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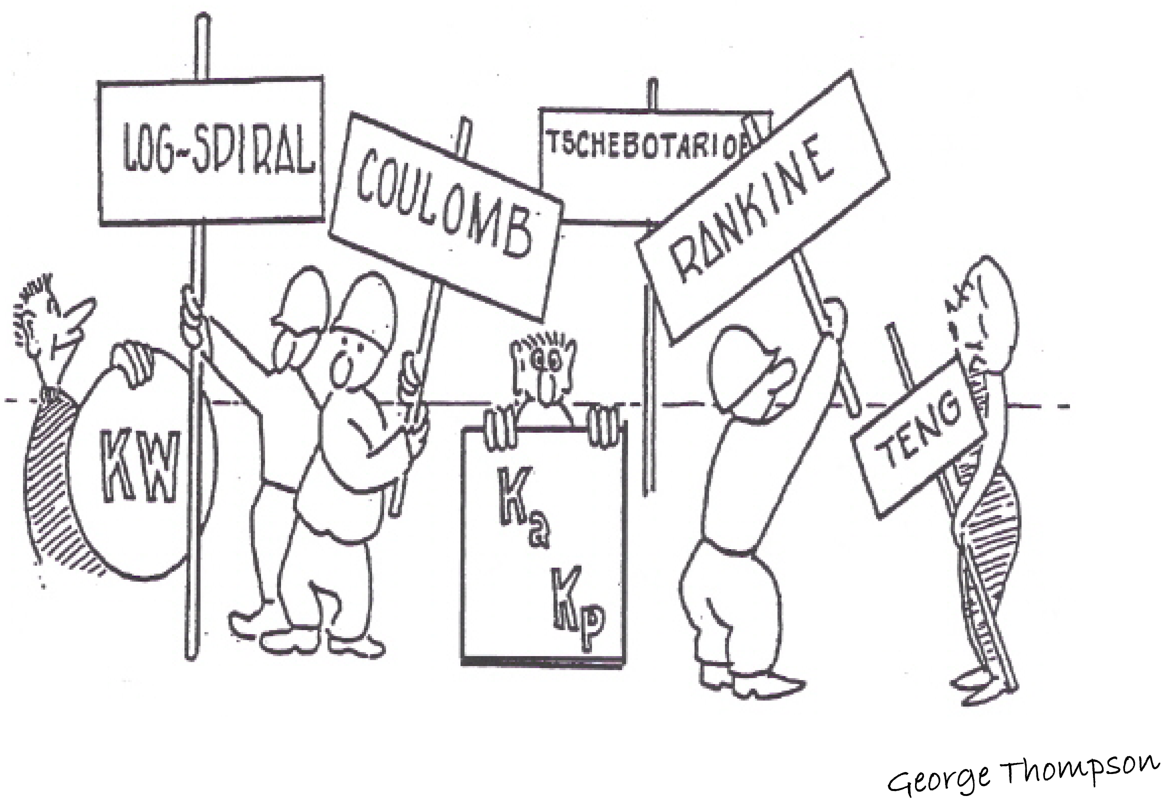
CHAPTER 4

EARTH PRESSURE THEORY AND APPLICATION



Chapter 4: Earth Pressure Theory and Application

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4-1 Introduction

All shoring systems must be designed to withstand lateral earth pressure, water pressure, and the effect of surcharge loads in accordance with the general principles and guidelines specified in this *California Department of Transportation Trenching & Shoring Manual*. The Simplified Method for cantilevered systems presented is developed around AASHTO and Transportation Research Board National Cooperative Highway Research Program (TRB NCHRP Report 611) equations. The Conventional Method for cantilevered systems presented in the 1984 *USS Steel Sheet Piling Design Manual* is presented here as the Rigorous Method (see Appendix D, *Sheet Piles,* for additional information on this reference). This method is a reintroduction of a past practice used in the pre-2006 *Trenching and Shoring Manual*. The Rigorous Method, as the name implies, produces a more refined or precise analysis.

This chapter will describe the two basic categories of shoring systems, the development of soil pressures behind the shoring, and how to use these pressures to develop the loads the shoring system must resist.

4-2 Shoring Types

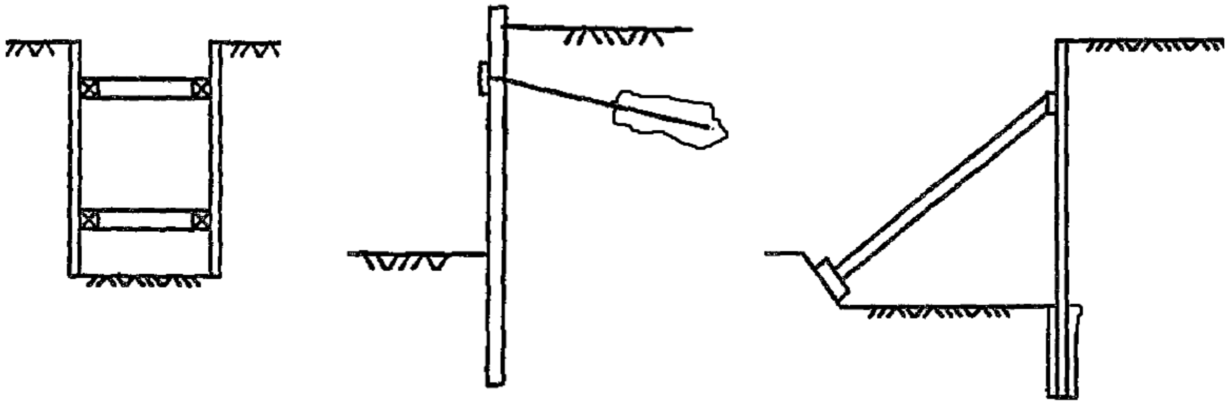
Shoring systems are generally classified as unrestrained (non-gravity cantilevered walls) and restrained (braced or anchored walls). Unrestrained shoring systems rely on structural components of the wall partially embedded in the foundation material to mobilize passive resistance to lateral loads. Restrained shoring systems derive their capacity to resist lateral loads by their structural components being restrained by tension or compression elements connected to the vertical structural members of the shoring system and, additionally, by the partial embedment (if any) of their structural components into the foundation material.

4-2.01 Unrestrained Shoring Systems

Unrestrained shoring systems (non-gravity cantilevered walls) are constructed of vertical structural members consisting of partially embedded soldier piles or continuous sheet piles. This type of system depends on the passive resistance of the foundation material and the moment-resisting capacity of the vertical structural members for stability; therefore, its maximum height is limited by the competence of the foundation material and the moment-resisting capacity of the vertical structural members. The economical height of this type of wall is generally limited to a maximum of 18 feet.

4-2.02 Restrained Shoring Systems

Restrained shoring systems are either anchored or braced walls, as illustrated in Figure 4-1. They are typically comprised of the same elements as unrestrained (non-gravity cantilevered) walls but derive additional lateral resistance from one or more levels of braces, rakers, or anchors. These systems would also include a trench system, where the sides are braced against each other with two or more levels of struts between the sides.



Braced trench with two rows of struts

Cantilever Wall restrained with a single ground anchor

Cantilever Wall restrained with a single brace (or raker)

Figure 4-1. Examples of Restrained Shoring Systems

These walls are typically constructed in cut situations in which construction proceeds from the top down to the base of the wall. The vertical wall elements should extend below the potential failure plane associated with the retained soil mass. For these types of walls, heights up to 80 feet are economically feasible.

Note - Soil Nail Walls and Mechanically Stabilized Earth (MSE) Walls are not included in this manual. Both of these types of systems are designed by other methods that can be found online with FHWA or AASHTO.

4-2.03 Shoring Movement and Loading

A major issue in providing a safe shoring system design is to determine the appropriate earth-pressure loading diagram. The loads are to be calculated using the appropriate earth pressure theories. The lateral horizontal stresses (σ_h) for both active and passive pressure are to be calculated based on the soil properties and the shoring system. Earth pressure loads on a shoring system are a function of the unit weight of the soil, location of the groundwater table, seepage forces, surcharge loads, and the shoring structure system. Shoring systems that cannot tolerate any movement should be designed for at-rest lateral earth pressure. At-rest lateral earth pressure is when a wall experiences no lateral movement. Shoring systems which can move away from the soil mass should be designed for active earth pressure conditions depending on the magnitude of the tolerable movement. Any movement, which is required to reach the minimum active pressure or the maximum passive pressure, is a function of the wall height and the soil type. A significant amount of movement is necessary to mobilize the full passive pressure. The variation of lateral stress between the active and passive earth pressure values can be brought about only through lateral movement within the soil mass of the backfill as shown in Figure 4-2.

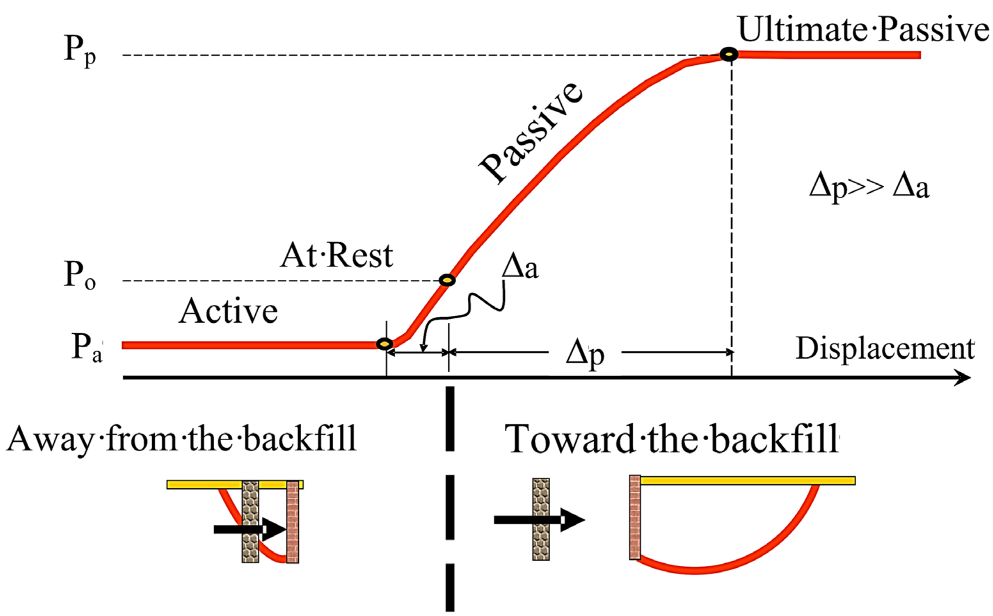


Figure 4-2. Active and Passive Earth Pressure Coefficient as a Function of Wall Displacement

Typical values of these mobilizing movements, relative to wall height, are given in Table 4-1 (Clough 1991).

Table 4-1. Mobilized Wall Movements

Type of Backfill	Value of **Δ/H**	
	Active	Passive
Dense Sand	0.001	0.01
Medium Dense Sand	0.002	0.02
Loose Sand	0.004	0.04
Compacted Silt	0.002	0.02
Compacted Lean Clay	0.01	0.05
Compacted Fat Clay	0.01	0.05

Where:

Δ = the movement of top of wall required to reach minimum active or maximum passive pressure, by tilting or lateral translation.

H = height of wall.

