	73.740
Extreme Event I (k_h=0.35)	14.630	27.862	121.143

4.6.3 Example 4-3: Multiple Tieback Solider Pile Wall

Design a multiple tieback solider pile wall with a single soil layers shown below with tieback spacing = 8 feet.

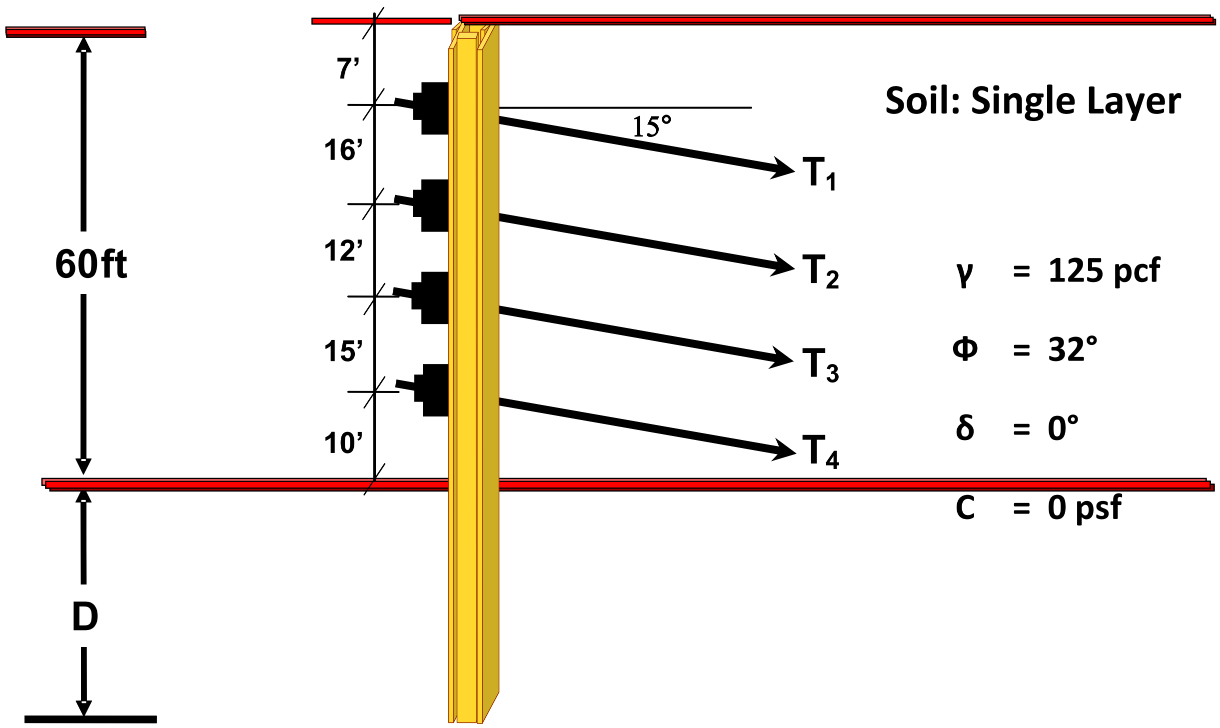


Figure 4-38: Multiple Tieback Soldier Pile Wall

Determine:

1. Active & Passive Earth Pressures
2. Pile Embedment *D*
3. Maximum Shear
4. Maximum Moment

PART A: SERVICE I CALCULATIONS

- Calculate active and passive earth pressure coefficients: Use Trial Wedge method to determine the active and passive earth pressure coefficients. The coefficients listed below are the horizontal components only.

  $K_{a1} = 0.307$

  $K_{a2} = 0.259$ $\qquad\qquad K_p = 6.471$ (horizontal component only)

  Note: Trial Wedge method tends to overestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to achieve a more conservative design.

- Calculate earth pressure distribution

Lateral load distribution at excavation level (LRFD Factor – 1.0):

$$\sigma^{+} = (1.0)(\gamma)(h=60)(K_{a1}) = (1.0)(0.125)(60)(0.307) = 2.303\,\text{ksf}$$

$$\sigma^{-} = (1.0)(\gamma)(h=60)(K_{a2}) = (1.0)(0.125)(60)(0.259) = 1.943\,\text{ksf}$$

Active Lateral load distribution at a depth D (LRFD Factor – 1.0):

$$\begin{aligned}\sigma_D &= \sigma^{-} + (1.0)(\gamma)(D)(K_{a2}) = 1.943 + (1.0)(0.125)(D)(0.259)\\ &= (1.943 + 0.0324D)\,\text{ksf}\end{aligned}$$

Passive Lateral load distribution for the second layer in the front at depth *D* (LRFD Factor – 1.0):

$$\begin{aligned}\boldsymbol{\sigma_P} &= \mathbf{(1.0)(\gamma)(D)(K_p)} = \mathbf{(1.0)(0.125)(D)(6.471)}\\ &= \mathbf{0.809D\,ksf}\end{aligned}$$

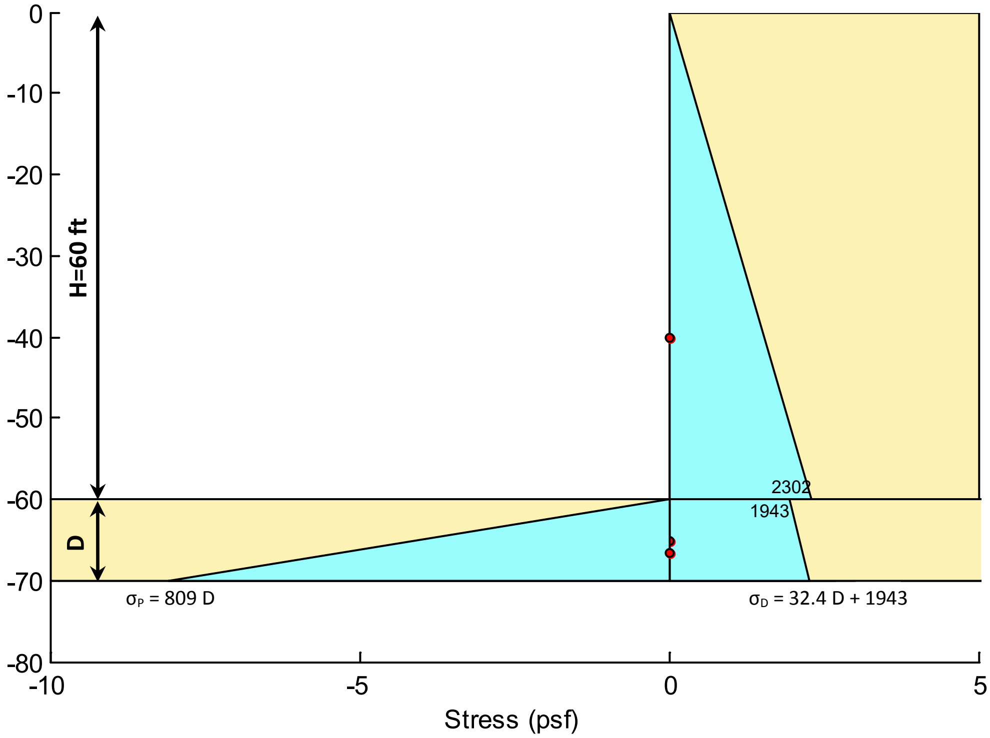


Figure 4-39: Triangular Earth Pressure Diagram for Service I Loading Condition

The lateral earth pressure distribution for the design of braced or anchored walls constructed in cohesionless soils may be determined using Figure 95. The maximum ordinate ($\sigma_{\text{Trapezoid}}$) of the pressure diagram is determined as follows:

$$\sigma_{Trapezoid} = \frac{P_T}{\left[H - \frac{1}{3}\left(H_1 + H_5\right)\right]}$$

Where the total active earth pressure is calculated as follow:

$$P = \frac{1}{2}\gamma H^2 K_{a1} = \left(\frac{1}{2}\right)(125)\left(60^2\right)(0.307) = 69{,}075 \text{ lb/ft}$$

$$P_T = 1.3P = 1.3(69{,}075) = 89{,}797.5 \text{ lb/ft}$$

Develop Trapezoidal loading:

$$\sigma_{Trapezoid} = \frac{89.798}{\left[60 - \frac{1}{3}(7 + 10)\right]}(1.0) = 1.653 \text{ ksf}$$

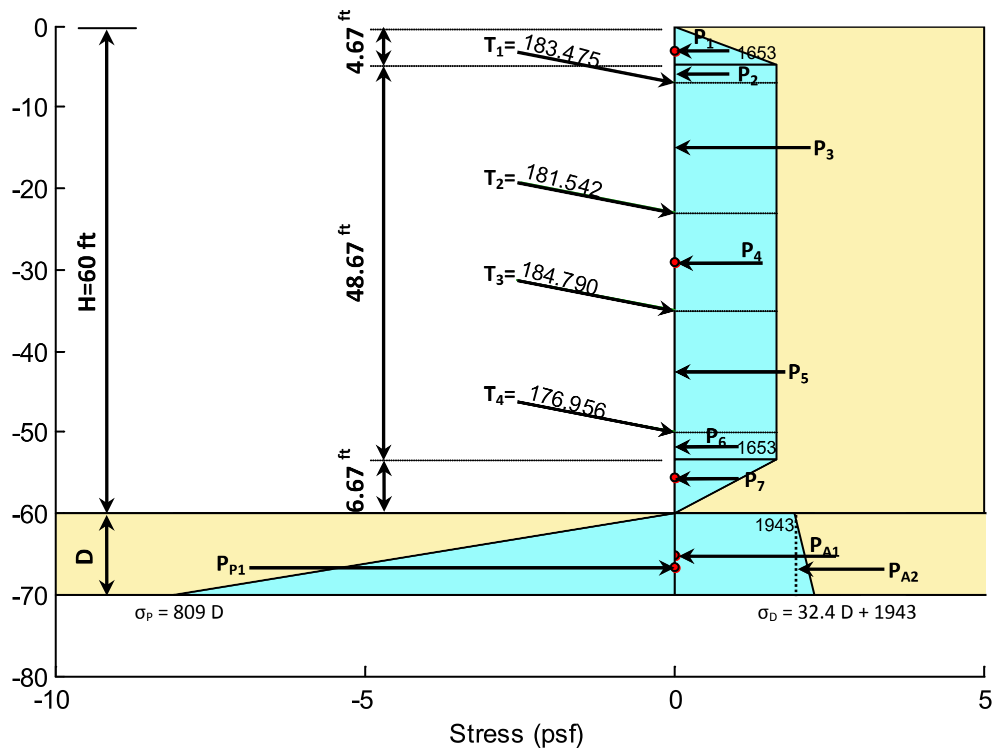


Figure 4-40: Developed Trapezoidal Distribution for Service I Loading Condition

Calculate Tieback Loads:

$$P_1 = \left(\frac{1}{2}\right)(4.667)(1.653)(8) = 30.850\ kips$$

$$P_2 = (2.333)(1.653)(8) = 30.850\ \text{kips}$$

$$P_3 = (16)(1.653)(8) = 211.546\ \text{kips}$$

$$P_4 = (12)(1.653)(8) = 158.659\ \text{kips}$$

$$P_5 = (15)(1.653)(8) = 198.324\ \text{kips}$$

$$P_6 = (3.333)(1.653)(8) = 44.072\ \text{kips}$$

$$P_7 = \left(\frac{1}{2}\right)(6.667)(1.653)(8) = 44.072\ \text{kips}$$

$$M_1 = 30.850\left[2.333 + 4.667\left(\frac{1}{3}\right) + 2.333\left(\frac{1}{2}\right)\right] = 155.966\ \text{kip-ft}$$

$$T_{1U} = P_1 + P_2 = 30.85 + 30.85 = 61.70 \text{ kips}$$

$$T_{1L} = \left(\frac{P_3}{2}\right) + \left(\frac{M_1}{16}\right) = \left(\frac{211.546}{2}\right) + \left(\frac{155.966}{16}\right) = 115.521 \text{ kips}$$

$$T_1 = \frac{T_{1U} + T_{1L}}{\cos(15°)} = \frac{(61.70 + 115.521)}{\cos(15°)} = 183.475 \text{ kips}$$

$$T_{2U} = \left(\frac{P_3}{2}\right) - \left(\frac{M_1}{16}\right) = \left(\frac{211.546}{2}\right) - \left(\frac{155.966}{16}\right) = 96.025 \text{ kips}$$

$$T_{2L} = \left(\frac{P_4}{2}\right) = \left(\frac{158.659}{2}\right) = 79.330 \text{ kips}$$

$$T_2 = \frac{T_{2U} + T_{2L}}{\cos(15°)} = \frac{(96.025 + 79.330)}{\cos(15°)} = 181.542 \; kips$$

$$T_{3U} = \left(\frac{P_4}{2}\right) = \left(\frac{158.659}{2}\right) = 79.330 \; kips$$

$$T_{3L} = \left(\frac{P_5}{2}\right) = \left(\frac{198.324}{2}\right) = 99.162 \; kips$$

$$T_3 = \frac{T_{3U} + T_{3L}}{\cos(15°)} = \frac{(79.330 + 99.162)}{\cos(15°)} = 184.790 \text{ kips}$$

Determine D to calculate T_4 by Taking a Moment about T_4

$$M_D = 44.072\left(\frac{3.333}{2} + \frac{6.667}{3} + 3.333\right) + 1.943D\left[\left(\frac{D}{2}\right) + 10\right](8)$$

$$+\left(\frac{0.0324D^2}{2}\right)\left(\frac{2}{3}D + 10\right)(8)$$

$$= 0.0863D^3 + 9.065D^2 + 155.4D + 318.301$$

$$M_R = \left(\frac{0.809D^2}{2}\right)\left(\frac{2}{3}D + 10\right)(8) \; = 2.157D^3 + 32.355D^2$$

$$M_R = M_D$$

$$2.071D^3 + 23.290D^2 - 155.400D - 318.301 = 0$$

$$D^3 + 11.248D^2 - 75.048D - 153.719 = 0$$

$$D \approx 5.898 \text{ ft}$$

$$T_{4U} = \left(\frac{P_5}{2}\right) = \left(\frac{198.324}{2}\right) = 99.162 \text{ kips}$$

$$T_{4L} = P_6 + P_7 + P_{a1} + P_{a2} - P_{p1}$$

$$= 44.072 + 44.072 + \left(1.943(5.898) + \frac{0.0324(5.898)^2}{2} - \frac{0.809(5.898)^2}{2}\right)(8) = 71.762 kips$$

$$T_4 = \frac{(99.162 + 71.762)}{\cos(15)} = 176.958 \text{ kips}$$

The Shear and moment Diagrams are shown below:

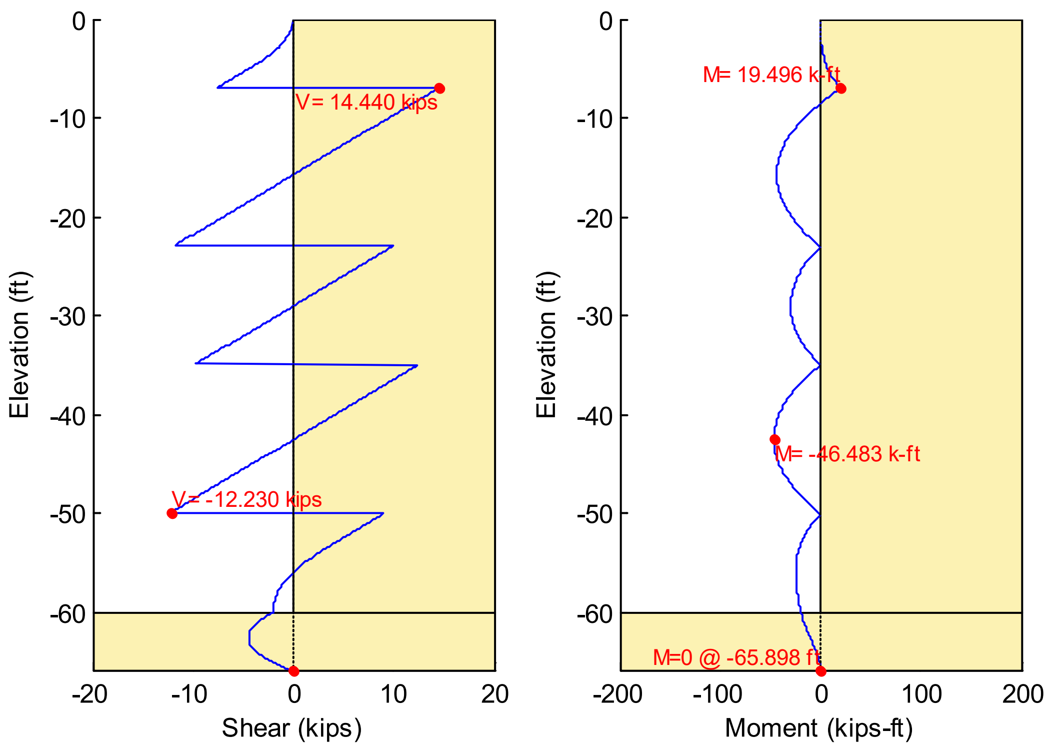


Figure 4-41: Shear and Moment Diagrams for Service I Loading Condition

PART B: STRENGTH I CALCULATIONS

- *Calculate active and passive earth pressure coefficients*:

Use Trial Wedge method to determine the active and passive earth pressure coefficients. The coefficients listed below are the horizontal components only.

$K_{a1} = 0.307$

$K_{a2} = 0.259 \qquad K_p = 6.471$ (horizontal component only)

Note: Trial Wedge method tends to overestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to achieve a more conservative design.

- *Calculate earth pressure distribution*

Lateral load distribution at excavation level (LRFD Factor – 1.35):

$$\sigma^+ = (1.35)(\gamma)(h = 60)(K_{a1}) = (1.35)(0.125)(60)(0.307) = 3.108\ \text{ksf}$$

$$\sigma^- = (1.35)(\gamma)(h = 60)(K_{a2}) = (1.35)(0.125)(60)(0.259) = 2.622\ \text{ksf}$$

Active Lateral load distribution at a depth D (LRFD Factor – 1.35):

$$\sigma_D = \sigma^- + (1.35)(\gamma)(D)(K_{a2}) = 2.622 + (1.35)(0.125)(D)(0.259)$$
$$= (2.622 + 0.0437D)\ \text{ksf}$$

Passive Lateral load distribution for the second layer in the front at depth *D* (LRFD Factor – 1.0):

$$\boldsymbol{\sigma_P = (1.0)(\gamma)(D)(K_p) = (1.0)(0.125)(D)(6.471) = 0.809D\ \text{ksf}}$$

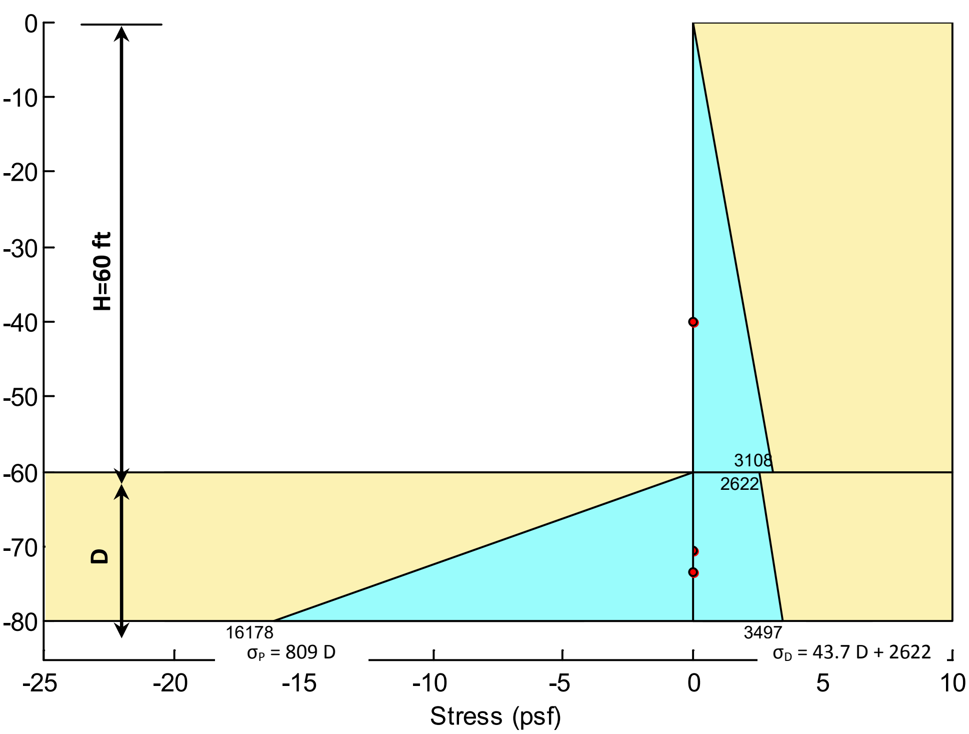


Figure 4-42: Triangular Earth Pressure Diagram for Strength I Loading Condition

The lateral earth pressure distribution for the design of braced or anchored walls constructed in cohesionless soils may be determined using Figure 95. The maximum ordinate ($\sigma_{\mathrm{Trapezoid}}$) of the pressure diagram is determined as follows:

$$\sigma_{Trapezoid} = \frac{P_T}{\left[H - \frac{1}{3}\left(H_1 + H_5\right)\right]}$$

Where the total active earth pressure is calculated as follow:

$$P = \frac{1}{2}\gamma H^2 K_{a1} = \left(\frac{1}{2}\right)(125)\left(60^2\right)(0.307) = 69{,}075 \text{ lb/ft}$$

$$P_T = 1.3P = 1.3(69{,}075) = 89{,}797.5 \text{ lb/ft}$$

Develop Trapezoidal loading:

$$\sigma_{Trapezoid} = \frac{89.798}{\left[60 - \frac{1}{3}(7+10)\right]}(1.35) = 2.231\,\text{ksf}$$

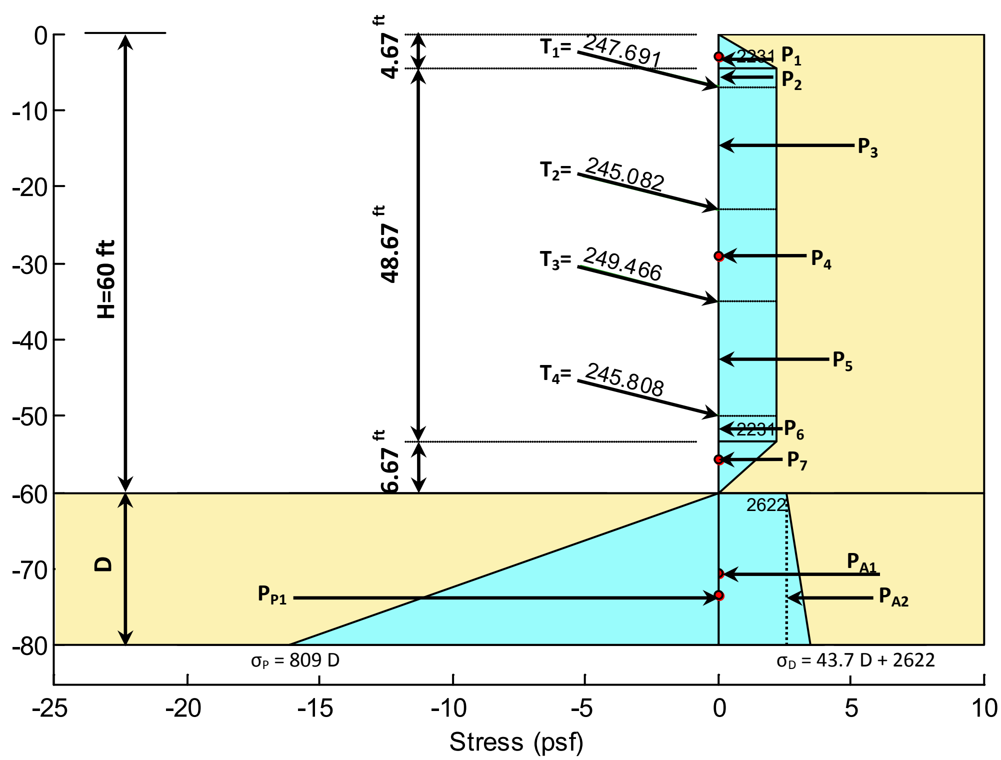


Figure 4-43: Developed Trapezoidal Distribution for Strength I Loading Condition

Calculate Tieback Loads:

$$P_1 = \left(\tfrac{1}{2}\right)(4.667)(2.231)(8) = 41.648\ \textit{kips}$$
$$P_2 = (2.333)(2.231)(8) = 41.648\ \text{kips}$$
$$P_3 = (16)(2.231)(8) = 285.594\ \text{kips}$$
$$P_4 = (12)(2.231)(8) = 214.195\ \text{kips}$$
$$P_5 = (15)(2.231)(8) = 267.744\ \text{kips}$$
$$P_6 = (3.333)(2.231)(8) = 59.496\ \text{kips}$$
$$P_7 = \left(\tfrac{1}{2}\right)(6.667)(2.231)(8) = 59.496\ \text{kips}$$

$$M_1 = 41.648\left[2.333 + 4.667\left(\tfrac{1}{3}\right) + 2.333\left(\tfrac{1}{2}\right)\right] = 210.552\ \text{kip-ft}$$

$$T_{1U} = P_1 + P_2 = 41.648 + 41.648 = 83.296\ \text{kips}$$
$$T_{1L} = \left(\frac{P_3}{2}\right) + \left(\frac{M_1}{16}\right) = \left(\frac{285.594}{2}\right) + \left(\frac{210.552}{16}\right) = 155.957\ \text{kips}$$
$$T_1 = \frac{T_{1U} + T_{1L}}{\cos(15°)} = \frac{(83.296 + 155.957)}{\cos(15°)} = 247.691\ \text{kips}$$
$$T_{2U} = \left(\frac{P_3}{2}\right) - \left(\frac{M_1}{16}\right) = \left(\frac{285.594}{2}\right) - \left(\frac{210.552}{16}\right) = 129.638\ \text{kips}$$
$$T_{2L} = \left(\frac{P_4}{2}\right) = \left(\frac{214.195}{2}\right) = 107.098\ \text{kips}$$
$$T_2 = \frac{T_{2U} + T_{2L}}{\cos(15°)} = \frac{(129.638 + 107.098)}{\cos(15°)} = 245.082\ \textit{kips}$$
$$T_{3U} = \left(\frac{P_4}{2}\right) = \left(\frac{214.195}{2}\right) = 107.098\ \textit{kips}$$
$$T_{3L} = \left(\frac{P_5}{2}\right) = \left(\frac{267.744}{2}\right) = 133.872\ \textit{kips}$$
$$T_3 = \frac{T_{3U} + T_{3L}}{\cos(15°)} = \frac{(107.098 + 133.872)}{\cos(15°)} = 249.466\ \text{kips}$$

Determine D to calculate T_4 by Taking a Moment about T_4

$$\boldsymbol{M_D} = 59.496\left(\frac{3.333}{2} + \frac{6.667}{3} + 3.333\right) + 2.622D\left[\left(\frac{D}{2}\right) + 10\right](8) + \left(\frac{0.0437D^2}{2}\right)\left(\frac{2}{3}D + 10\right)(8)$$
$$= 0.117D^3 + 12.238D^2 + 209.790D + 429.706$$
$$\boldsymbol{M_R} = \left(\frac{0.809D^2}{2}\right)\left(\frac{2}{3}D + 10\right)(8) = 2.157D^3 + 32.355D^2$$

$$M_R = M_D$$
$$2.040D^3 + 20.117D^2 - 209.790D - 429.706 = 0$$
$$D^3 + 9.859D^2 - 102.816D - 210.594 = 0$$
$$D \approx 7.524 \text{ ft}$$

$$T_{4U} = \left(\frac{P_5}{2}\right) = \left(\frac{267.744}{2}\right) = 133.872 \text{ kips}$$

$$T_{4L} = P_6 + P_7 + P_{a1} + P_{a2} - P_{p1}$$
$$= 59.493 + 59.502 + \left(2.622(7.524) + \frac{0.0437(7.524)^2}{2} - \frac{0.809(7.524)^2}{2}\right)(8) = 103.522\ kips$$

$$T_4 = \frac{(133.872 + 103.522)}{\cos(15)} = 245.808 \text{ kips}$$

The shear and moment Diagrams are shown below:

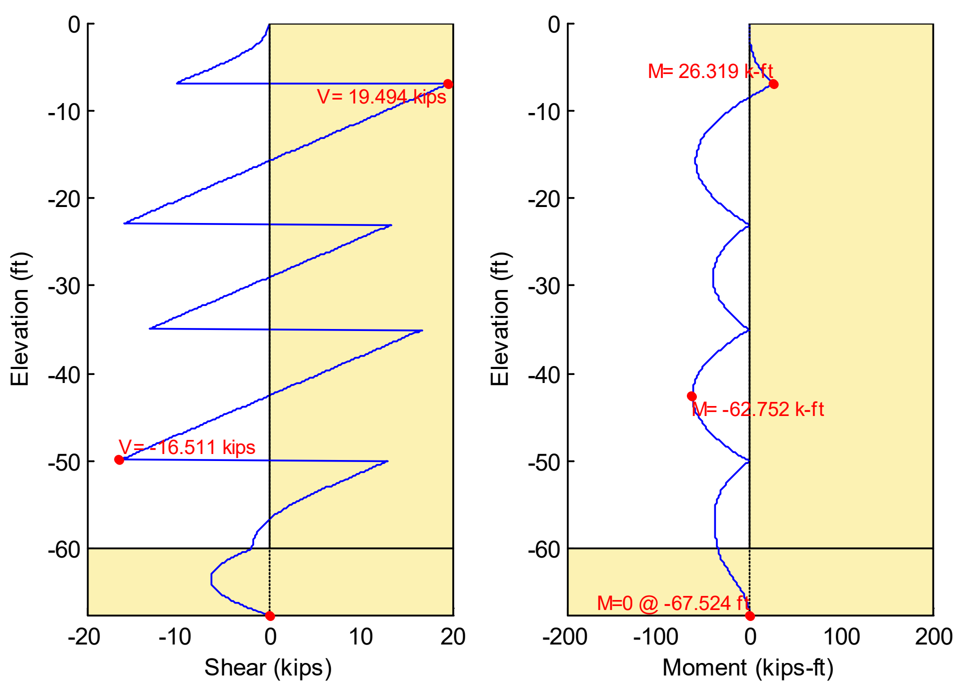


Figure 4-44: Shear and Moment Diagrams for Strength I Loading Condition

PART C: EXTREME EVENT CALCULATIONS (k_h = 0.35)

- Calculate active and passive earth pressure coefficients: Use Trial Wedge method to determine the active and passive earth pressure coefficients. The coefficients listed below are the horizontal components only.

$$K_{ae1} = 0.585$$

$$K_{ae2} = 0.564 \qquad\qquad K_{pe} = 4.413$$

Note: Trial Wedge method tends to overestimates the passive earth pressure. It is recommended to use the Log-Spiral-Rankine Model to achieve a more conservative design.

- Calculate earth pressure distribution

Lateral load distribution at excavation level:

$$\sigma^{+} = (\gamma)(h=60)(K_{ae1}) = (0.125)(60)(0.585) = 4.388 \text{ ksf}$$

$$\sigma^{-} = (\gamma)(h=60)(K_{ae2}) = (0.125)(60)(0.564) = 4.230 \text{ ksf}$$

Active Lateral load distribution at a depth *D*:

$$\sigma_D = \sigma^{-} + (\gamma)(D)(K_{ae2}) = 4.230 + (0.125)(D)(0.564) = (4.230 + 0.0705D) \text{ ksf}$$

Passive Lateral load distribution for the second layer in the front at depth *D*:

$$\sigma_P = (\gamma)(D)(K_{pe}) = (0.125)(D)(4.413) = 0.552D \text{ ksf}$$

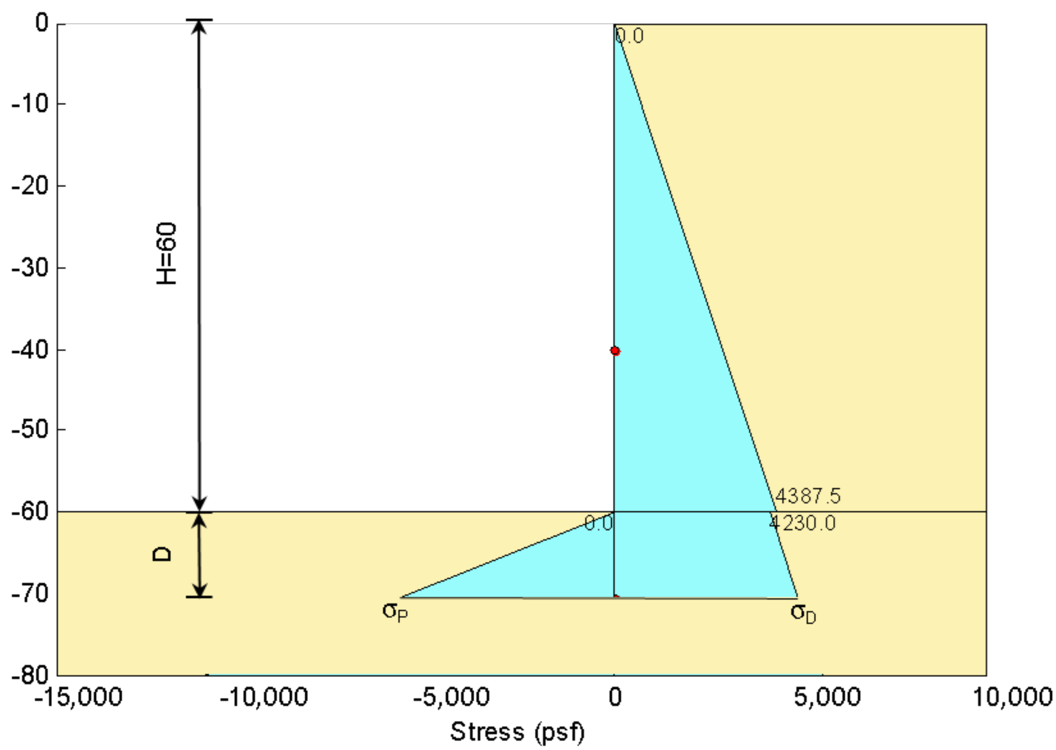


Figure 4-45: Triangular Earth Pressure Diagram for Extreme Event Loading Condition

The lateral earth pressure distribution for the design of braced or anchored walls constructed in cohesionless soils may be determined using Figure 4-15. The maximum ordinate ($\sigma_{Trapezoid}$) of the pressure diagram is determined as follows:

$$\sigma_{Trapezoid} = \frac{P_T}{\left[H - \frac{1}{3}(H_1 + H_5)\right]}$$

Where the total active earth pressure is calculated as follow:

$$P = \frac{1}{2}\gamma H^2 K_{ae1} = \left(\frac{1}{2}\right)(0.125)(60^2)(0.585) = 131.625 \text{ klf}$$

$$P_T = 1.3P = 1.3(131.625) = 171.113 \text{ klf}$$

Develop Trapezoidal loading:

$$\sigma_{Trapezoid} = \frac{171.113}{\left[60 - \frac{1}{3}(7 + 10)\right]} = 3.149 \text{ ksf}$$

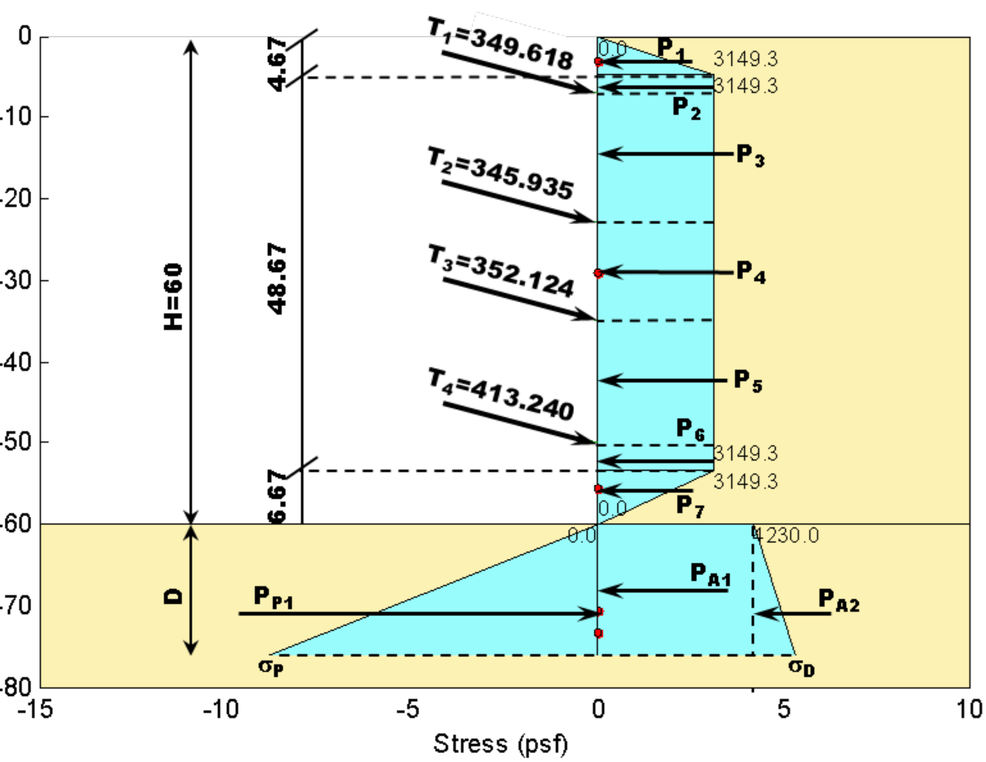


Figure 4-46: Developed Trapezoidal Distribution for Tieback System

Calculate Tieback Loads:

$$P_1 = \left(\tfrac{1}{2}\right)(4.667)(3.149)(8) = 58.787 \text{ kips}$$

$$P_2 = (2.333)(3.149)(8) = 58.787 \text{ kips}$$

$$P_3 = (16)(3.149)(8) = 403.112 \text{ kips}$$

$$P_4 = (12)(3.149)(8) = 302.334 \text{ kips}$$

$$P_5 = (15)(3.149)(8) = 377.917 \text{ kips}$$

$$P_6 = (3.333)(3.149)(8) = 83.982 \text{ kips}$$

$$P_7 = \left(\tfrac{1}{2}\right)(6.667)(3.149)(8) = 83.982 \text{ kips}$$

$$M_1 = 58.787\left[2.333 + 4.667\left(\tfrac{1}{3}\right) + 2.333\left(\tfrac{1}{2}\right)\right] = 297.202 \text{ kip-ft}$$

$$T_{1U} = P_1 + P_2 = 58.787 + 58.787 = 117.574 \text{ kips}$$

$$T_{1L} = \left(\frac{P_3}{2}\right) + \left(\frac{M_1}{16}\right) = \left(\frac{403.112}{2}\right) + \left(\frac{297.202}{16}\right) = 220.131 \text{ kips}$$

$$T_1 = \frac{T_{1U} + T_{1L}}{\cos(15)} = \frac{(117.574 + 220.131)}{\cos(15)} = 349.618 \text{ kips}$$

$$T_{2U} = \left(\frac{P_3}{2}\right) - \left(\frac{M_1}{16}\right) = \left(\frac{304.90}{2}\right) - \left(\frac{297.202}{16}\right) = 182.981 \text{ kips}$$

$$T_{2L} = \left(\frac{P_4}{2}\right) = \left(\frac{302.334}{2}\right) = 151.167\, kips$$

$$T_2 = \frac{T_{2U} + T_{2L}}{\cos(15)} = \frac{(182.981 + 151.167)}{\cos(15)} = 345.935 \text{ kips}$$

$$T_{3U} = \left(\frac{P_4}{2}\right) = \left(\frac{302.334}{2}\right) = 151.167\, kips$$

$$T_{3L} = \left(\frac{P_5}{2}\right) = \left(\frac{377.917}{2}\right) = 188.959\, kips$$

$$T_3 = \frac{T_{3U} + T_{3L}}{\cos(15)} = \frac{(151.167 + 188.959)}{\cos(15)} = 352.124 \text{ kips}$$

Determine D to calculate T_4 by Taking a Moment about T_4

$$M_D = 83.982\left(\frac{3.333}{2} + \frac{6.667}{3} + 3.333\right) + 4.230D\left[\left(\frac{D}{2}\right) + 10\right](8)$$
$$+\left(\frac{0.0705D^2}{2}\right)\left(\frac{2}{3}D + 10\right)(8) = 0.188D^3 + 19.74D^2 + 338.4D + 606.504$$

$$M_R = \left(\frac{0.552D^2}{2}\right)\left(\frac{2}{3}D + 10\right)(8) = 1.471D^3 + 22.065D^2$$

$$M_R = M_D$$
$$D^3 + 1.812D^2 - 263.757D - 472.746 = 0$$
$$D \approx 16.232 \text{ ft}$$

$$T_{4U} = \left(\frac{P_5}{2}\right) = \left(\frac{377.917}{2}\right) = 188.959 \text{ kips}$$

$$T_{4L} = P_6 + P_7 + P_{a1} + P_{a2} - P_{p1}$$
$$= 83.982 + 83.982 + \left(4.230(16.232) + \frac{0.0705(16.232)^2}{2} - \frac{0.552(16.232)^2}{2}\right)(8) = 210.20 \text{ kips}$$

$$T_4 = \frac{(188.959 + 210.200)}{\cos(15)} = 413.24 \text{ kips}$$

The shear and moment Diagrams are shown below:

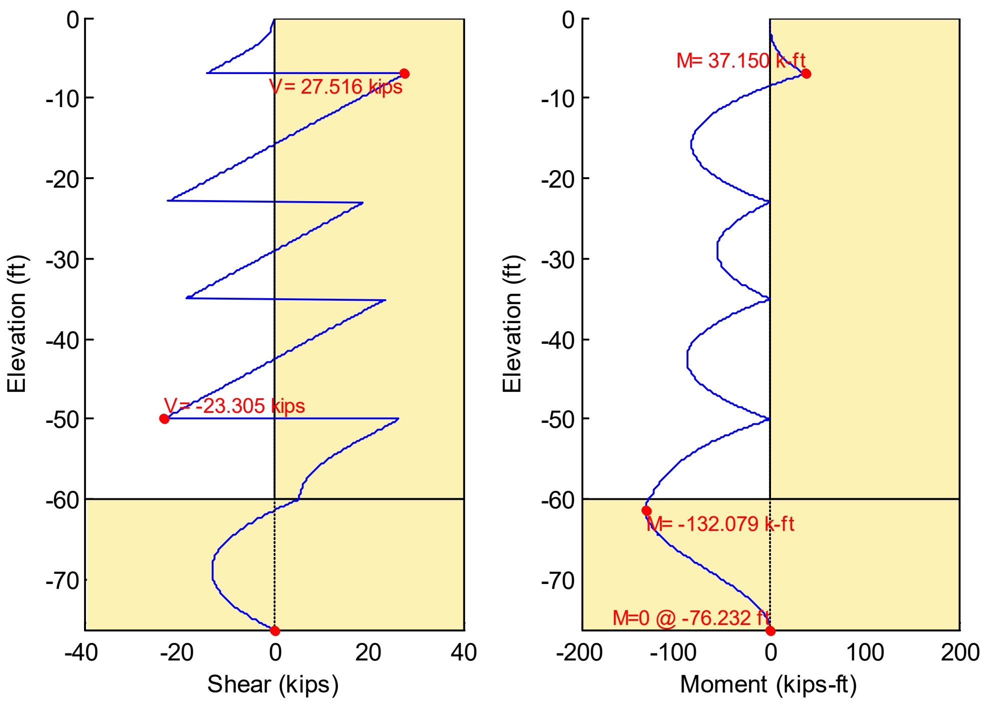
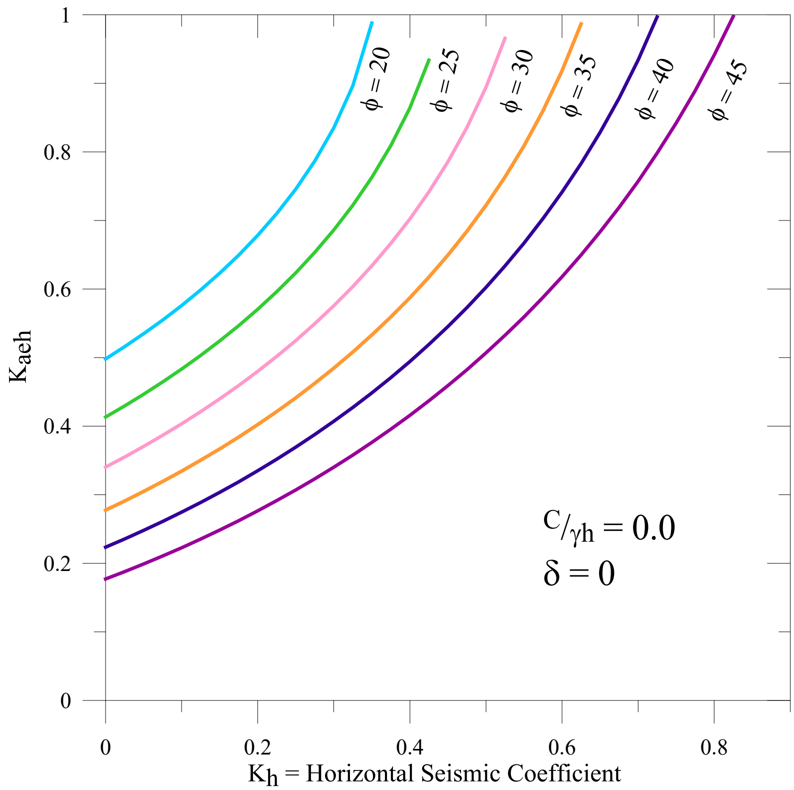


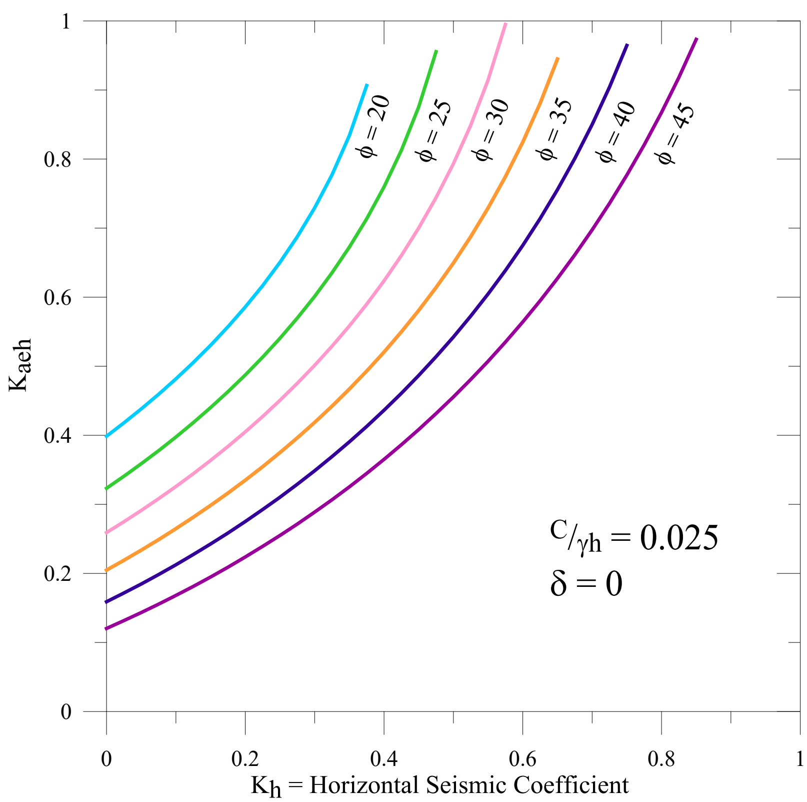
Figure 4-47: Shear and Moment Diagrams for Extreme Event Loading Condition

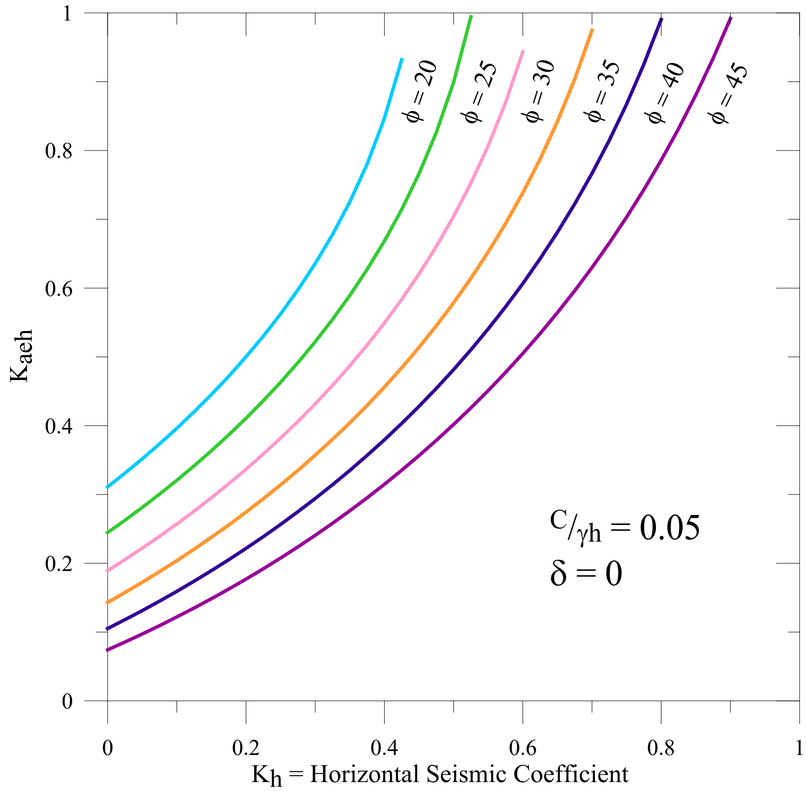
When $k_h > 0.17$, the Extreme Event Limit State controls the design, as summarized in the table below.

Limit State	D_0 (ft)	$V_{\max}$ (kips)	$M_{\max}$ (kips-ft)
Service I	5.898	14.440	46.483
Strength I	7.524	19.494	62.752
Extreme Event I (k_h=0.17)	9.05	19.624	62.910
Extreme Event I (k_h=0.35)	16.232	27.516	132.079

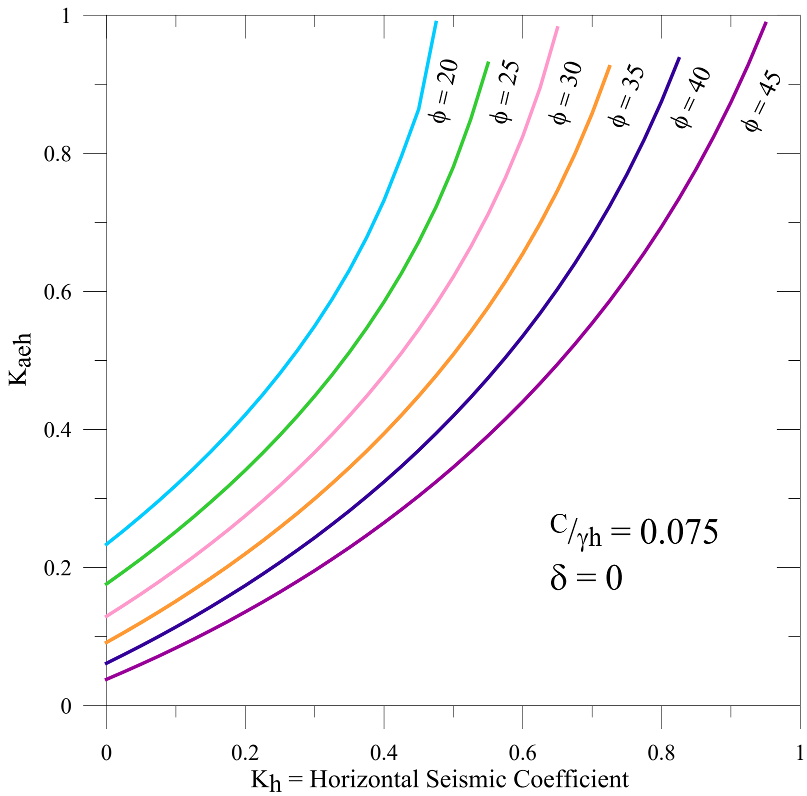
APPENDIX A. ACTIVE SEISMIC EARTH PRESSURE COEFFICIENTS



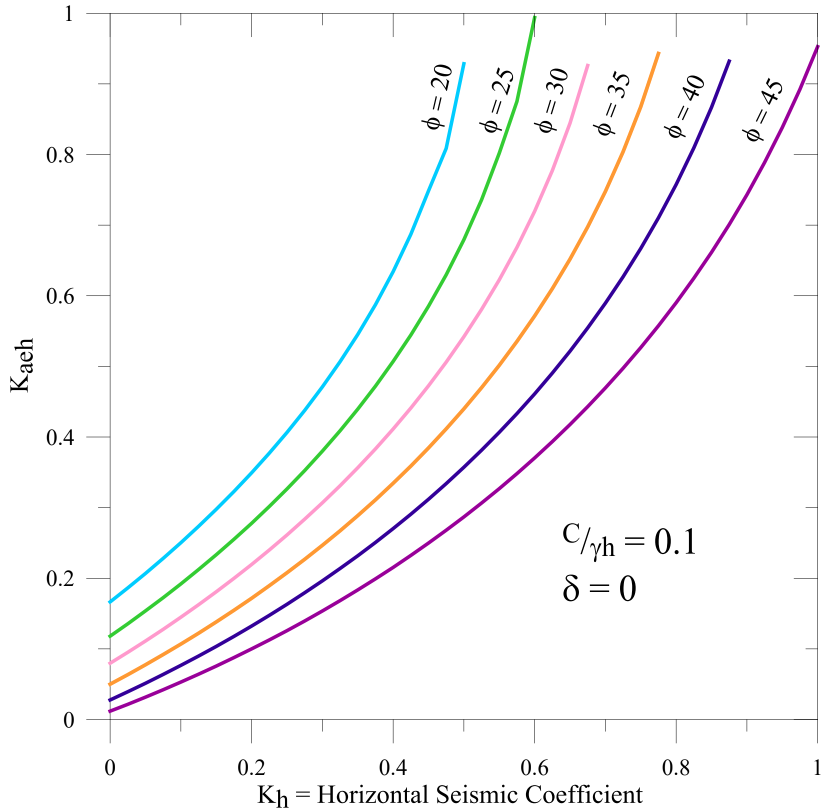




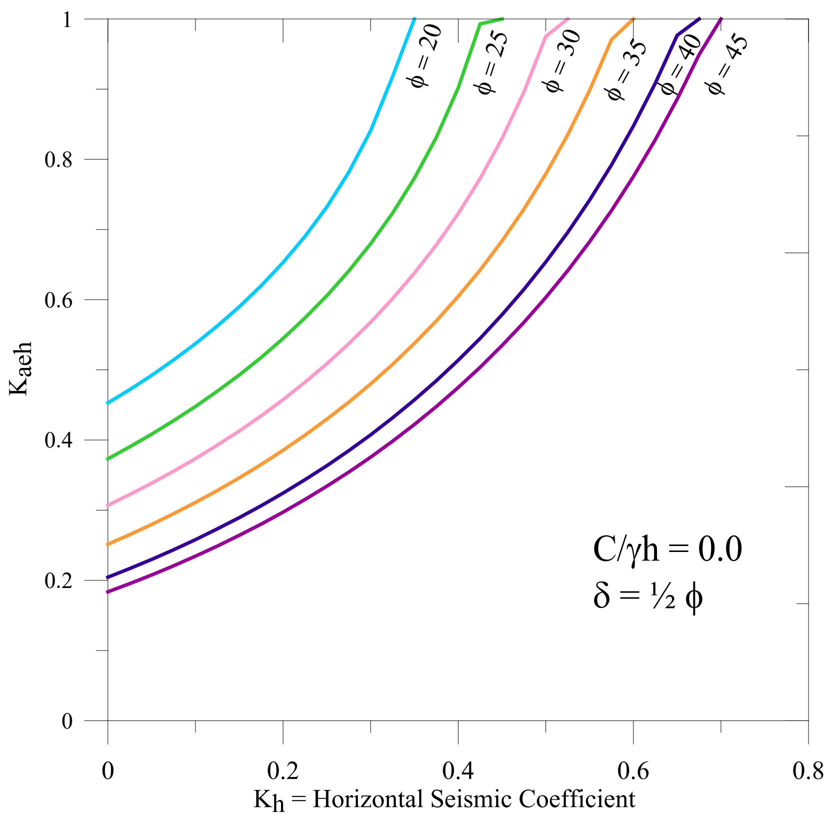
1
0.8
0.6
0.4
0.2
0
Kaeh
0
0.2
0.4
0.6
0.8
1
Kh = Horizontal Seismic Coefficient
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.05
δ = 0