4-3 Developing Earth Pressures for Granular Soil

At present, the methods of analysis in common use for retaining structures are based on Rankine (1857) and Coulomb (1776) theories. Both methods are based on the limit equilibrium approach with an assumed planar failure surface. Developments since 1920, largely due to the influence of Terzaghi (1943), have led to a better understanding of the limitations and appropriate applications of classical earth pressure theories. Terzaghi assumed a logarithmic failure surface. Many experiments have been conducted to validate Coulomb's wedge theory, and it has been found that the sliding surface is not a plane, but a curved surface as shown in Figure 4-3**.** (Terzaghi 1943).

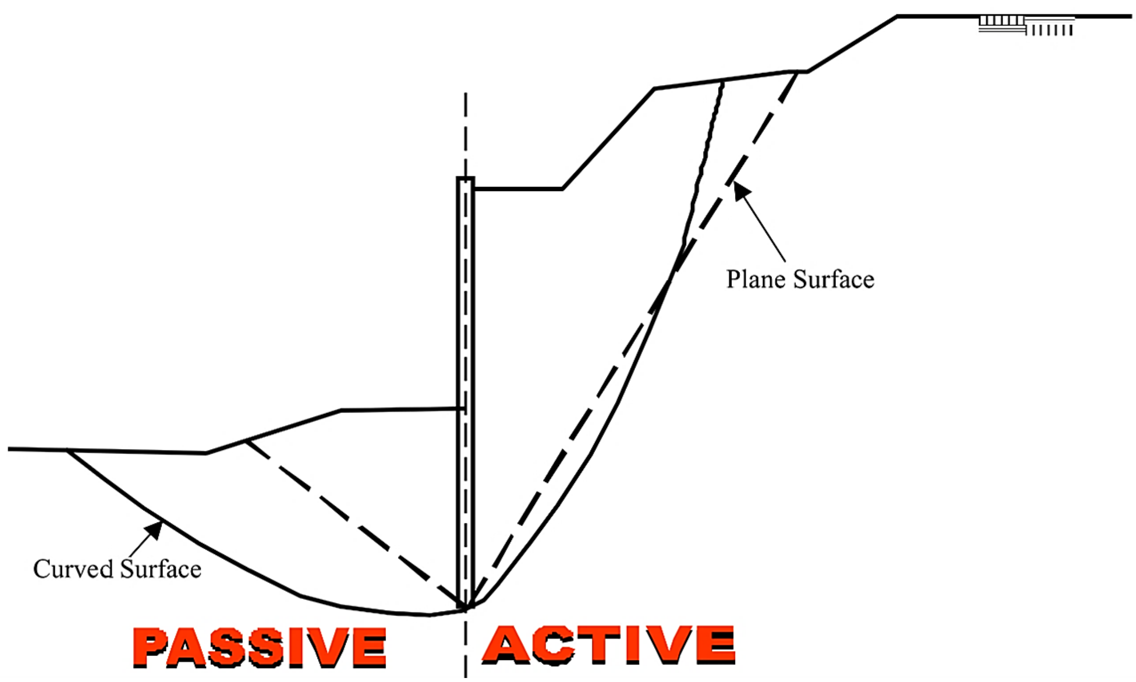


Figure 4-3. Comparison of Plane versus Curved Failure Surfaces

Furthermore, these experiments have shown that the Rankine (1857) and Coulomb (1776) earth pressure theories lead to quite accurate results for the active earth pressure. However, for the passive earth pressure, these theories are accurate only for the backfill of clean, dry sand which would lead to a low wall-interface friction angle between the material and the wall.

For the purpose of the initial discussion, it is assumed that the backfill is level, homogeneous, and the distribution of vertical stress (σ_v) with depth is hydrostatic, as shown in Figure 4-4. The horizontal stress (σ_h) is linearly proportional to depth and is a multiple of vertical stress (σ_v) as shown in Equation 4-3-1.

$$\sigma_h = \sigma_v K = \gamma h K \quad \text{where gamma, } \gamma \text{, is the unit weight of the soil.} \qquad (4\text{-}3\text{-}1)$$

Depending on the wall movement, the coefficient **K** represents the active (**K_a**), passive (**K_p**), or at-rest (**K_o**) earth pressure coefficient in the above equation.

The resultant lateral earth load, **P**, which is equal to the area of the load diagram as calculated in Equation 4-3-2 and illustrated in Figure 4-4, acts at a height of **h/3** above the base of the wall, where **h** is the height of the pressure surface, measured from the surface of the ground to the base of the wall. **P** is the force that causes bending, sliding, and overturning in the wall.

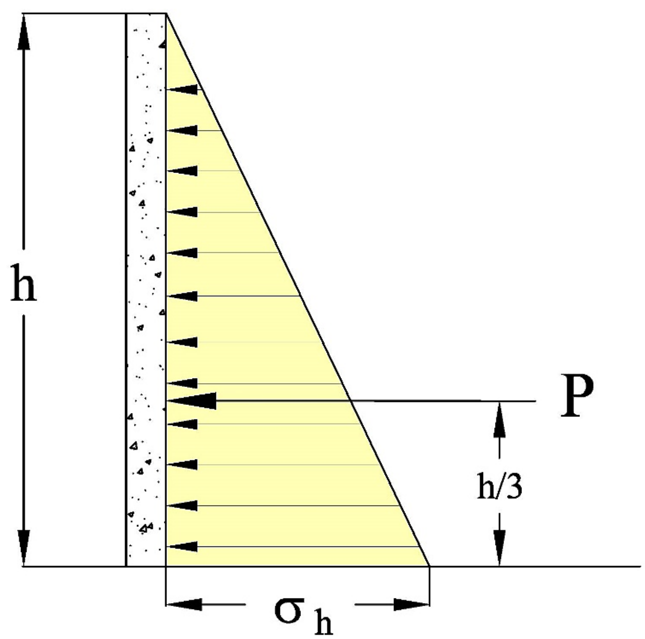


Figure 4-4. Lateral Earth Pressure Variation with Depth

$$\mathbf{P} = \frac{1}{2}\sigma_h \mathbf{h} \quad (4\text{-}3\text{-}2)$$

Depending on the shoring system, the value of the active and/or passive pressure will be determined using either the Rankine, Coulomb, Log-Spiral, or Trial Wedge methods as appropriate.

The state of the active and passive earth pressure depends on the expansion or compression transformation of the backfill from elastic state to state of plastic equilibrium. The concept of the active and passive earth pressure theory can be explained using a continuous anchor block buried near the ground surface for the stability of a sheet pile wall, as shown in Figure 4-5. As a result of wall deflection, Δ, the tie rod is pulled until the active and passive wedges are formed behind and in front of the anchor block. Element P, in front of the anchor block, and element A, at the back of the anchor block, are acted on by two principal stresses: a vertical stress (σ_v) and a horizontal stress (σ_h). In the active case, the horizontal stress (σ_a) is the minor principal stress, and the vertical stress (σ_v) is the major principal stress. In the passive case, the horizontal stress (σ_p) is the major principal stress, and the vertical stress (σ_v) is the minor principal stress. The resulting failure surface within the soil mass corresponding to active and passive earth pressure for the cohesionless soil is shown in Figure 4-6.

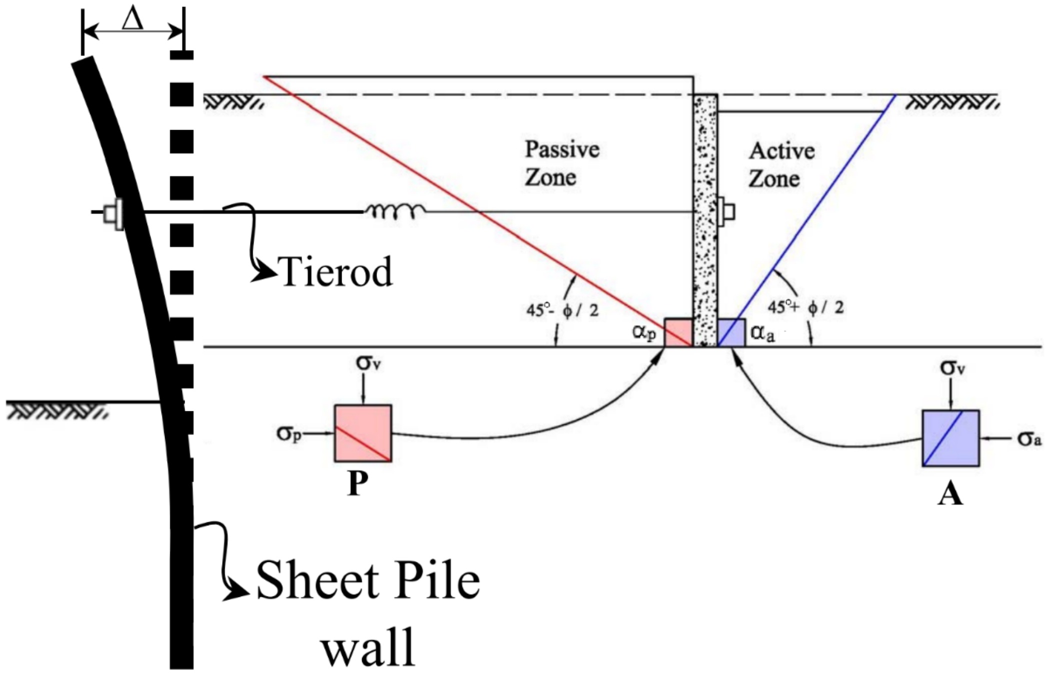


Figure 4-5. Concept of Active and Passive Earth Pressure Theory

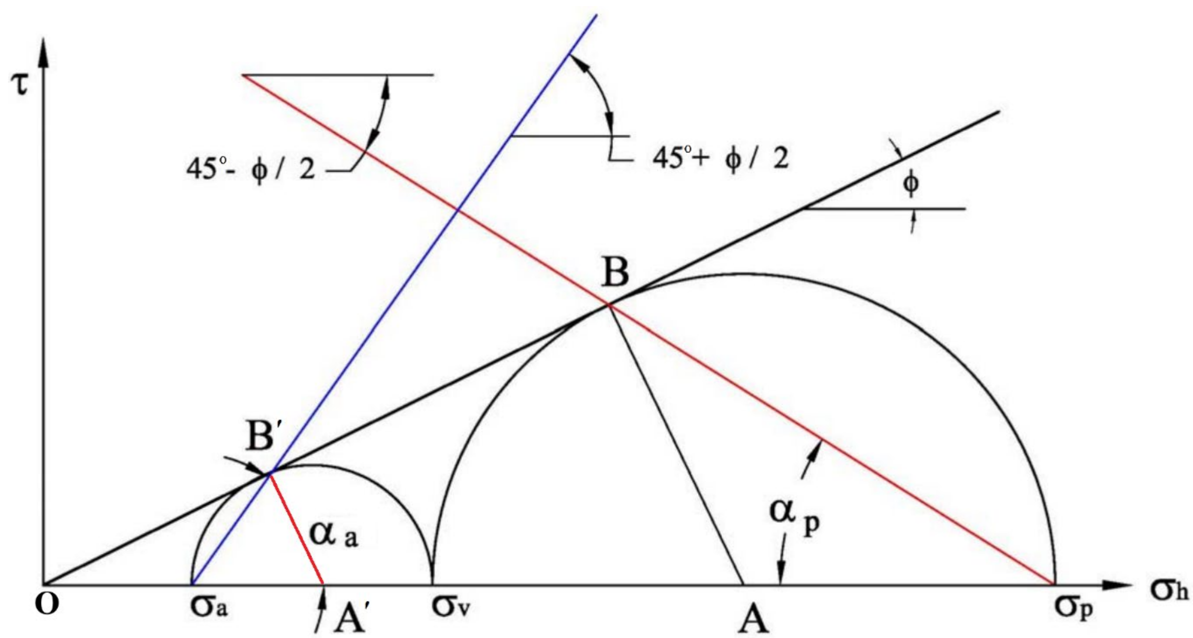


Figure 4-6. Mohr Circle Representation of Earth Pressure for Cohesionless Backfill

From Figure 4-6 above:

$$\sin\emptyset = \frac{A'B'}{OA'} = \frac{\frac{\sigma_v - \sigma_a}{2}}{\frac{\sigma_v + \sigma_a}{2}} \qquad (4\text{-}3\text{-}3)$$

Where **A'B'** is the radius of the small circle:

$$\sin\emptyset = \frac{A'B'}{OA'} = \frac{\sigma_v - \sigma_a}{\sigma_v + \sigma_a} \qquad (4\text{-}3\text{-}4)$$

$$\sigma_v \sin\,\phi + \sigma_a \sin\,\phi = \sigma_v - \sigma_a \qquad (4\text{-}3\text{-}5)$$

Collecting Terms:

$$\sigma_a + \sigma_a(\sin\,\phi) = \sigma_v - \sigma_v(\sin\phi) \qquad (4\text{-}3\text{-}6)$$

$$\sigma_a(1 + \sin\,\phi) = \sigma_v(1 - \sin\,\phi) \qquad (4\text{-}3\text{-}7)$$

$$\frac{\sigma_a}{\sigma_v} = \frac{(1 - \sin\,\phi)}{(1 + \sin\,\phi)} \qquad (4\text{-}3\text{-}8)$$

From trigonometric identities:

$$\frac{(1 - \sin\,\phi)}{(1 + \sin\,\phi)} = \tan^2(45^\circ - \phi/2) \qquad (4\text{-}3\text{-}9)$$

$$\frac{(1 + \sin\,\phi)}{(1 - \sin\,\phi)} = \tan^2(45^\circ + \phi/2) \qquad (4\text{-}3\text{-}10)$$

$$K_a = \tan^2(45^\circ - \phi/2), \text{ where } K_a = \frac{\sigma_a}{\sigma_v} \qquad (4\text{-}3\text{-}11)$$

For the passive case:

$$K_p = \frac{\sigma_p}{\sigma_v} = \frac{1 + \sin\,\phi}{1 - \sin\,\phi} = \tan^2(45^\circ + \phi/2) \qquad (4\text{-}3\text{-}12)$$

4-3.01 At-Rest Lateral Earth Pressure Coefficient (K_0)

For a zero lateral strain condition, horizontal and vertical stresses are related by the Poisson's ratio (μ) as follows:

$$K_0 = \frac{\mu}{1 - \mu} \qquad (4\text{-}3\text{-}13)$$

For normally consolidated soils and vertical walls, the coefficient of at-rest lateral earth pressure may be taken as:

$$\mathbf{K_0 = (1 - \sin\ \phi)(1 - \sin\ \beta)} \tag{4-3-14}$$

ϕ = effective friction angle of soil

$\mathbf{K_0}$ = coefficient of at-rest lateral earth pressure

β = slope angle of backfill surface behind retaining wall.

For over-consolidated soils, level backfill, and a vertical wall, the coefficient of at-rest lateral earth pressure may be assumed to vary as a function of the over-consolidation ratio or stress history, and may be taken as:

$$\mathbf{K_0 = (1 - \sin\phi)(OCR)^{\sin\phi}} \tag{4-3-15}$$

Where:

OCR = over consolidation ratio

Note that the equations for the coefficient of at-rest lateral earth pressure are empirical.

4-3.02 Active and/or Passive Earth Pressure

Depending on the shoring system, the value of the active and/or passive pressure can be determined using either the Rankine, Coulomb, or trial wedge methods.

4-3.02A Rankine's Theory

Rankine's theory is the simplest formulation proposed for earth pressure calculations and is based on the following assumptions:

1. The wall is smooth and vertical.
2. There is no friction or adhesion between the wall and the soil.
3. The failure wedge is a plane surface and is a function of soil's friction ϕ and the backfill slope β as shown in Equation 4-3-16 and Equation 4-3-19.
4. Lateral earth pressure varies linearly with depth.
5. The direction of the lateral earth pressure is parallel to the slope of the backfill as shown in Figure 4-7 and Figure 4-8.
6. The resultant earth pressure acts at a distance equal to one-third of the wall height from the base.

Values for the coefficient of active lateral earth pressure using the Rankine's theory may be calculated as shown in Equation 4-3-16.

$$\mathbf{K_a = \cos\ \beta \frac{\cos\ \beta - \sqrt{\cos^2\ \beta - \cos^2\ \phi}}{\cos\ \beta + \sqrt{\cos^2\ \beta - \cos^2\ \phi}}} \qquad (4\text{-}3\text{-}16)$$

And the magnitude of active earth pressure can be determined as shown in Figure 4-7 and Equation 4-3-17:

$$\mathbf{P_a = \frac{1}{2}(\gamma)(h^2)(K_a)} \qquad (4\text{-}3\text{-}17)$$

The failure plane angle α can be determined as shown in Equation 4-3-18:

$$\boldsymbol{\alpha = \left(45^\circ + \frac{\phi}{2}\right) - \frac{1}{2}\left(\mathrm{Arcsin}\left(\frac{\sin\ \beta}{\sin\ \phi}\right) - \beta\right)} \qquad (4\text{-}3\text{-}18)$$

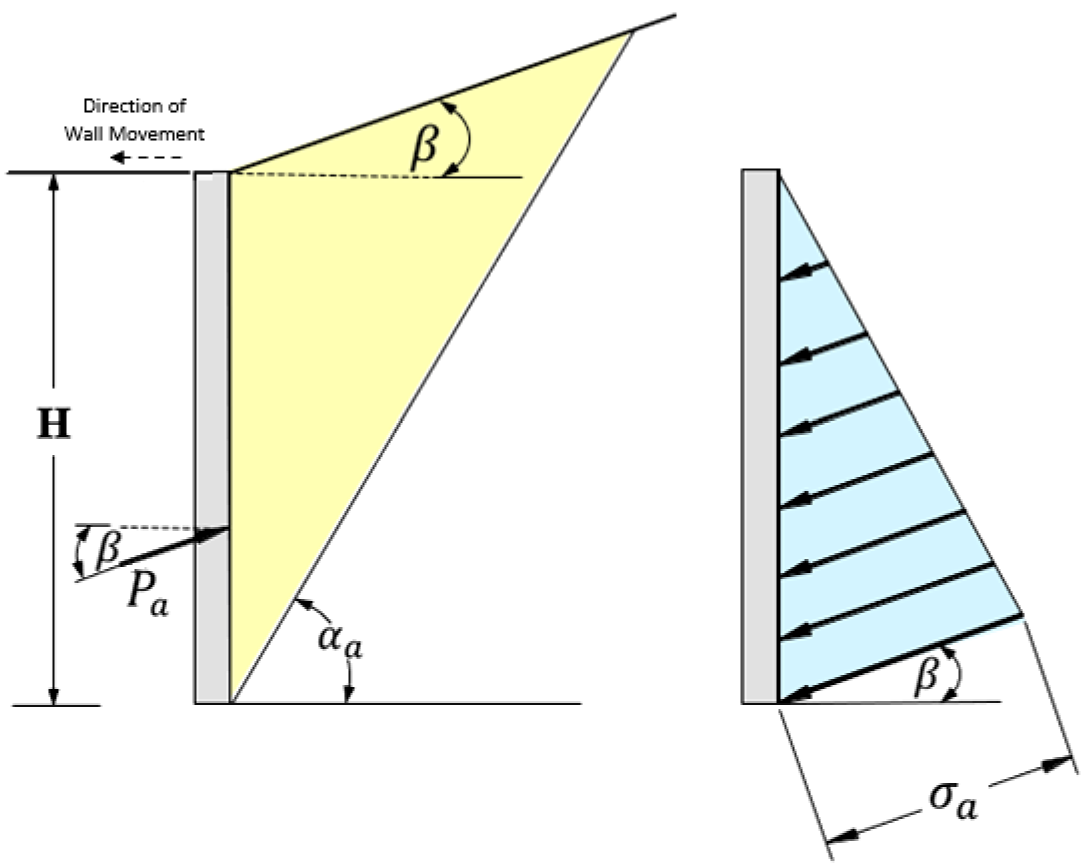


Figure 4-7. Rankine's Active Wedge

Rankine made similar assumptions to his active earth pressure theory to calculate the passive earth pressure. Values for the coefficient of passive lateral earth pressure may be calculated as:

$$\mathbf{K_p = \cos\ \beta \frac{\cos\ \beta + \sqrt{\cos^2\ \beta - \cos^2\ \phi}}{\cos\ \beta - \sqrt{\cos^2\ \beta - \cos^2\ \phi}}} \quad (4\text{-}3\text{-}19)$$

And the magnitude of passive earth pressure can be determined as shown in Figure 4-8 and Equation 4-3-20:

$$\mathbf{P_P = \frac{1}{2}(\gamma)(h^2)(K_p)} \quad (4\text{-}3\text{-}20)$$

The failure plane angle α can be determined as shown in Equation 4-3-21:

$$\boldsymbol{\alpha = \left(45^\circ - \frac{\phi}{2}\right) + \frac{1}{2}\left(\mathrm{Arcsin}\ \left(\frac{\sin\ \beta}{\sin\ \phi}\right) + \beta\right)} \quad (4\text{-}3\text{-}21)$$

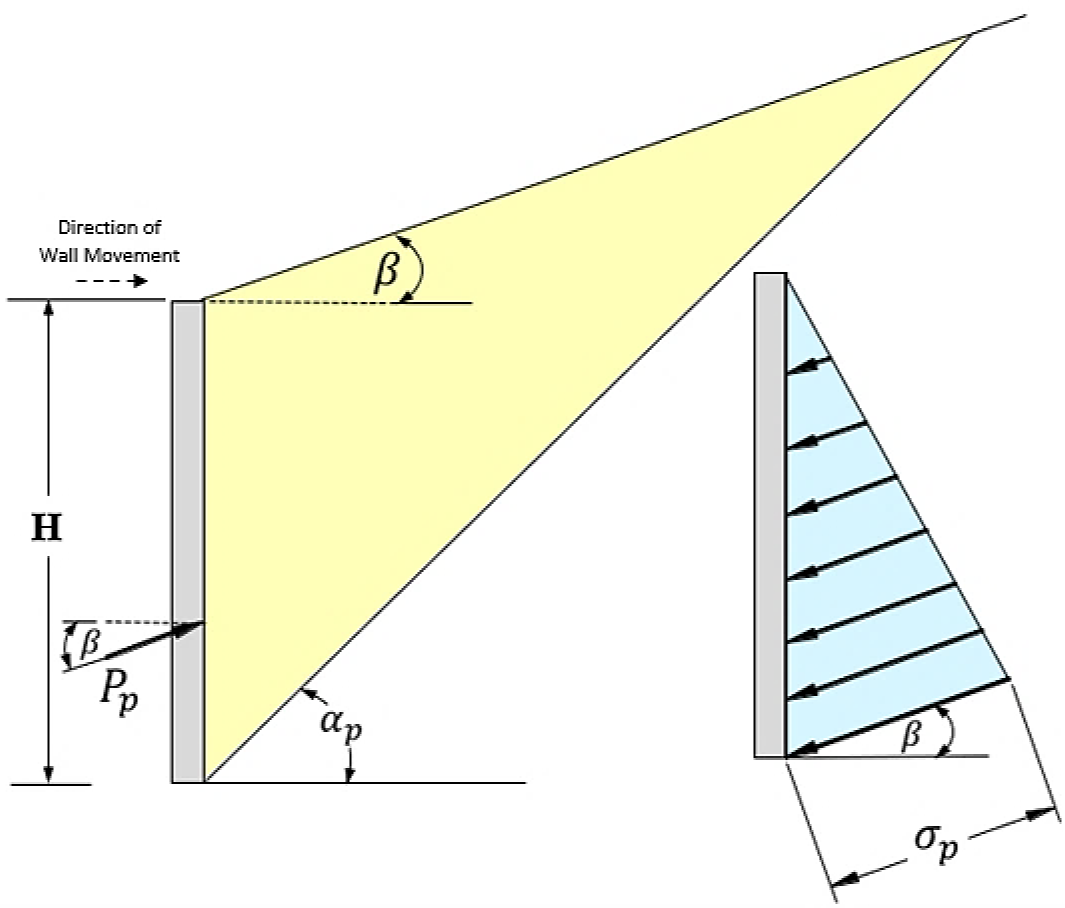


Figure 4-8. Rankine's Passive Wedge

Where:

H	=	Height of pressure surface on the wall
$\mathbf{P_a}$	=	Active lateral earth pressure resultant per unit width of wall
$\mathbf{P_p}$	=	Passive lateral earth pressure resultant per unit width of wall
β	=	Angle from backfill surface to the horizontal
α	=	Failure plane angle with respect to horizontal
ϕ	=	Effective friction angle of soil
$\mathbf{K_a}$	=	Coefficient of active lateral earth pressure
$\mathbf{K_p}$	=	Coefficient of passive lateral earth pressure
γ	=	Unit weight of soil

Although Rankine's equation for the passive earth pressure is provided above, one should not use the Rankine method to calculate the passive earth pressure when the backfill angle is greater than zero ($\beta>0$). As a matter of fact, the $\mathbf{K_p}$ value for both positive ($\beta>0$) and negative ($\beta<0$) backfill slope is identical. This is clearly not correct. Therefore, avoid using the Rankine equation to calculate the passive earth pressure coefficient for sloping ground.

4-3.02B Coulomb's Theory

Coulomb's (1776) earth pressure theory is based on the following assumptions:

1. The wall is rough.
2. There is friction and/or adhesion between the wall and the soil; refer to Table 4-2 for typical values of wall friction.
3. The failure wedge has a plane surface and is a function of the soil friction ϕ, wall friction δ, the backfill slope β, and the slope of the wall ω.
4. Lateral earth pressure varies linearly with depth.
5. The lateral earth pressure acts at an angle δ with a line that is normal to the wall.
6. The resultant earth pressure acts at a distance equal to one-third of the wall height from the base.

Values for the coefficient of active lateral earth pressure may be taken as shown in Equation 4-3-22.

$$\mathrm{K_a} = \frac{\cos^2(\phi - \omega)}{\cos^2\omega\,\cos(\delta + \omega)\left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi - \beta)}{\cos(\delta + \omega)\cos(\omega - \beta)}}\right]^2} \tag{4-3-22}$$

The magnitude of active earth pressure can be determined as shown in Figure 4-9 and Equation 4-3-23.

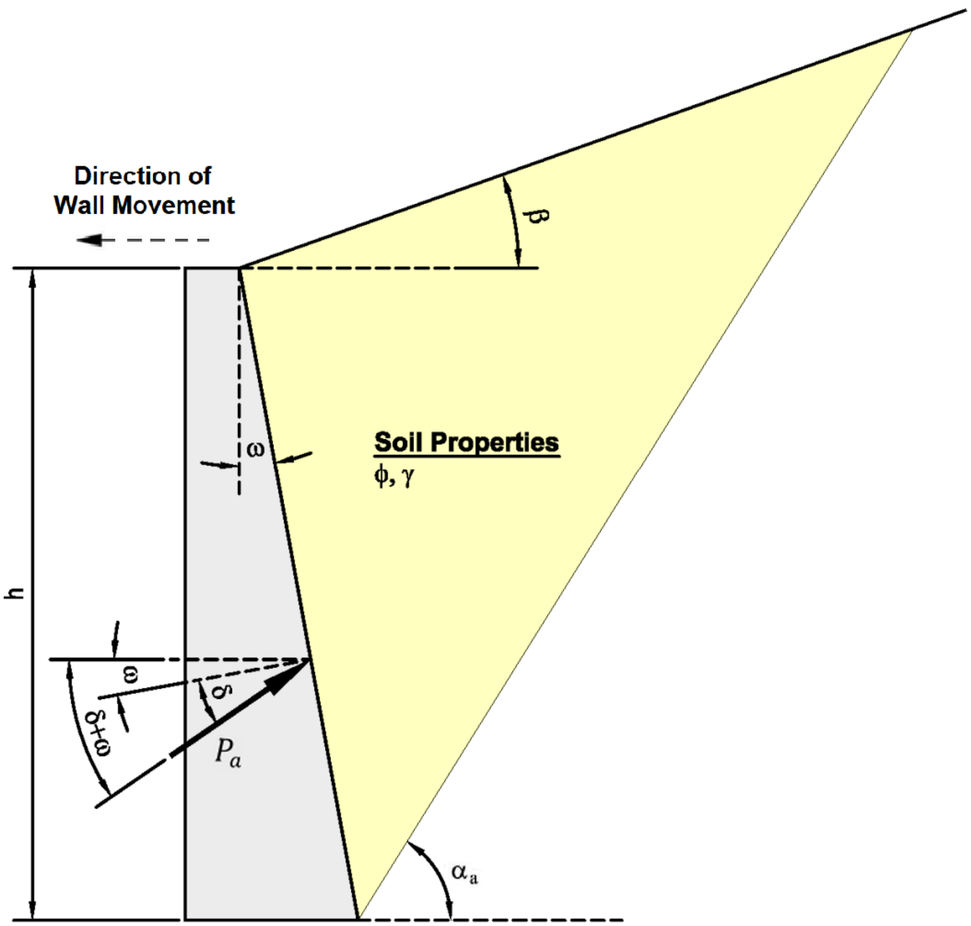


Figure 4-9. Coulomb's Active Wedge

$$\mathbf{P_a = \frac{1}{2}(\gamma)(h^2)(K_a)} \tag{4-3-23}$$

Coulomb's passive earth pressure is derived similarly to his active earth pressure except the inclination of the force is as shown in Figure 4-10. Values for the coefficient of passive lateral earth pressure may be taken as calculated in Equation 4-3-24.

$$\mathbf{K_p = \frac{\cos^2(\phi+\omega)}{\cos^2\omega\cos(\delta-\omega)\left[1-\sqrt{\frac{\sin(\delta+\phi)\sin(\phi+\beta)}{\cos(\delta-\omega)\cos(\beta-\omega)}}\right]^2}} \tag{4-3-24}$$

The magnitude of passive earth pressure can be determined as shown in Figure 4-10 and Equation 4-3-25.

$$\mathbf{P_p = \frac{1}{2}(\gamma)(h^2)(K_p)} \tag{4-3-25}$$

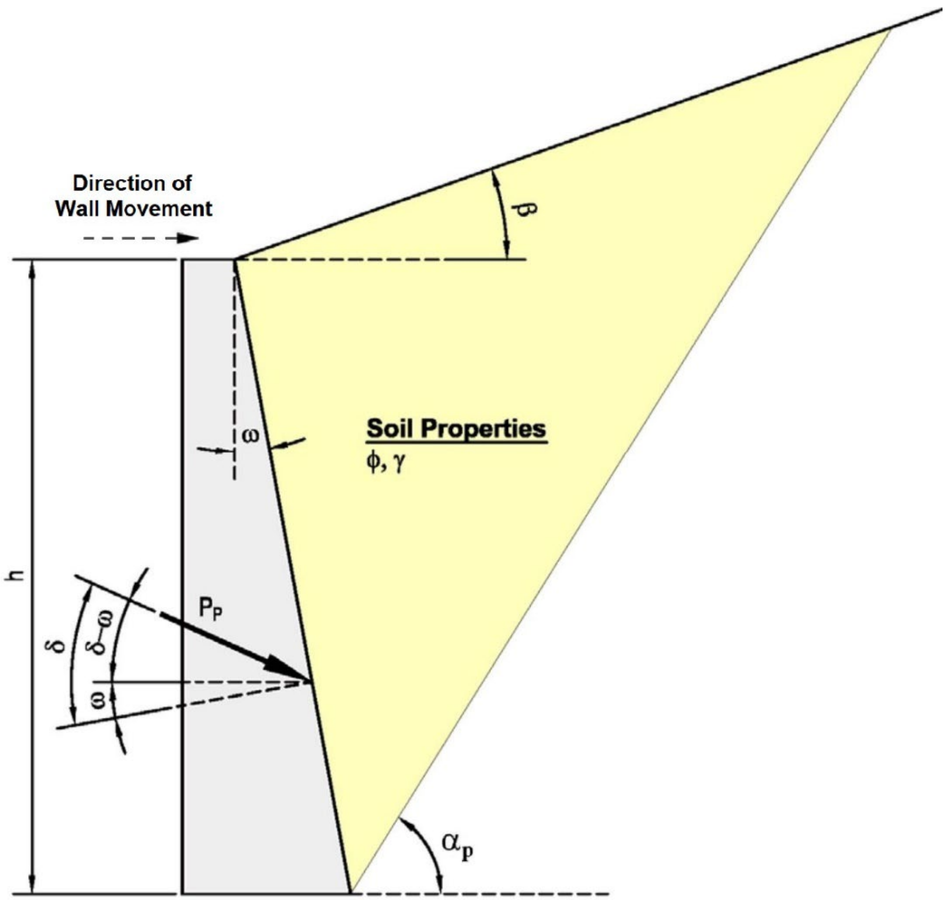


Figure 4-10. Coulomb's Passive Wedge

Where:

h	=	Height of pressure surface on the wall.
P_a	=	Active lateral earth pressure resultant per unit width of wall.
P_p	=	Passive lateral earth pressure resultant per unit width of wall.
δ	=	Friction angle between backfill material and face of wall.
β	=	Angle from backfill surface to the horizontal.
α	=	Failure plane angle with respect to the horizontal.
ω	=	Angle from the face of wall to the vertical.
ϕ	=	Effective friction angle of soil.
K_a	=	Coefficient of active lateral earth pressure.
K_p	=	Coefficient of passive lateral earth pressure.
γ	=	Unit weight of soil.