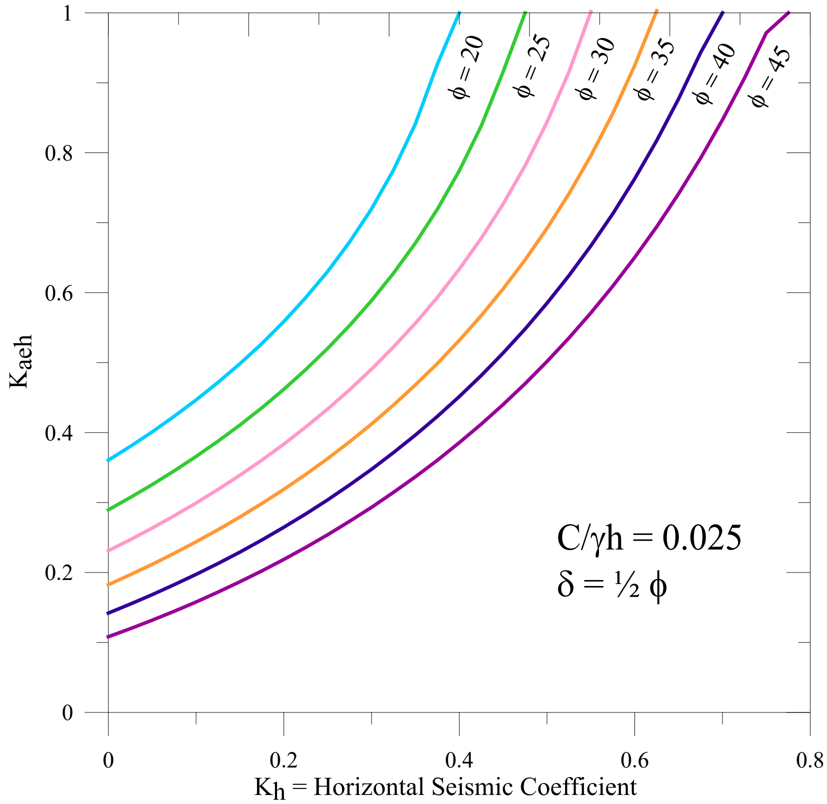
1
0.8
0.6
0.4
0.2
0
Kaeh
0
0.2
0.4
0.6
0.8
1
Kh = Horizontal Seismic Coefficient
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.075
δ = 0

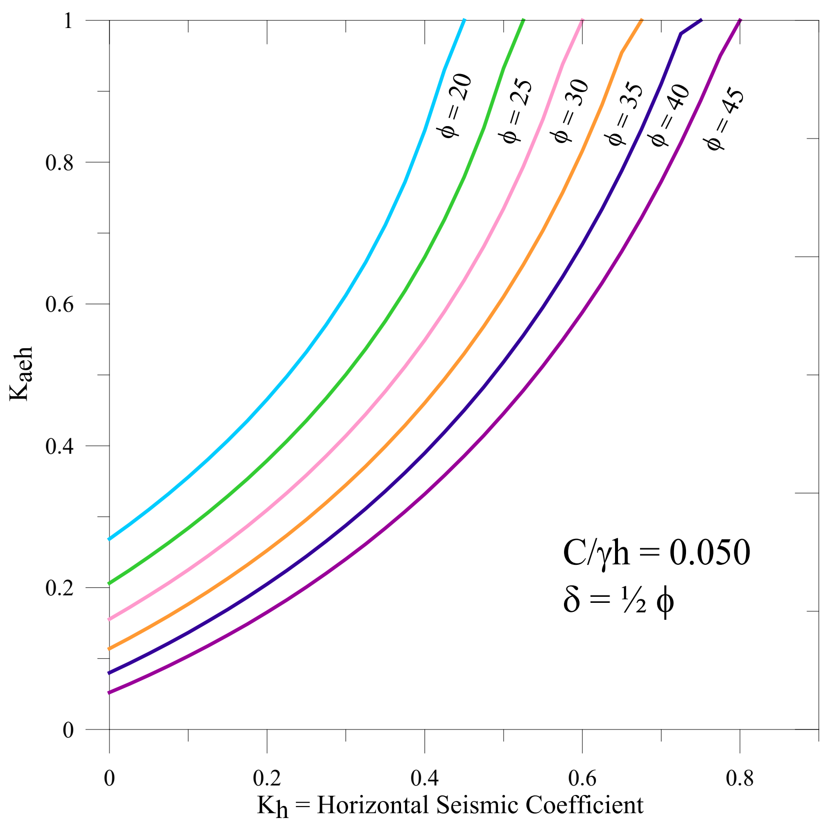


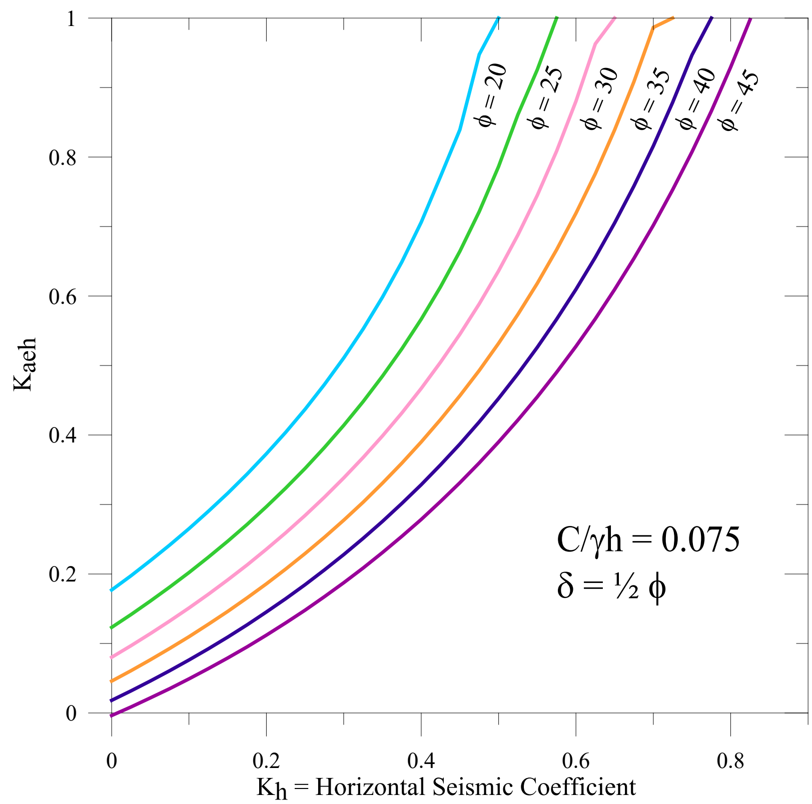
1
0.8
0.6
0.4
0.2
0
Kaeh
ϕ = 20
ϕ = 25
ϕ = 30
ϕ = 35
ϕ = 40
ϕ = 45
C/γh = 0.1
δ = 0
0
0.2
0.4
0.6
0.8
1
Kh = Horizontal Seismic Coefficient



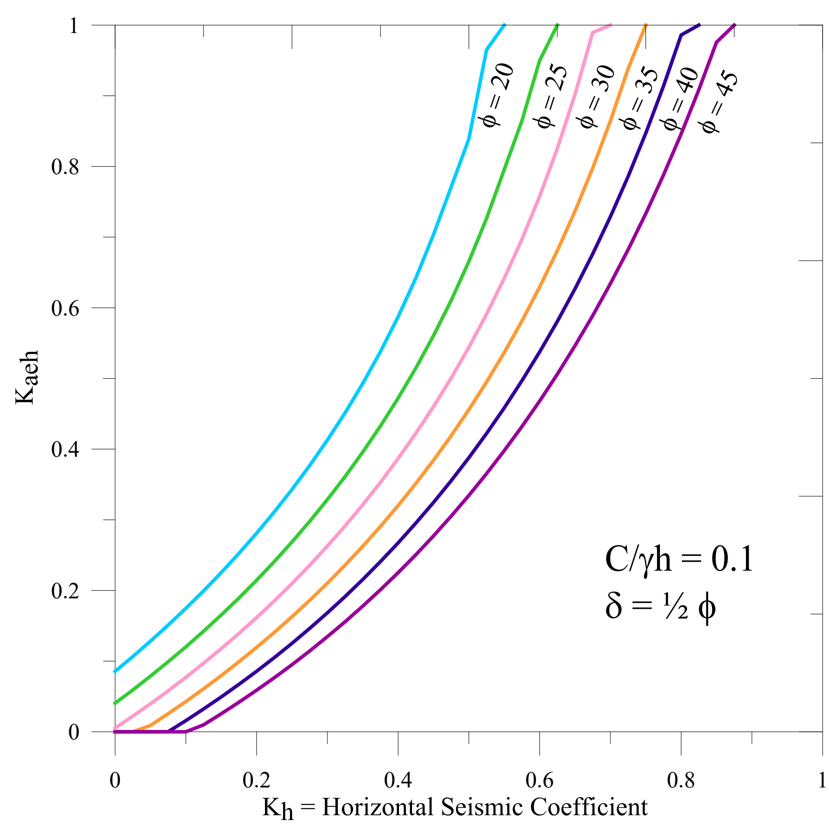
1
0.8
0.6
0.4
0.2
0
Kaeh
ϕ = 20
ϕ = 25
ϕ = 30
ϕ = 35
ϕ = 40
ϕ = 45
C/γh = 0.0
δ = ½ ϕ
0
0.2
0.4
0.6
0.8
Kh = Horizontal Seismic Coefficient


1
0.8
0.6
0.4
0.2
0
Kaeh
0
0.2
0.4
0.6
0.8
Kh = Horizontal Seismic Coefficient
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.025
δ = ½ φ

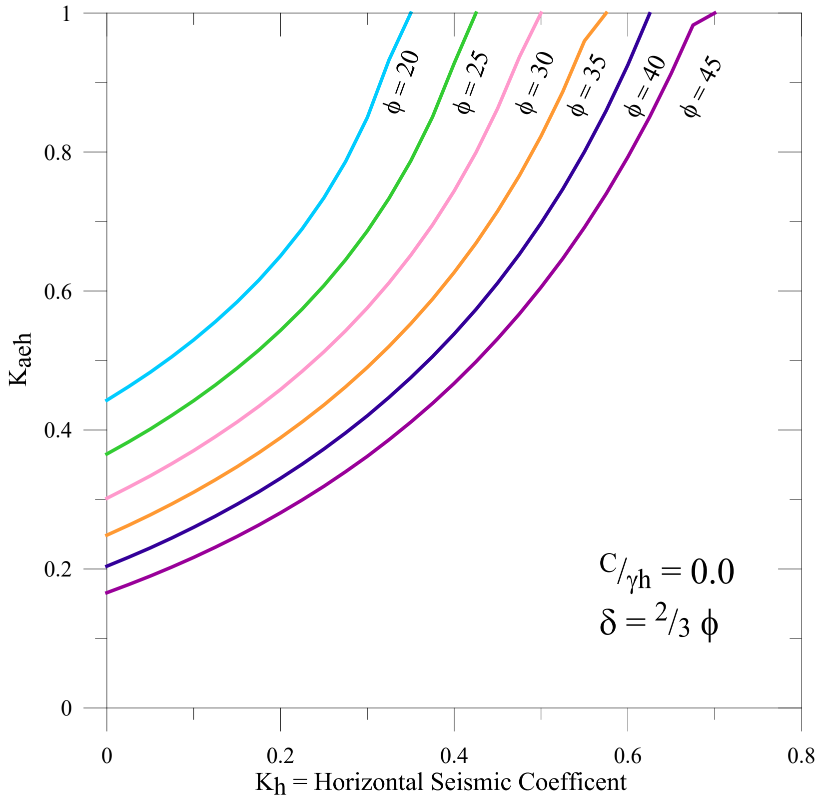

1
0.8
0.6
0.4
0.2
0
Kaeh
0
0.2
0.4
0.6
0.8
Kh = Horizontal Seismic Coefficient
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.050
δ = ½ φ



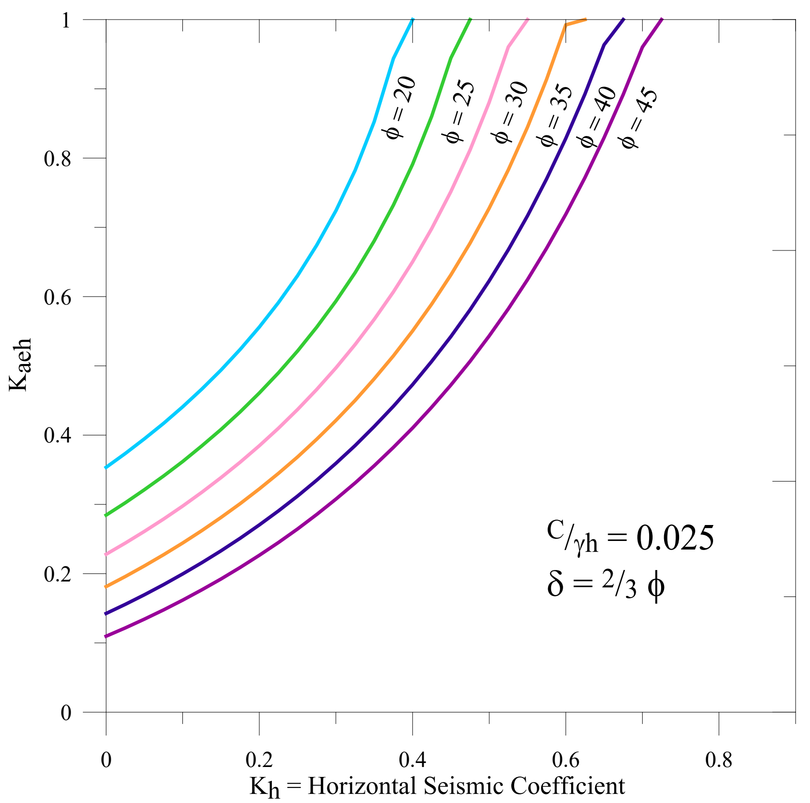
1
0.8
0.6
0.4
0.2
0
Kaeh
0
0.2
0.4
0.6
0.8
Kh = Horizontal Seismic Coefficient
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.075
δ = ½ φ