Table 4-2. Wall Friction

Ultimate Friction Factor for Dissimilar Materials	
INTERFACE MATERIALS	**Friction Angle, δ (°)**
Mass concrete on the following foundation materials:	
• Clean sound rock	35
• Clean gravel, gravel-sand mixtures, coarse sand	29 to 31
• Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel	24 to 29
• Clean fine sand, silty or clayey fine to medium sand	19 to 24
• Fine sandy silt, nonplastic silt	17 to 19
• Very stiff and hard residual or preconsolidated clay..	22 to 26
• Medium stiff and stiff clay and silty clay	17 to 19
Note: Masonry on foundation materials has similar friction factors.	
Steel sheet piles against the following soils:	
• Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls........	22
• Clean sand, silty sand-gravel mixture, single-size hard rock fill	17
• Silty sand, gravel or sand mixed with silt or clay..	14
• Fine sandy silt, nonplastic silt	11
Formed or precast concrete or concrete sheet piling against the following soils:	
• Clean gravel, gravel-sand mixture, well-graded rock fill with spalls..........	22 to 26
• Clean sand, silty sand-gravel mixture, single-size hard rock fill	17 to 22
• Silty sand, gravel or sand mixed with silt or clay..	17
• Fine sandy silt, nonplastic silt	14
Various structural materials:	
• Masonry on wood in direction of cross grain..	26
• Steel on steel at sheet pile interlocks..	17
• Masonry on masonry, igneous and metamorphic rocks:	
○ Dressed soft rock on dressed soft rock..	35
○ Dressed hard rock on dressed soft rock	33
○ Dressed hard rock on dressed hard rock	29

Note: This table is a reprint of Table C3.11.5.3-1, AASHTO LRFD BDS, 8th ed, 2017

Further discussion of wall friction is included in Section 4-6, *Log-Spiral Passive Earth Pressure.*

4-3.03 Summary of Earth Pressure Theories

There are various pros and cons to the individual earth theories when it comes to practical application of slope angles, friction angles, and failure planes. A summary for each theory is presented below:

- The Rankine formula for passive pressure can only be used correctly when the embankment slope angle, β, equals zero (i.e., level) or is negative. If a large wall friction value can develop, the Rankine theory is not correct and will give less conservative results. Rankine's theory is not intended to be used for determining earth pressures directly against a wall (friction angle does not appear in equations above). The theory is intended to be used for determining earth pressures on a vertical plane within a mass of soil, and therefore its use is to be avoided for passive earth pressure.
- For the Coulomb coefficient of passive earth pressure equation, if the shoring system is vertical and the wall friction angle is equal to zero degrees, the result will be the same as Rankine's for a level ground condition. Since wall friction requires a curved surface of sliding to satisfy equilibrium, the Coulomb formula will give only approximate results since it assumes planar failure surfaces. The accuracy for Coulomb will diminish with increased depth. For passive pressures, the Coulomb formula can also give inaccurate results when there is a large back slope or wall friction angle. Because of these limitations it is also recommended not to use Coulomb for the passive earth pressure.

The Log-Spiral theory was developed because of the unrealistic values of earth pressures that are obtained by theories that assume a straight-line failure plane. The difference between the Log-Spiral curved failure surface and the straight-line failure plane can be large and on the unsafe side for Coulomb passive pressures (especially when wall friction exceeds $\phi/3$). Figure 4-3 illustrates a comparison of the Coulomb and Rankine failure surfaces (planar) versus the Log-Spiral failure surface (curvilinear). More information on Log-Spiral theory can be found in in Section 4-6, *Log-Spiral Passive Earth Pressure,* of this manual.

4-4 Developing Earth Pressures for Cohesive Soil

Neither Coulomb's nor Rankine's theories explicitly incorporated the effect of cohesion in the lateral earth pressure computations. Bell (1952) modified Rankine's solution to include the effect of a backfill with cohesion. The derivation of Bell's equations for the active and passive earth pressure follows the same steps as were used in Section 4-3, *Developing Earth Pressures for Granular Soil*. The derivation is shown below.

For the cohesive soil, Figure 4-11 can be used to derive the relationship for the active and passive earth pressures.

Where: (for Equations 4-4-1 through 4-4-9)

σ_v	=	Vertical stress.
σ_a	=	Horizontal stress.
h	=	Height of pressure back of wall.
P_a	=	Active lateral earth pressure resultant per unit width of wall.
P_p	=	Passive lateral earth pressure resultant per unit width of wall.
ϕ	=	Effective friction angle of soil.
c	=	Effective soil cohesion.
K_a	=	Coefficient of active lateral earth pressure.
K_p	=	Coefficient of passive lateral earth pressure.
γ	=	Unit weight of soil.
h_{cr}	=	Height of the tension crack

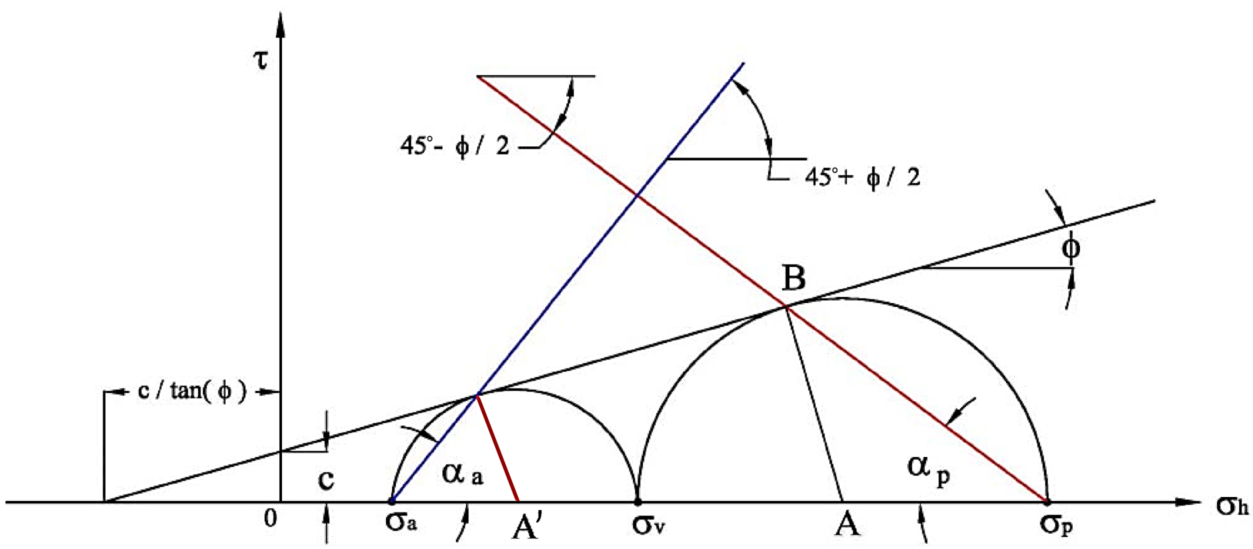


Figure 4-11. Mohr Circle Representation of Earth Pressure for Cohesive Backfill

For the active case:

$$\sin \phi = \frac{\frac{\sigma_v - \sigma_a}{2}}{\frac{\sigma_v + \sigma_a}{2} + \frac{c}{\tan \phi}} \tag{4-4-1}$$

Then,

$$\sigma_v \sin \phi + \sigma_a \sin \phi + 2c(\cos \phi) = \sigma_v - \sigma_a \tag{4-4-2}$$

Collecting terms:

$$\sigma_v(1-\sin\phi)=\sigma_a(1+\sin\phi)+2c(\cos\phi) \quad (4\text{-}4\text{-}3)$$

Solving for σ_a:

$$\sigma_a=\frac{\sigma_v(1-\sin\phi)}{(1+\sin\phi)}-\frac{2c(\cos\phi)}{(1+\sin\phi)} \quad (4\text{-}4\text{-}4)$$

Using the trigonometric identities from above:

$$\sigma_a=\sigma_v\tan^2\left(45^\circ-\frac{\phi}{2}\right)-2c\tan\left(45^\circ-\phi/2\right) \quad (4\text{-}4\text{-}5)$$

$$\sigma_a=\sigma_v K_a-2c\sqrt{K_a},\quad \text{where}\quad \sigma_v=\gamma z \quad (4\text{-}4\text{-}6)$$

For the passive case:

Solving for σ_p:

$$\sigma_p=\frac{\sigma_v(1+\sin\phi)}{(1-\sin\phi)}+\frac{2c(\cos\phi)}{(1-\sin\phi)} \quad (4\text{-}4\text{-}7)$$

$$\sigma_p=\sigma_v\tan^2\left(45^\circ+\frac{\phi}{2}\right)+2c\tan\left(45^\circ+\phi/2\right) \quad (4\text{-}4\text{-}8)$$

$$\sigma_p=\sigma_v K_p+2c\sqrt{K_p},\quad \text{where}\quad \sigma_v=\gamma z \quad (4\text{-}4\text{-}9)$$

Extreme caution is advised when using the cohesion value (**c**) to evaluate soil stresses. The evaluation of the stress induced by cohesive soils is highly uncertain due to the soil's sensitivity to shrinkage-swell, wet-dry, and degree of saturation. Tension cracks (gaps) can form, which may considerably alter the assumptions for the estimation of stress. The development of the tension cracks from the surface to depth, h_{cr}, is shown in Figure 4-12 and the depth of tension crack zone, h_{cr}, can be estimated by Equation 4-4-13.

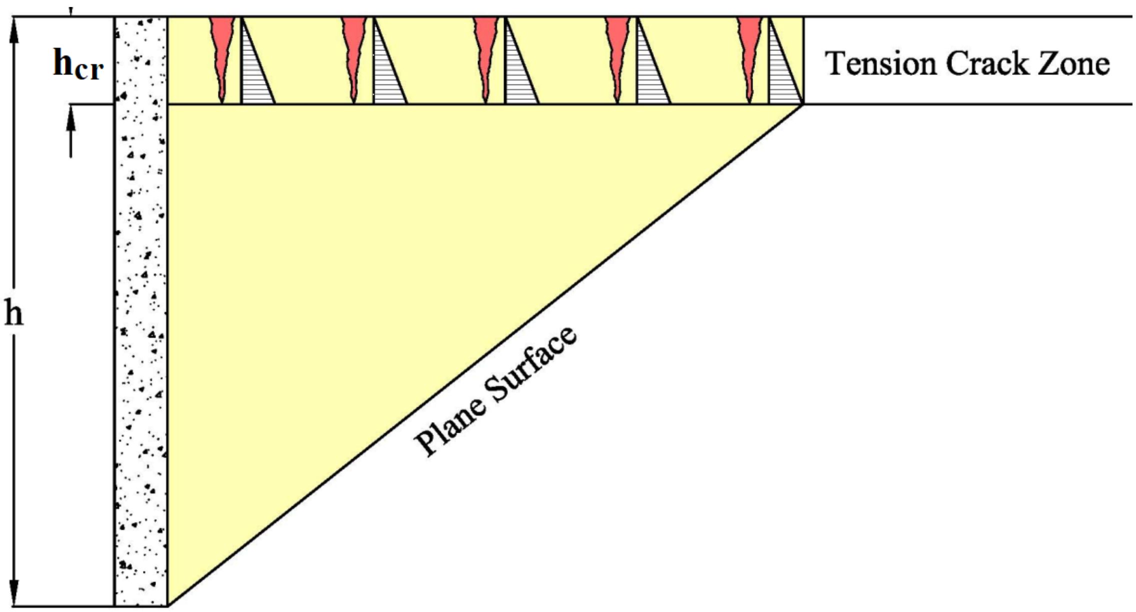


Figure 4-12. Tension Crack with Hydrostatic Water Pressure

As shown in Figure 4-13, the active earth pressure (**σ_a**) normal to the back of the wall at depth, **h**, is equal to:

$$\boldsymbol{\sigma_a = \gamma h K_a - 2c\sqrt{K_a}} \quad (4\text{-}4\text{-}10)$$

$$\boldsymbol{P_a = \frac{1}{2}\gamma h^2 K_a - 2c\sqrt{K_a}(h)} \quad (4\text{-}4\text{-}11)$$