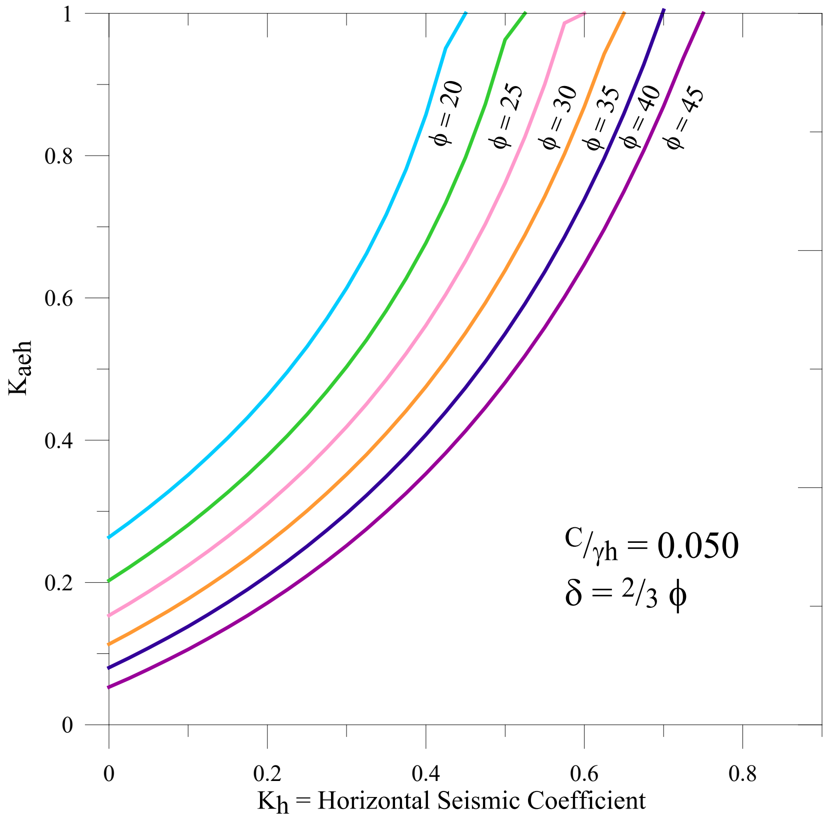
1
0.8
0.6
0.4
0.2
0
Kaeh
0
0.2
0.4
0.6
0.8
1
Kh = Horizontal Seismic Coefficient
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.1
δ = ½ φ

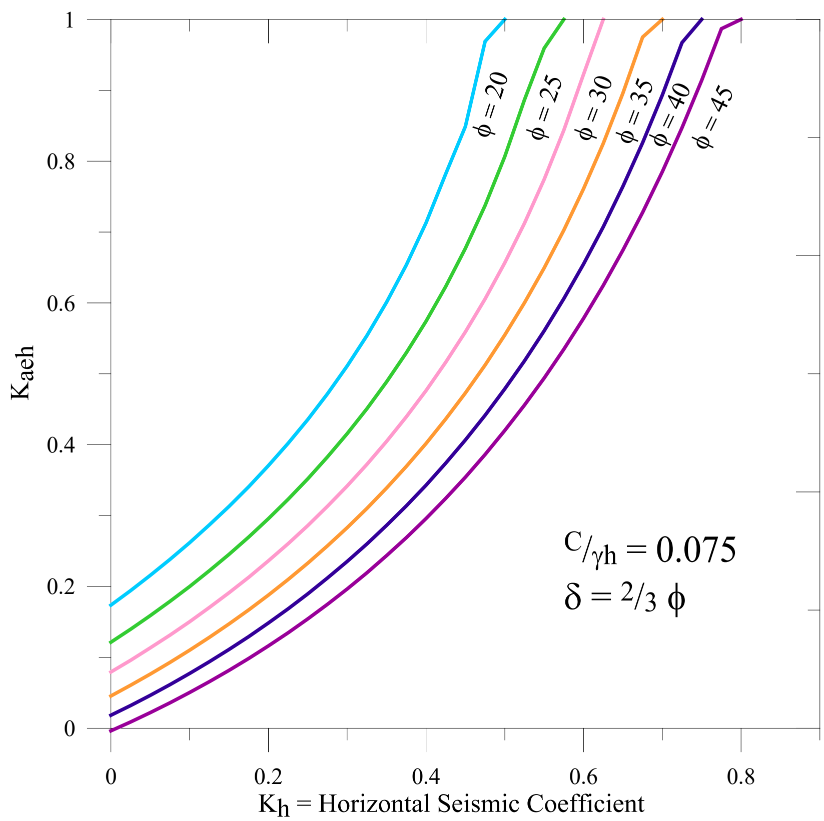


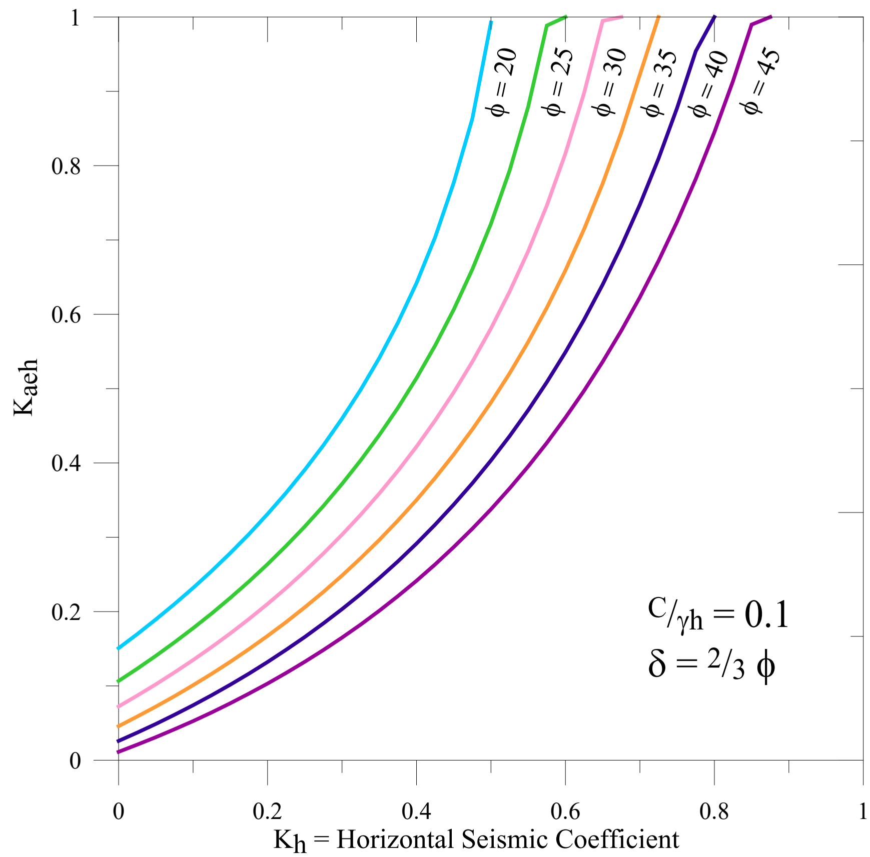
1
0.8
0.6
0.4
0.2
0
K_{aeh}
$\phi = 20$
$\phi = 25$
$\phi = 30$
$\phi = 35$
$\phi = 40$
$\phi = 45$
$^C/_{\gamma h} = 0.0$
$\delta = {}^2/_3\ \phi$
0
0.2
0.4
0.6
0.8
K_h = Horizontal Seismic Coefficent



1
0.8
0.6
0.4
0.2
0
K_{aeh}
$\phi = 20$
$\phi = 25$
$\phi = 30$
$\phi = 35$
$\phi = 40$
$\phi = 45$
$^C/_{\gamma h} = 0.025$
$\delta = {}^2/_3\ \phi$
0
0.2
0.4
0.6
0.8
K_h = Horizontal Seismic Coefficient

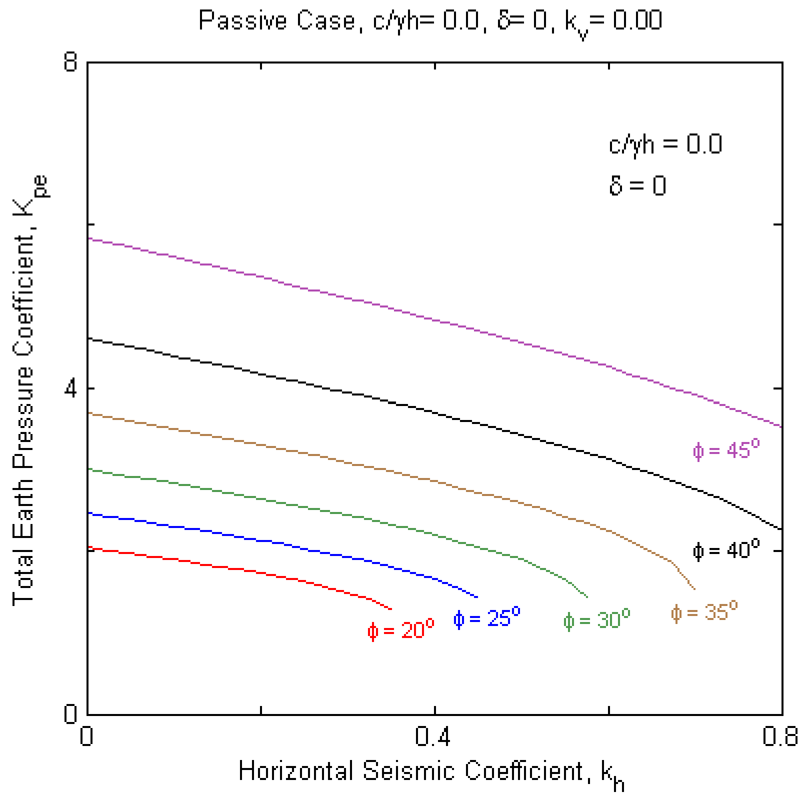

1
0.8
0.6
0.4
0.2
0
Kaeh
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.050
δ = 2/3 φ
0
0.2
0.4
0.6
0.8
Kh = Horizontal Seismic Coefficient

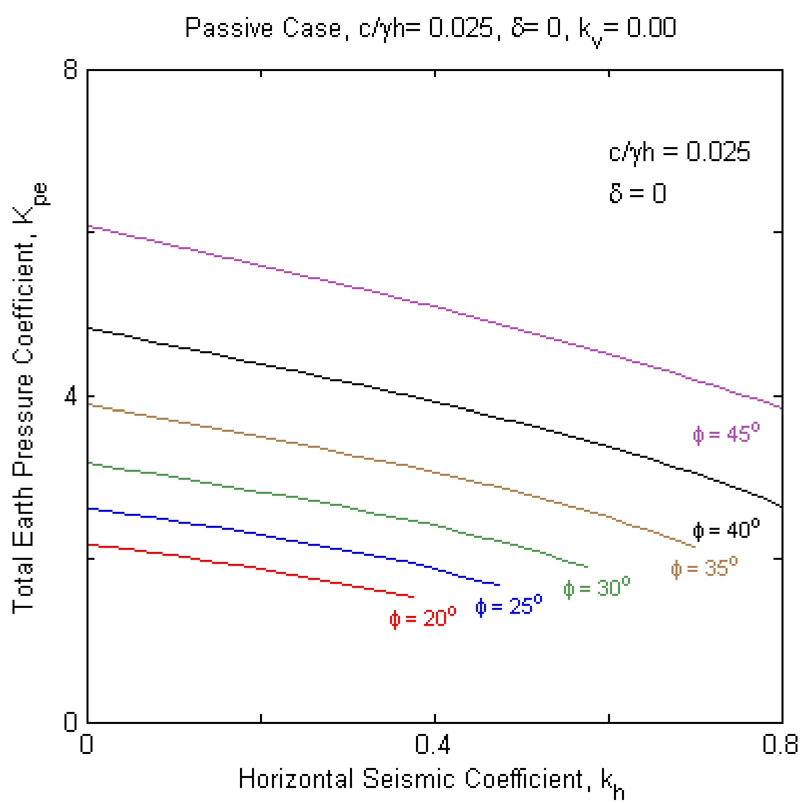

1
0.8
0.6
0.4
0.2
0
Kaeh
φ = 20
φ = 25
φ = 30
φ = 35
φ = 40
φ = 45
C/γh = 0.075
δ = 2/3 φ
0
0.2
0.4
0.6
0.8
Kh = Horizontal Seismic Coefficient



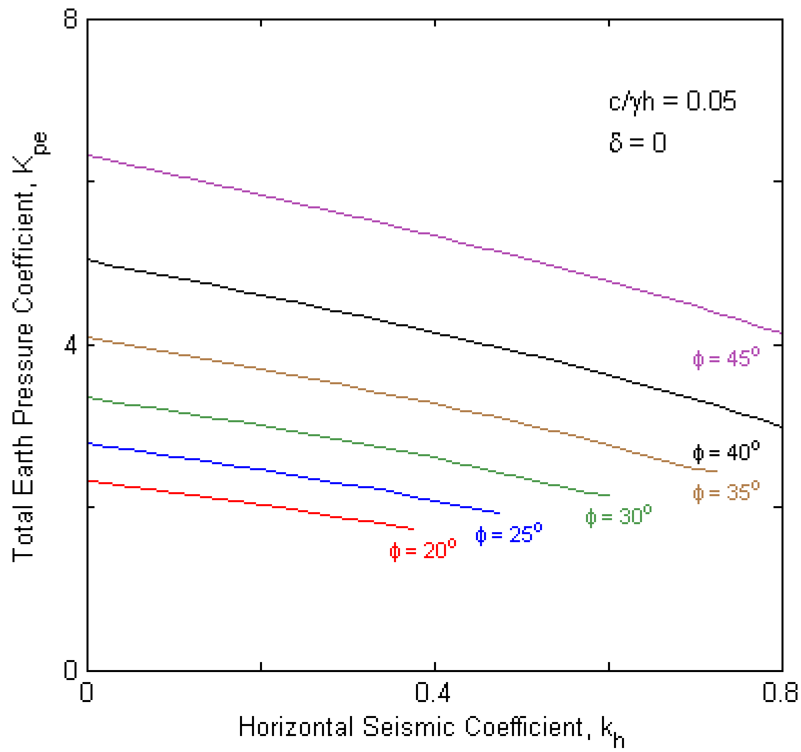
1
0.8
0.6
0.4
0.2
0
K_{aeh}
0
0.2
0.4
0.6
0.8
1
K_h = Horizontal Seismic Coefficient
$\phi = 20$
$\phi = 25$
$\phi = 30$
$\phi = 35$
$\phi = 40$
$\phi = 45$
$C/\gamma h = 0.1$
$\delta = 2/3\ \phi$