According to Equation 4-4-10, the lateral stress (**σ_a**) at some point along the wall is equal to zero, therefore:

$$\boldsymbol{\gamma h K_a - 2c\sqrt{K_a} = 0} \quad (4\text{-}4\text{-}12)$$

$$\boldsymbol{h = h_{cr} = \frac{2c\sqrt{K_a}}{\gamma K_a}} \quad (4\text{-}4\text{-}13)$$

As shown in Figure 4-13, the passive earth pressure (**σ_p**) normal to the back of the wall at depth, **h**, is equal to:

$$\boldsymbol{\sigma_p = \gamma h K_p + 2c\sqrt{K_p}} \quad (4\text{-}4\text{-}14)$$

$$\boldsymbol{P_p = \frac{1}{2}\gamma h^2 K_p + 2c\sqrt{K_p}(h)} \quad (4\text{-}4\text{-}15)$$

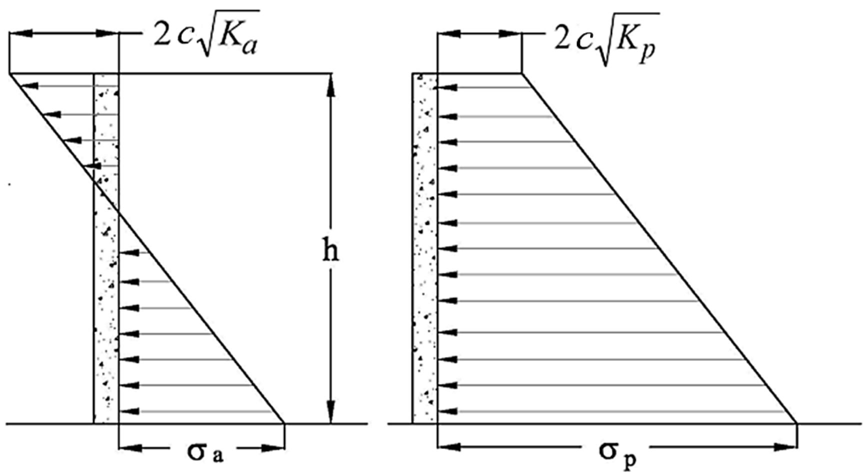


Figure 4-13. Cohesive Soil Active and Passive Earth Pressure Distribution

For shoring systems which support cohesive backfill, the height of the tension zone, $\mathbf{h_{cr}}$, should be ignored, as it is undeterminable when a crack develops, and the cohesive tension in the soil is no longer present. Thus, modified lateral earth pressure distribution acting along the entire wall height should be used (see Figure 4-14 below). The active lateral earth pressure (σ_a) at the base of the pressure diagram may be used provided its value is not less than the minimum $\mathbf{K_a}$ value of 0.25, times the effective vertical stress ($\sigma_v = \gamma \mathbf{h}$). The vertical particle stress has not changed ($\sigma_v = \gamma \mathbf{h}$), however, because cohesion shifts the pressure diagram over to the left, there is a lower horizontal stress seen by the shoring. Thus, a revised earth pressure coefficient can be calculated and then used for the pressure at any intermediate depth. This new value is referred to as the "apparent active earth pressure coefficient", $\mathbf{K_{apparent}}$. As seen in Equation 4-4-16, $\mathbf{K_{apparent}}$ must be greater than or equal to the established minimum of 0.25.

Any design based on a $\mathbf{K_{apparent}}$ lower than 0.25 must have justification that could include multiple laboratory tests verifying higher values for "**c**", an explanation of the time frame for the excavation, and an explanation of other conditions that may affect the "**c**" value while the shoring is in place.

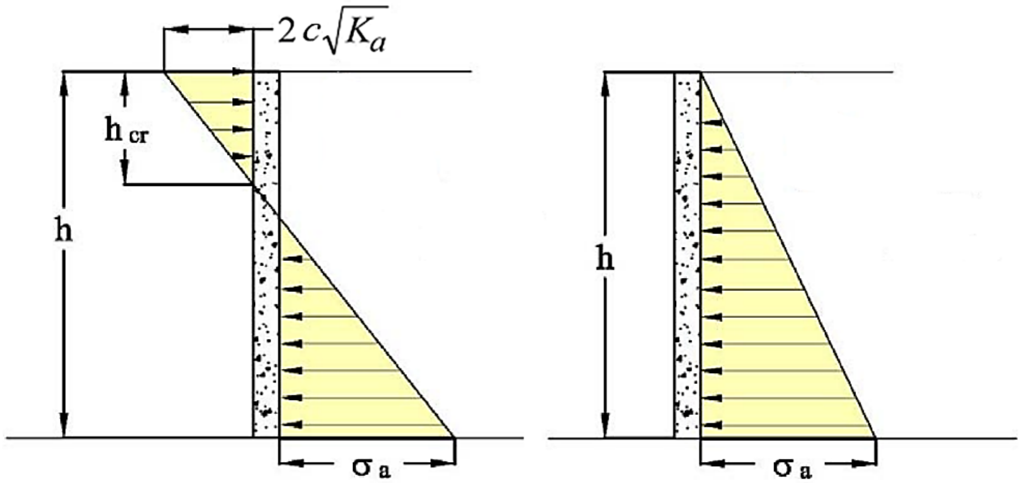


Figure 4-14. Pressure for K apparent

$$\mathbf{K_{apparent}} = \frac{\boldsymbol{\sigma_a}}{\boldsymbol{\gamma}\,\mathbf{h}} \geq \mathbf{0.25} \qquad (4\text{-}4\text{-}16)$$

The effect of surcharges and ground water is not included in the above figure. When water is present, the tension crack will fill with water, and the hydrostatic pressure needs to be considered.

Including the presence of water pressure within the tension zone, the pressure diagram is shown below, in Figure 4-15.

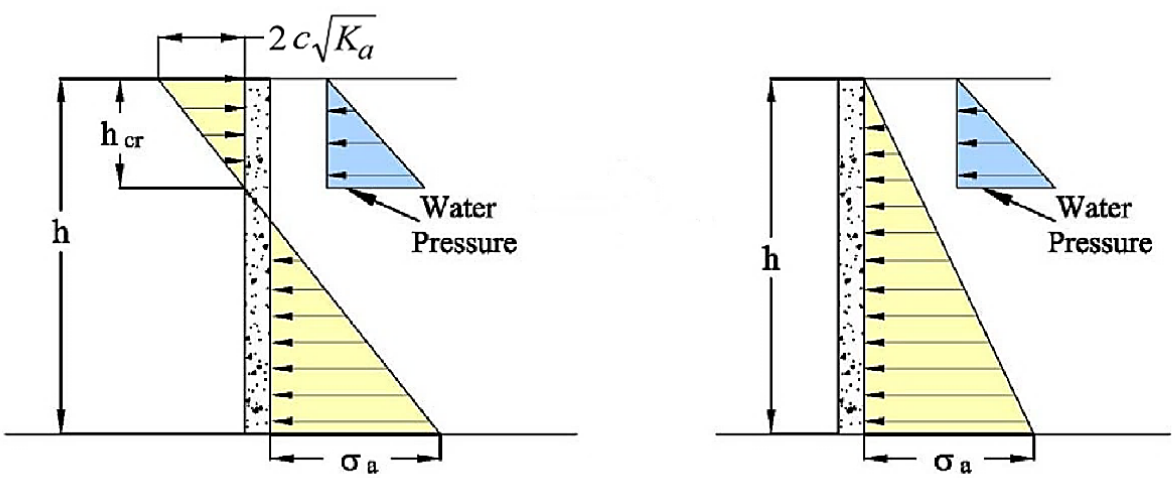


Figure 4-15. Pressure Diagram Depicting Hydrostatic Water Pressure in the Tension Crack Zone

4-5 Estimating Maximum Allowable Embankment Slope Angle

In nature, there are many stable slopes that have slope angles (β) that are larger than the angles of internal friction (ϕ). This is due to the presence of cohesion (**c**). None of the earth pressure theories will work when the slope angle β is larger than friction angle ϕ, even if the shoring system is to be installed in cohesive soil. Mohr Circle representation of the **C-ϕ** soil backfill, where the slope angle β is less than or equal to ϕ, is shown in Figure 4-16.

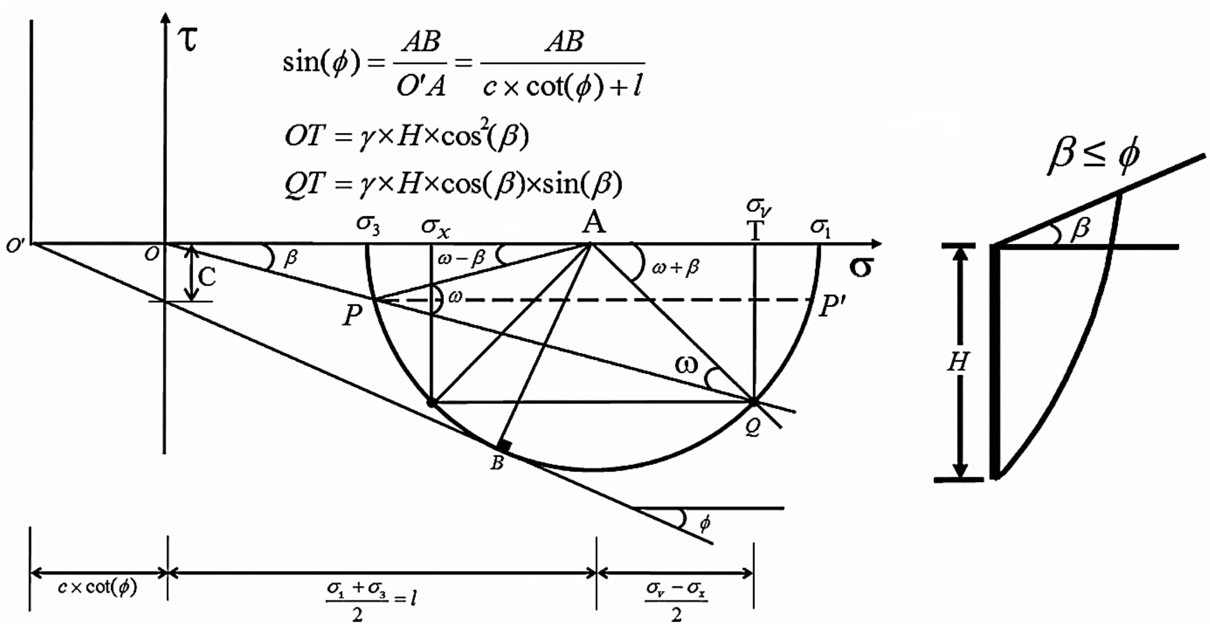


Figure 4-16. Mohr's Circles for Sloping Ground

The following equations are based on the *ASCE Journal of Geotechnical* and *Geoenvironmental Engineering* (February 1997) and are used to estimate the maximum allowable embankment slope angle for **C-ϕ** soil backfill.

$$\sin \beta \le \sin \phi + \frac{c}{l} \cos \phi \qquad (4\text{-}5\text{-}1)$$

Where:

$$l = \frac{1}{\cos^2 \phi}\left[\sigma_x + \frac{1}{2}\sin(2\phi) - \sqrt{\sigma_v(\cos^2 \beta - \cos^2 \phi) + \sigma_x[c \sin(2\phi)] + c^2\cos^2 \phi}\right]$$

(4-5-2)

$$\boldsymbol{\sigma_V = \gamma(H\cos\beta)} \tag{4-5-3}$$

$$\boldsymbol{\sigma_x = \gamma(H\cos^2\beta)} \tag{4-5-4}$$

The following sections outline various methods for analyzing shoring systems that have sloping ground conditions.