APPENDIX B. PASSIVE SEISMIC EARTH PRESSURE COEFFICIENTS

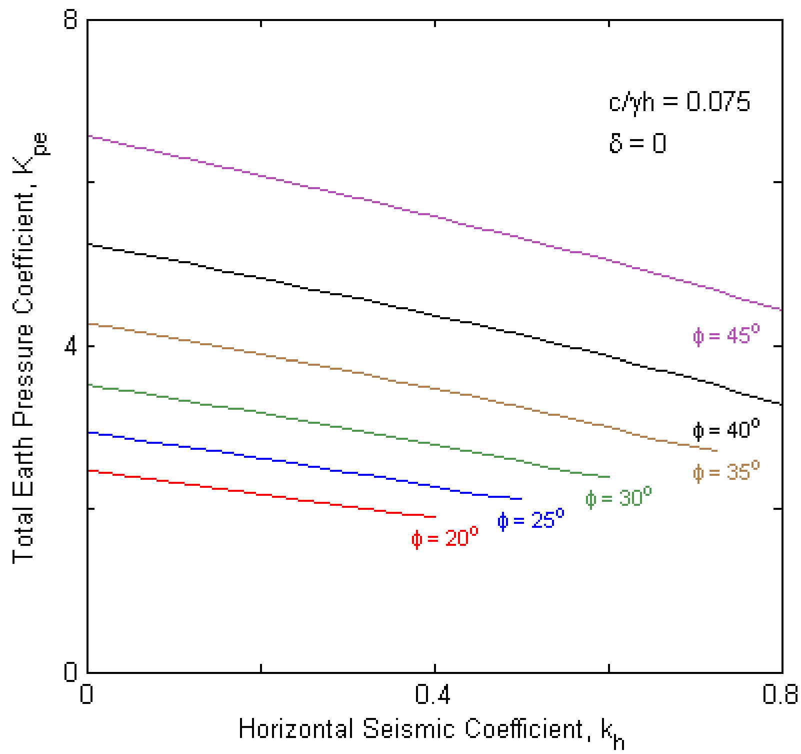


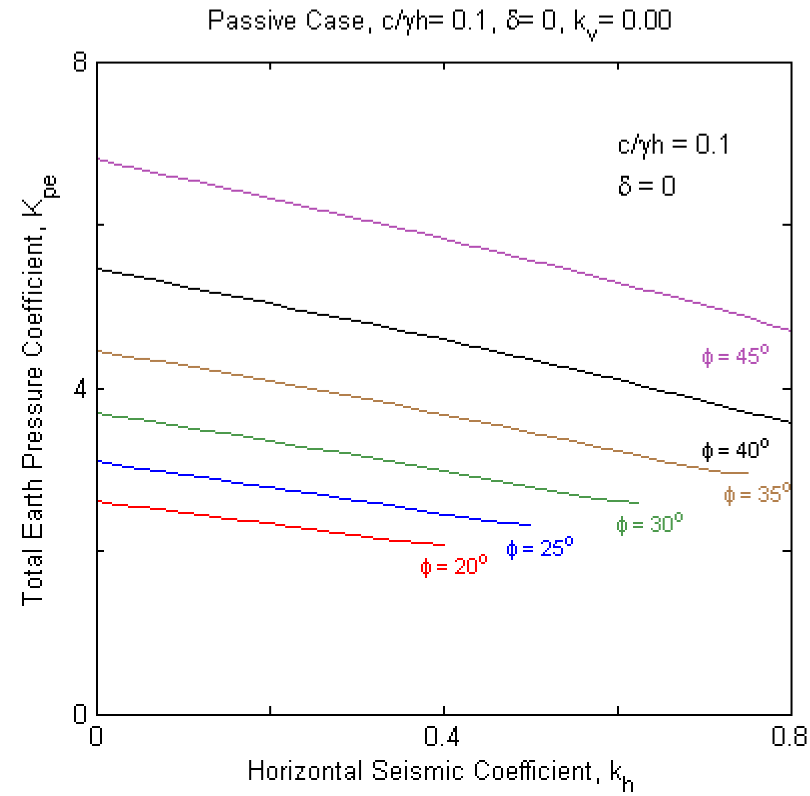


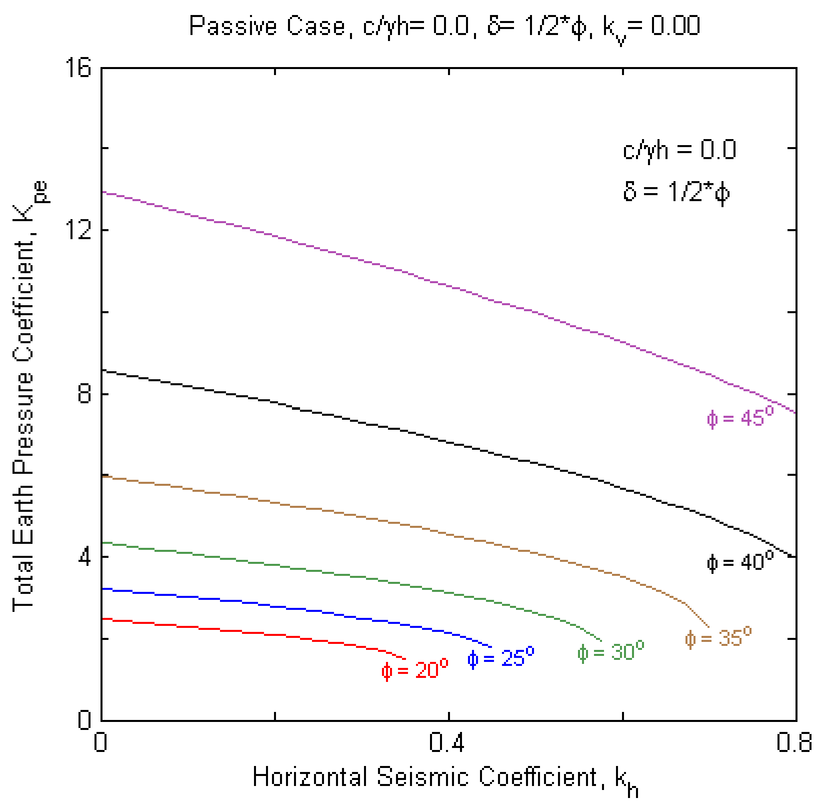
Passive Case, c/γh= 0.05, δ= 0, k_v= 0.00

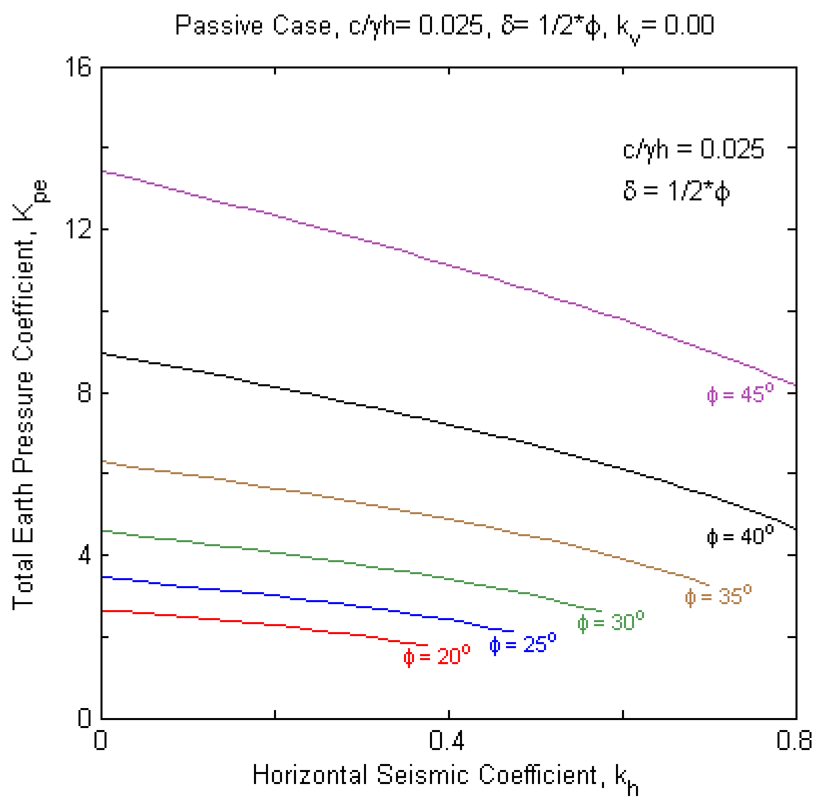


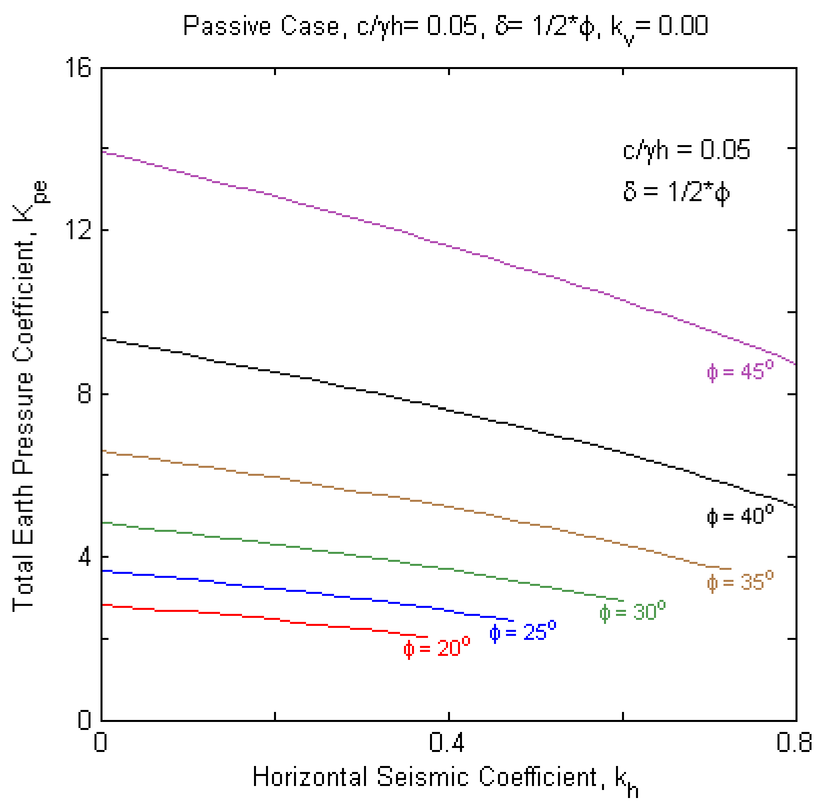
Passive Case, c/γh= 0.075, δ= 0, k_v= 0.00

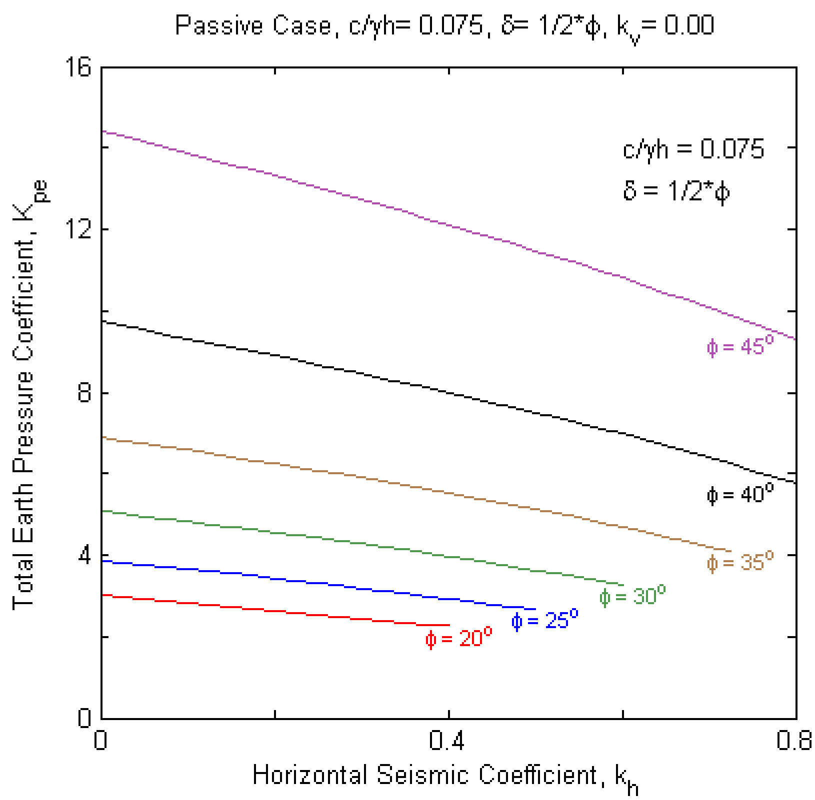


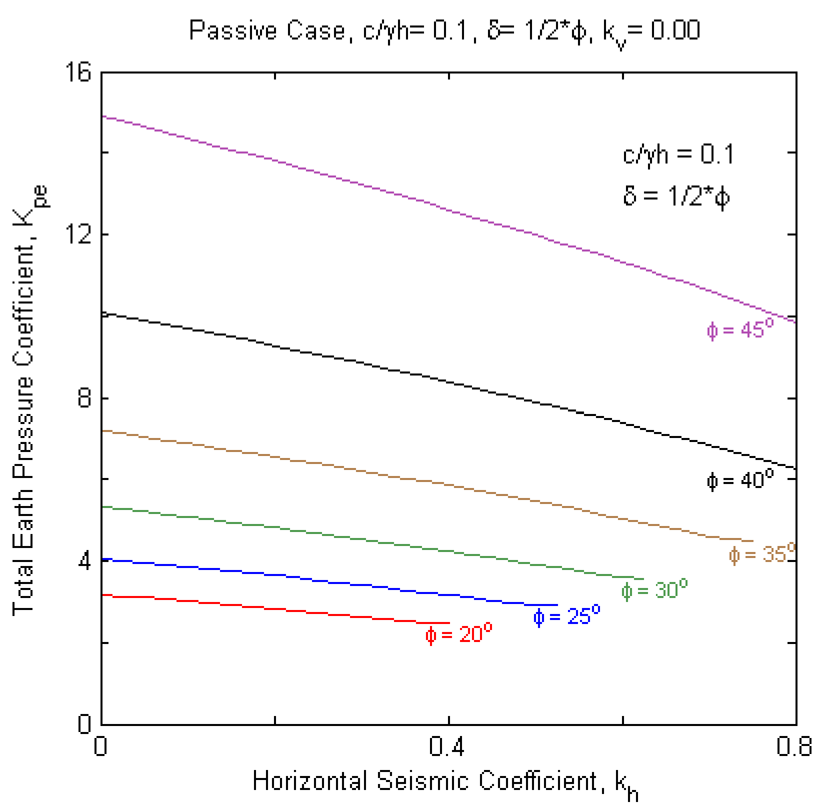

Passive Case, c/γh= 0.1, δ= 0, kv= 0.00
c/γh = 0.1
δ = 0
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
8
4
0
0
0.4
0.8


Passive Case, c/γh= 0.0, δ= 1/2*ϕ, kv= 0.00
c/γh = 0.0
δ = 1/2*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
16
12
8
4
0
0
0.4
0.8