4-5.01 Active Trial Wedge Method

Figure 4-17 shows the assumptions used to determine the resultant active pressure for sloping ground with an irregular backfill condition applying the wedge theory. This is an iterative process. The failure plane angle (α_n) for the wedge varies until the maximum value of the active earth pressure is computed using Equation 4-5-5. The development of Equation 4-5-5 is based on the limiting equilibrium for a general soil wedge. It is assumed that the soil wedge moves downward along the failure surface and along the wall surface to mobilize the active wedge. This wedge is held in equilibrium by the resultant force equal to the resultant active pressure (**P_a**) acting on the face of the wall. Since the wedge moves downward along the face of the wall, this force acts with an assumed wall friction angle (δ) below the normal to the wall to oppose this movement.

For any assumed failure surface defined by angle α_n from the horizontal and the length of the failure surface **L_c**, the magnitude of the wedge weight (**W_n**) is the weight of the soil wedge plus the relevant surcharge load. For any failure wedge, the maximum value of **P_a** can be determined using Equation 4-5-5.

$$\mathbf{P_a} = \frac{W_n[\tan(\alpha-\phi)] - C_oL_c[\sin\alpha\tan(\alpha-\phi)+\cos\alpha] - C_aL_a[\tan(\alpha-\phi)\cos(-\omega)+\sin\omega]}{[1+\tan(\delta+\omega)\tan(\alpha-\phi)]\cos(\delta+\omega)}$$

(4-5-5)

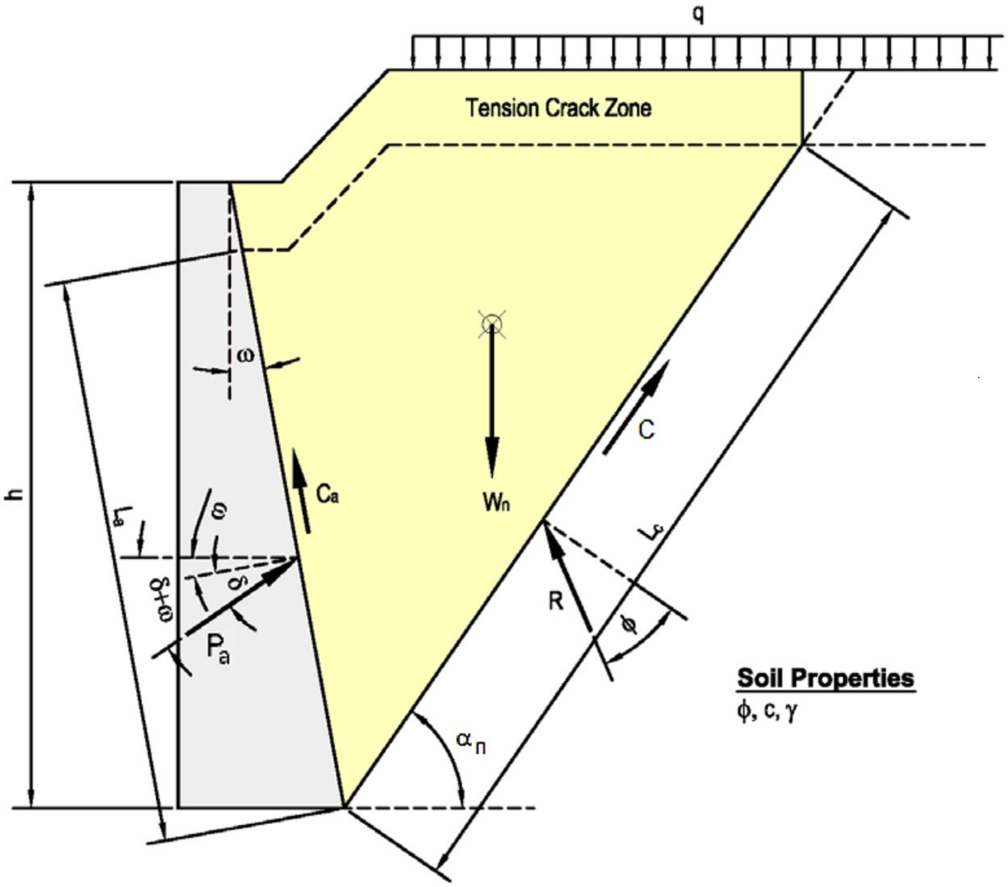


Figure 4-17. Active Trial Wedge

Where:

P_a	=	Active lateral earth pressure resultant per unit width of wall.
W_n	=	Weight of soil wedge plus the relevant surcharge loads.
δ	=	Friction angle between backfill material and back of wall.
ϕ	=	Effective friction angle of soil.
α_n	=	Failure plane angle with respect to horizontal.
C	=	Soil cohesion resultant force.
C_a	=	Wall-backfill adhesion resultant force.
L_c	=	Length of the failure plane on which cohesion acts.
L_a	=	Length of the active wedge along the backwall on which adhesion acts.

Similar to wall friction for granular soils, adhesion is the resistance to slippage along the interface of a wall and a cohesive soil. The textbook, *Foundation Analysis and Design* (4^{th} edition, 1988, by Joseph Bowles) discusses adhesion and places its value as 50 percent to 70 percent of the cohesion.

4-5.02 Passive Trial Wedge Method

Figure 4-18 shows the assumptions used to determine the resultant passive pressure for a broken back slope condition applying the trial wedge theory. Using the limiting equilibrium for a given wedge, Equation 4-5-6 calculates the passive earth pressure on a wall. The same iterative procedure is used as in the active case. However, the failure surface angle (α_n) is varied until the minimum value of passive pressure $\mathbf{P_p}$ is attained.

$$\mathbf{P_p} = \frac{W_n[\tan(\alpha+\phi)] + C_o L_c[\sin\alpha\tan(\alpha+\phi) + \cos\alpha] + C_a L_a[\tan(\alpha+\phi)\cos(-\omega) + \sin\omega]}{[1-\tan(\delta+\omega)\tan(\alpha+\phi)]\cos(\delta+\omega)}$$

(4-5-6)

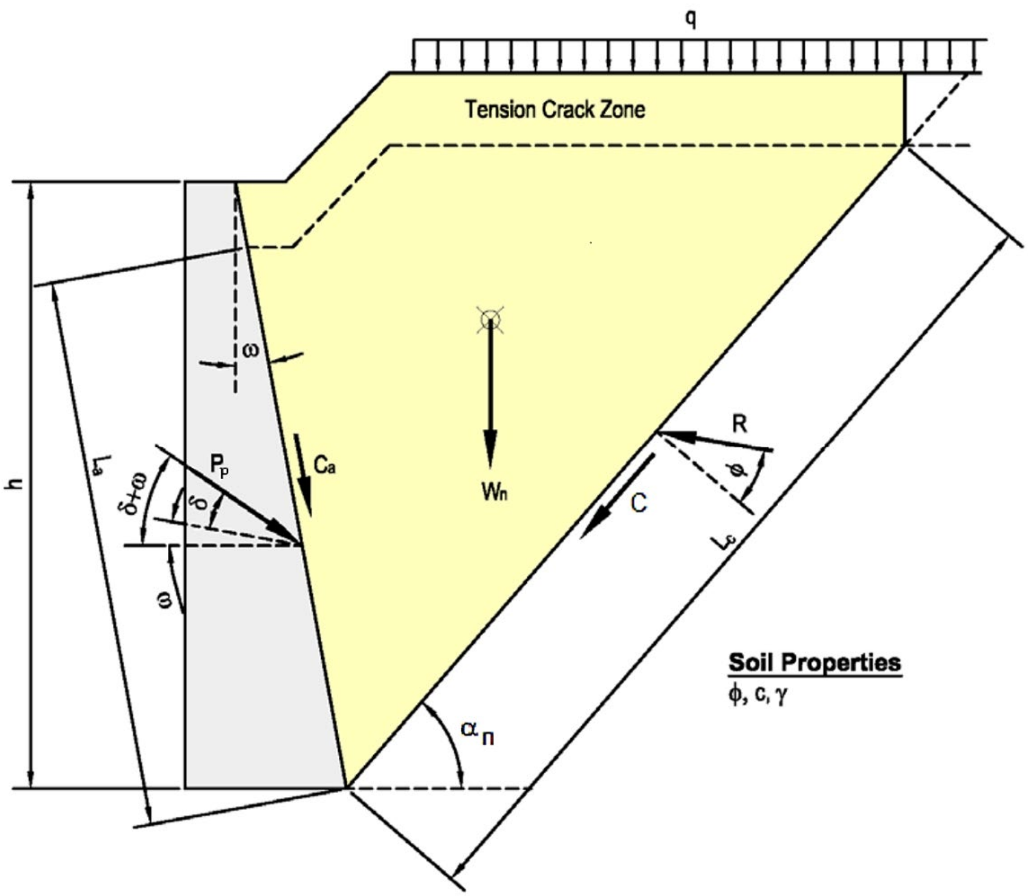


Figure 4-18. Passive Trial Wedge

Where:

- P_p = Passive lateral earth pressure resultant per unit width of wall.
- W_n = Weight of soil wedge plus the relevant surcharge loads.
- δ = Friction angle between backfill material and back of wall.
- ϕ = Effective friction angle of soil.
- α_n = Failure plane angle with respect to horizontal.
- C = Soil cohesion resultant force.
- C_a = Wall-backfill adhesion resultant force.
- L_c = Length of the failure plane on which cohesion acts.
- L_a = Length of the active wedge along the backwall on which adhesion acts.

4-5.03 Culmann's Graphical Solution for Active Earth Pressure

Culmann (1866) developed a convenient graphical solution procedure to calculate the active earth pressure for retaining walls for irregular backfill and surcharges. Figure 4-19 shows a failure wedge and a force polygon acting on the wedge. The forces per unit width of the wall to be considered for equilibrium of the wedge are as follows:

Where:

- c = Soil cohesion value
- K_a = Rankine active earth pressure coefficient
- ϕ = Soil friction angle (displayed as φ in some images)
- γ = Unit weight of soil