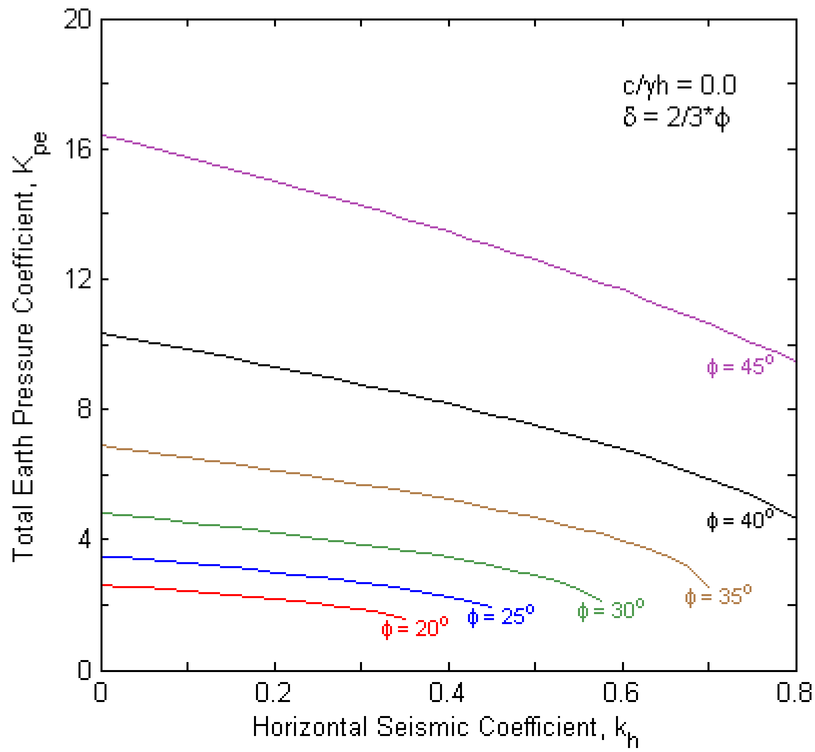

Passive Case, c/γh= 0.025, δ= 1/2*ϕ, kv= 0.00
c/γh = 0.025
δ = 1/2*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
16
12
8
4
0
0
0.4
0.8


Passive Case, c/γh= 0.05, δ= 1/2*ϕ, kv= 0.00
c/γh = 0.05
δ = 1/2*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
16
12
8
4
0
0
0.4
0.8

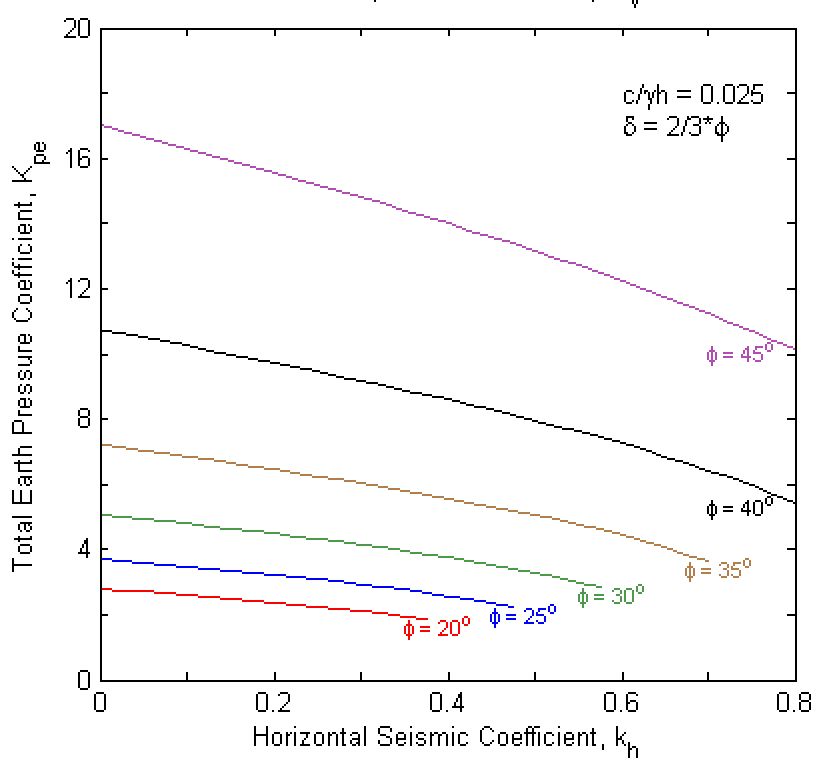

Passive Case, c/γh= 0.075, δ= 1/2*ϕ, kv= 0.00
c/γh = 0.075
δ = 1/2*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
16
12
8
4
0
0
0.4
0.8

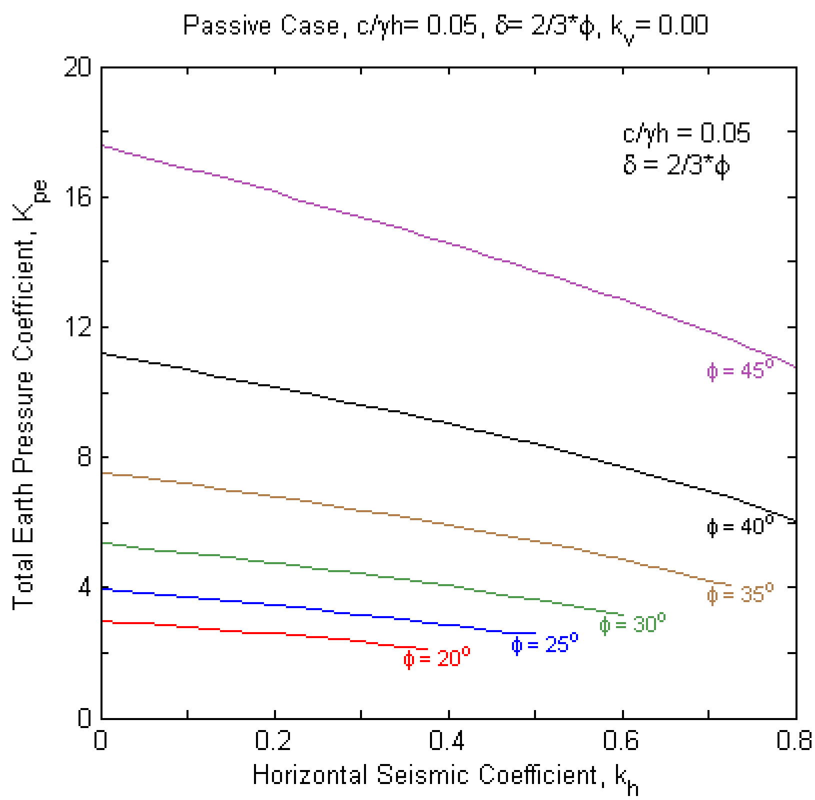

Passive Case, c/γh= 0.1, δ= 1/2*ϕ, kv= 0.00
c/γh = 0.1
δ = 1/2*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
16
12
8
4
0
0
0.4
0.8

Passive Case, c/γh= 0.0, δ= 2/3*ϕ, k_v= 0.00

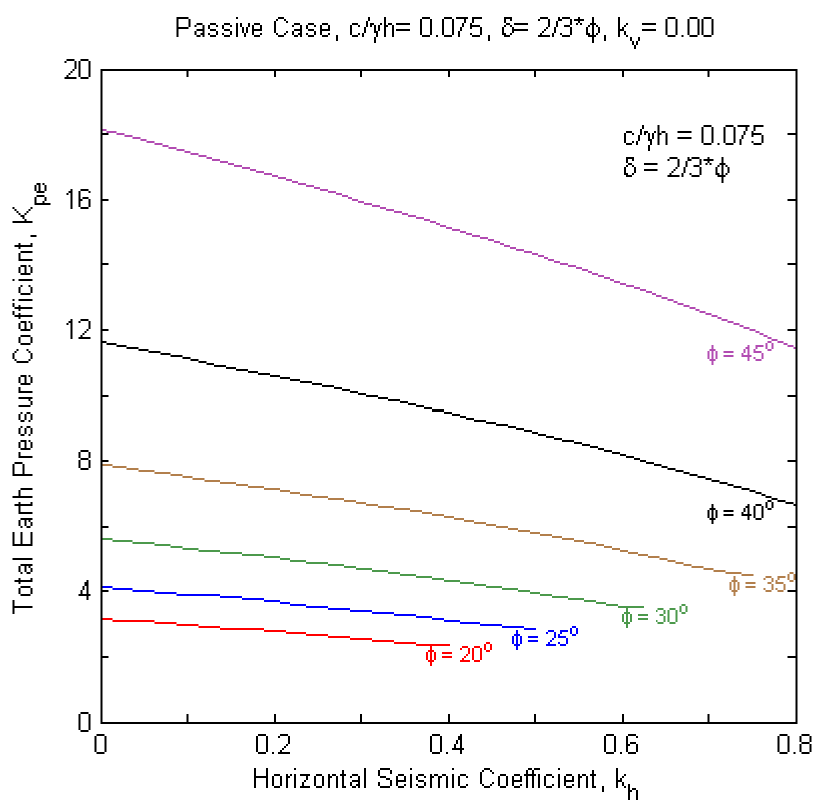


Passive Case, c/γh= 0.025, δ= 2/3*ϕ, k_v= 0.00

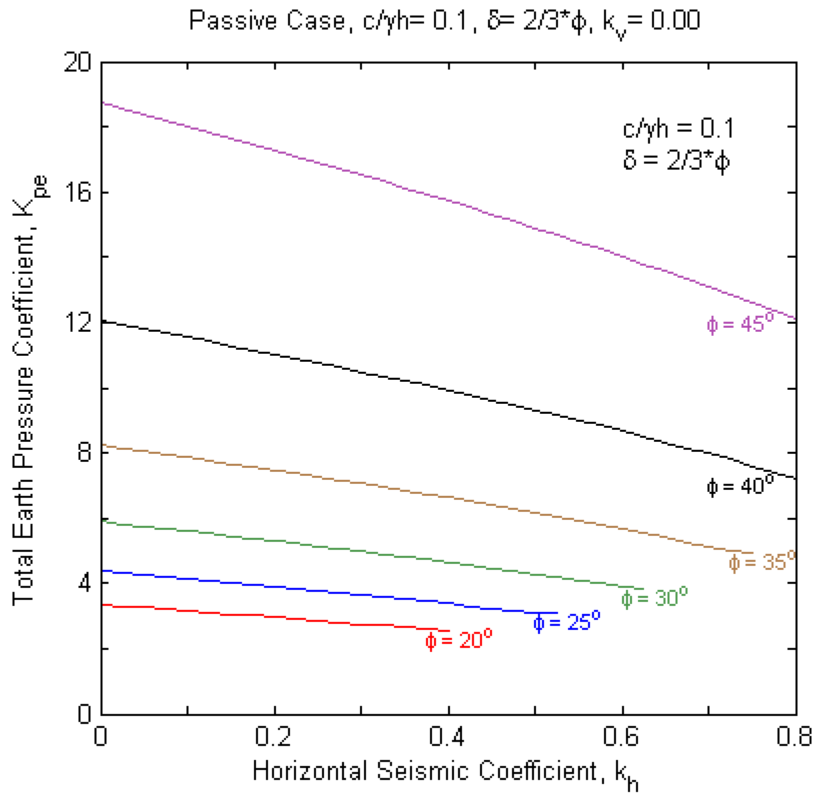




Passive Case, c/γh= 0.05, δ= 2/3*ϕ, kv= 0.00
c/γh = 0.05
δ = 2/3*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
0
4
8
12
16
20
0.2
0.4
0.6
0.8



Passive Case, c/γh= 0.075, δ= 2/3*ϕ, kv= 0.00
c/γh = 0.075
δ = 2/3*ϕ
ϕ = 45°
ϕ = 40°
ϕ = 35°
ϕ = 30°
ϕ = 25°
ϕ = 20°
Total Earth Pressure Coefficient, Kpe
Horizontal Seismic Coefficient, kh
0
4
8
12
16
20
0.2
0.4
0.6
0.8



Remark: The seismic passive earth pressure coefficients as presented in Appendix B are computed based on the simplified Log-Spiral-Rankine Method (Shamsabadi et al., 2013) outlined in section §2.3.3.

REFERENCES

Clough, G.W. and Duncan, J.M. (1991). Earth pressures, chapter in Foundation Engineering Handbook, 2nd edition, edited by Hsai-Yang Fang, van Nostrand Reinhold, New York, NY, pp. 223-235.

Coulomb, C.A. (1776). Essai sur une application des regles des maximis et minimis a quelques problems de statique relatives a l'architecture. *Memoires d'Academie Roy. Pres. Divers Savants*, **7**, Paris.

Kavazanjian, E., Matasovic, N., Hadj-Hamou, T., and Wang, J. (1998). Geotechnical and Foundation Engineering, Module 9 – Geotechnical Earthquake Engineering. Principal Investigator: George Munfakh, NHI Course No. 13239, U.S. Department of Transportation, Federal Highway Administration, National Highway Institute, Arlington, Virginia. Publication No. FHWA NHI-99-012.

Mayne, P.W., Kulhawy F.H. (1982). Ko-OCR Relationship in Soil, Journal of Geotechnical Engineering Division, **108**: 851-872. (K_0 vs *OCR* for overconsolidated soil).

Mononobe, N., Matsuo, H. (1929). On the Determination of Earth-Pressures during Earthquakes. *Proceedings of World Engineering Conference*, **9**: 179–187.

Okabe, S. (1926). General Theory of Earth-Pressures. *J. of Japanese Society of Civil Engineers*, **12**(1): 123–134.

Rankine, W.J.M. (1857). On the Stability of Loose Earth. *Phil. Trans. Roy. Soc. London*, **147**(1): 9-27.

Richards, R. Jr., Shi, X. (1994). Seismic Lateral Pressures in Soils with Cohesion. Journal of Geotechnical Engineering, **120**(7): 1230-1251.

Shamsabadi, A., Xu, S.-Y., and Taciroglu, E. (2013). A Generalized Log-Spiral-Rankine Limit Equilibrium Model for Seismic Earth Pressure Analysis. *Soil Dynamics and Earthquake Engineering*, **49**: 197–209

Xu, S.-Y., Shamsabadi, A., and Taciroglu, E. (2013). Evaluation of Active and Passive Seismic Earth Pressures Considering Internal Friction and Cohesion of Backfill Soil. *Soil Dynamics and Earthquake Engineering* (submitted for publication).

Zarrabi-Kashani, K. (1979)._Sliding of Gravity Retaining Wall during Earthquakes Considering Vertical Acceleration and Changing Inclination of Failure Surface. Master thesis, Massachusetts Institute of Technology.