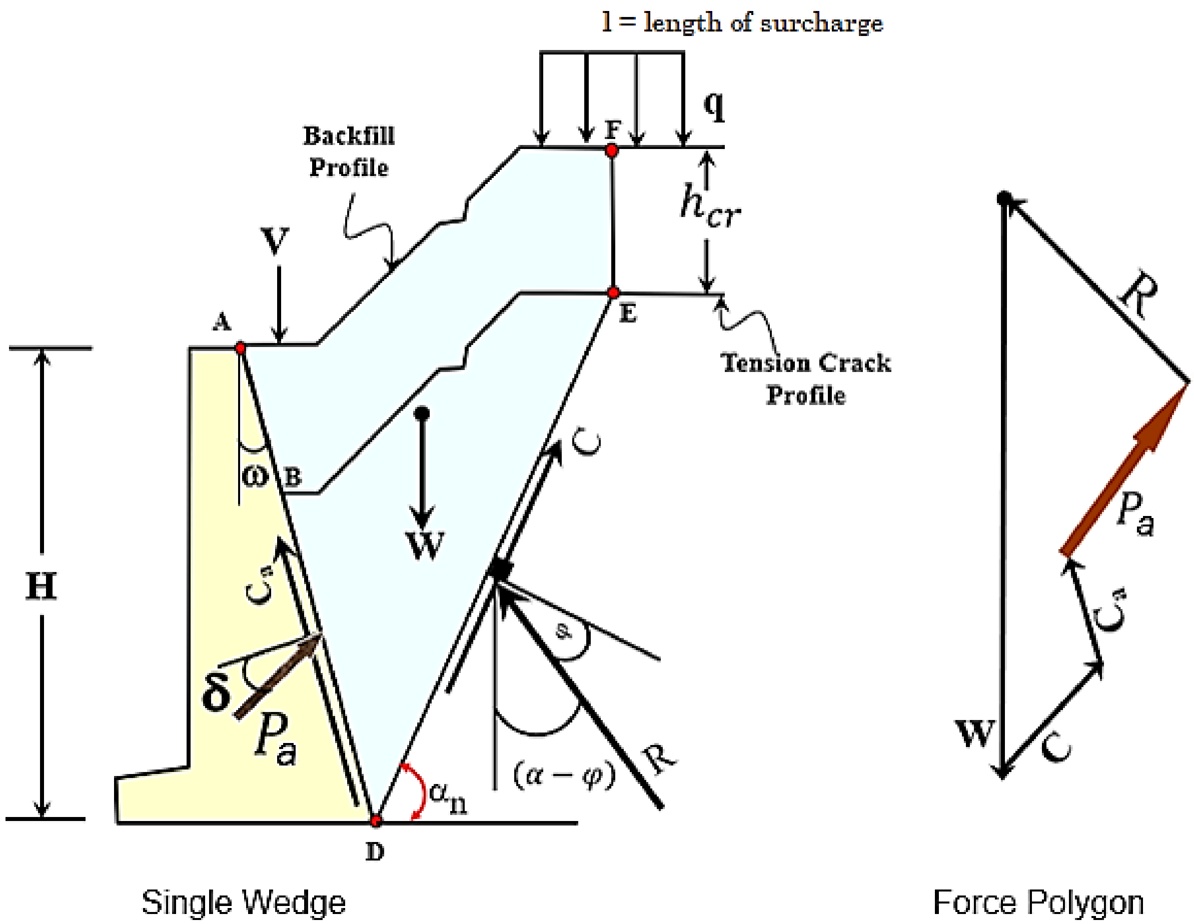


Figure 4-19. Single Wedge and Force Polygon

1. **W** = Weight of the wedge including weight of the tension crack zone and the surcharges with a known direction and magnitude.

$$\mathbf{W} = \mathbf{ABDEFA}_{\mathbf{area}}\ (\boldsymbol{\gamma}) + \mathbf{q}(\mathbf{1}) + \mathbf{V} \qquad (4\text{-}5\text{-}7)$$

2. **C**$_a$ = Adhesive force along the backfill of the wall with a known direction and magnitude.

$$\mathbf{C_a} = \mathbf{c}(\mathbf{BD}) \qquad (4\text{-}5\text{-}8)$$

3. **C** = Cohesive force along the failure surface with a known direction and magnitude.

$$\mathbf{C} = \mathbf{c}(\mathbf{DE}) \qquad (4\text{-}5\text{-}9)$$

4. **h_{cr}** = Height of the tension crack from Equation 4-4-13.

$$\mathbf{h_{cr}} = \frac{\mathbf{2c\sqrt{K_a}}}{\boldsymbol{\gamma}\mathbf{K_a}} \qquad (4\text{-}5\text{-}10)$$

5. **R** = Resultant of the shear and normal forces acting on the failure surface DE with the direction known only.

6. **P_a** = Active force of wedge with only the direction known.

To determine the maximum active force against a retaining wall, several trial wedges must be considered and the force polygons for all the wedges must be drawn to scale; this is illustrated in Appendix A, *Additional Theory For Inquiring Minds*, where further explanation of the Culmann's Graphical Solution for Active Earth Pressure is included.

4-6 Log-Spiral Passive Earth Pressure

Figure 4-3 from earlier in this chapter shows a simplified shoring system that has been sufficiently extended below the dredge line. The shoring system is stable when the active earth pressure developed on the high side of the wall is opposed by much higher passive earth pressure on the low side. It can be seen that the sliding surface for active earth pressure is practically a straight line, whereas a straight line cannot approximate the sliding surface for passive earth pressure. A detailed discussion on the nonlinear failure surfaces for passive pressures is included in Appendix A, *Additional Theory for Inquiring Minds*. A comparison between the "composite spiral-straight line" and the full Log-Spiral analysis is presented.

As mentioned in previous sections, Rankine's theory should not be used to calculate the passive earth pressure forces for a shoring system because it does not account for wall friction. While Coulomb's theory to determine the passive earth pressure force accounts for the angle of wall friction (δ), the theory assumes a linear failure surface. The result is an error in Coulomb's calculated force since the actual sliding surface is curved rather than planar. Coulomb's theory gives increasingly erroneous values of passive earth pressure as the wall friction (δ) increases. Therefore, Coulomb's theory could lead to unsafe shoring system designs because the calculated value of passive earth pressure would become higher than the soil could generate.

Terzaghi (1943) suggested that combining a logarithmic spiral and a straight line could represent the failure surface. Morrison and Ebeling (1995) suggested a single arc of the logarithmic spiral could realistically represent the failure surface.

Estimating the value of **K_p** for the shoring system in granular soils can be accomplished using the chart in Figure 4-20. This procedure requires that the values of δ, β, and ϕ are known. The failure surface is represented by a logarithmic spiral and a straight line. The procedure is shown below. For conditions that deviate from those described in Figure 4-20, the passive pressure may be calculated by using a trial procedure based on the trial wedge theory or a logarithmic spiral method.

1. Given δ, β, and ϕ.
2. Calculate ratios $^{\delta}/_{\phi}$ and $^{\beta}/_{\phi}$.
3. Determine initial $\mathbf{K_p}$ for $^{\beta}/_{\phi}$ from Figure 4-20.
4. Determine reduction factor **R** using the ratio of $^{\delta}/_{\phi}$.
5. Calculate final $\mathbf{K_p}$' = **R** x $\mathbf{K_p}$.

Example:

Wall Friction δ = 14° Slope above β = 27° Soil Friction ϕ = 32°

$^{\beta}/_{\phi}$ = 27/32 = 0.84

Enter the chart with the values of ϕ = 32° and $^{\beta}/_{\phi}$ = 0.84 to determine the initial $\mathbf{K_p}$.

Interpolating between the +0.8 and +1 lines, the initial value for $\mathbf{K_p}$ is approximately 19.

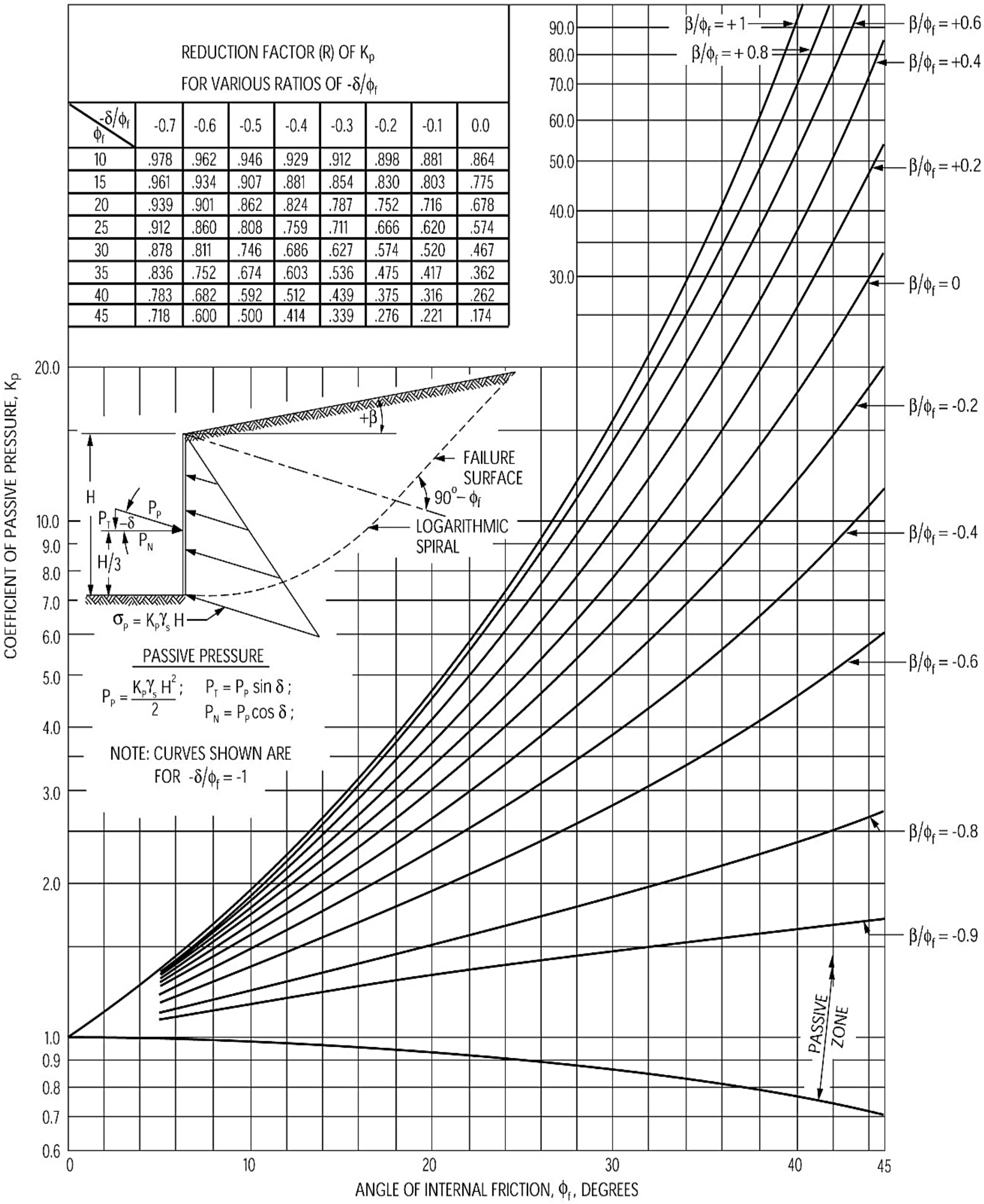
Use the table in the upper left portion of the chart to determine the reduction value, **R**, using $^{\delta}/_{\phi}$ = 0.44 and ϕ = 32°. Using interpolation, the value for **R** is 0.679.

	0.5	0.44	0.4
30	0.746	0.710	0.686
32	0.717	0.679	0.653
35	0.674	0.631	0.603

Matrix 4-1. Matrix for Interpolation of R

Multiply the initial value of $\mathbf{K_p}$ by the **R** value.

The $\mathbf{K_p}$ value used for your shoring check is $\mathbf{K_p}$' = $\mathbf{RK_p}$ = 0.679(19) = 13



ϕ_f \ $-\delta/\phi_f$	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
15	.961	.934	.907	.881	.854	.830	.803	.775
20	.939	.901	.862	.824	.787	.752	.716	.678
25	.912	.860	.808	.759	.711	.666	.620	.574
30	.878	.811	.746	.686	.627	.574	.520	.467
35	.836	.752	.674	.603	.536	.475	.417	.362
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

Figure 4-20. Passive Earth Pressure Coefficient (Caquot and Kerisel, 1948)