APPENDIX E – EXAMPLE CALCULATIONS

Example 1 – Continuous Two Span CIP Box-Girder Stressed from One End:

Information given on contract plans:
- 270 ksi low relaxation prestressing strand.
- \( E = 28,000 \text{ ksi} \).
- \( P_{\text{jack}} = 202.5 \text{ ksi} \).

The equation for stress in the prestressing steel at a distance \( x \) from the jacking end of the frame is:

\[
T_x = T_0 e^{-(\mu x + KL)} \quad \text{(Equation 1)}
\]

Where:
- \( \mu = 0.15 \) for frame lengths < 600 feet.
- \( K = 0.0002 \).
- \( \mu = \) Cumulative angle change at point of interest \( x \) from jacking end.
- \( L = \) Distance to point of interest \( x \) from jacking end.
Find the measurable elongation for the prestressing path in Figure 1:

![Diagram of vertical angle changes](image)

**Figure 2 – Vertical Angle Change.**

**Step 1:** Tendon elongations during the stressing operation are a function of both the average stress in the strands, and the length of the tendon. The stress in the strands vary along the tendon path due to angular friction between the tendon and the inside surface of the duct. Since there is no horizontal curvature given in this exercise, the angle changes are based on the vertical tendon profile only.

<table>
<thead>
<tr>
<th>Segment</th>
<th>$y$ (feet)</th>
<th>$L$ (feet)</th>
<th>$\alpha = 2(y/L)$ (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>2.500</td>
<td>64</td>
<td>0.0781</td>
</tr>
<tr>
<td>BC</td>
<td>3.333</td>
<td>80</td>
<td>0.0833</td>
</tr>
<tr>
<td>CD</td>
<td>0.666</td>
<td>16</td>
<td>0.0833</td>
</tr>
<tr>
<td>DE</td>
<td>0.666</td>
<td>14</td>
<td>0.0952</td>
</tr>
<tr>
<td>EF</td>
<td>3.333</td>
<td>70</td>
<td>0.0952</td>
</tr>
<tr>
<td>FG</td>
<td>2.500</td>
<td>56</td>
<td>0.0893</td>
</tr>
</tbody>
</table>
Step 2: Now that the vertical angle change within each parabolic segment has been calculated, it is time to compute the initial friction coefficients. These coefficients represent a decimal percentage of the jacking stress at the end of each parabolic segment. Based on the results in the following table, there is slightly more than 87 percent of $P_{j\text{ack}}$ in the strands at the dead end.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Friction $\mu$</th>
<th>$\alpha = 2(y/L)$ (radians)</th>
<th>$\Sigma \alpha$ (radians)</th>
<th>Wobble $K$</th>
<th>$L$ (feet)</th>
<th>$\Sigma L$ (feet)</th>
<th>$(\mu \Sigma \alpha + K \Sigma L)$</th>
<th>$e^{(\mu \Sigma \alpha + K \Sigma L)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.15</td>
<td>0.0781</td>
<td>0.0781</td>
<td>0.0002</td>
<td>64</td>
<td>64</td>
<td>0.0245</td>
<td>0.976</td>
</tr>
<tr>
<td>BC</td>
<td>0.15</td>
<td>0.0833</td>
<td>0.1614</td>
<td>0.0002</td>
<td>80</td>
<td>144</td>
<td>0.0530</td>
<td>0.948</td>
</tr>
<tr>
<td>CD</td>
<td>0.15</td>
<td>0.0833</td>
<td>0.2447</td>
<td>0.0002</td>
<td>16</td>
<td>160</td>
<td>0.0687</td>
<td>0.934</td>
</tr>
<tr>
<td>DE</td>
<td>0.15</td>
<td>0.0952</td>
<td>0.3399</td>
<td>0.0002</td>
<td>14</td>
<td>174</td>
<td>0.0858</td>
<td>0.918</td>
</tr>
<tr>
<td>EF</td>
<td>0.15</td>
<td>0.0952</td>
<td>0.4351</td>
<td>0.0002</td>
<td>70</td>
<td>244</td>
<td>0.1141</td>
<td>0.892</td>
</tr>
<tr>
<td>FG</td>
<td>0.15</td>
<td>0.0893</td>
<td>0.5244</td>
<td>0.0002</td>
<td>56</td>
<td>300</td>
<td>0.1387</td>
<td>0.870</td>
</tr>
</tbody>
</table>

Step 3: With the initial friction coefficients in hand, it is now possible to compute the average stress in the strands in each segment. Knowing the stress distribution along the entire length of the frame, and assuming a Young’s modulus for prestressing steel of $E = 28,000$ ksi, the tendon elongation can be calculated using the following equation:

$$\Delta = \frac{T_{\text{avg}} L}{E}$$

<table>
<thead>
<tr>
<th>Segment</th>
<th>$e^{-(\mu \alpha K L)}$</th>
<th>$T_o$ (ksi)</th>
<th>$T_o = T_e e^{(\mu \alpha K L)}$ (ksi)</th>
<th>$T_{\text{avg}}$ (ksi)</th>
<th>$L$ (feet)</th>
<th>$L$ (in)</th>
<th>$\Delta_e = T_{\text{avg}} L / E$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.976</td>
<td>202.5</td>
<td>197.6</td>
<td>200.1</td>
<td>64</td>
<td>768</td>
<td>5.49</td>
</tr>
<tr>
<td>BC</td>
<td>0.948</td>
<td>202.5</td>
<td>192.0</td>
<td>194.8</td>
<td>80</td>
<td>960</td>
<td>6.68</td>
</tr>
<tr>
<td>CD</td>
<td>0.934</td>
<td>202.5</td>
<td>189.1</td>
<td>190.6</td>
<td>16</td>
<td>192</td>
<td>1.31</td>
</tr>
<tr>
<td>DE</td>
<td>0.918</td>
<td>202.5</td>
<td>185.9</td>
<td>187.5</td>
<td>14</td>
<td>168</td>
<td>1.13</td>
</tr>
<tr>
<td>EF</td>
<td>0.892</td>
<td>202.5</td>
<td>180.6</td>
<td>183.3</td>
<td>70</td>
<td>840</td>
<td>5.50</td>
</tr>
<tr>
<td>FG</td>
<td>0.870</td>
<td>202.5</td>
<td>176.2</td>
<td>178.4</td>
<td>56</td>
<td>672</td>
<td>4.28</td>
</tr>
</tbody>
</table>

Total Elongation 24.39
Note that the length of the strand in the jack was not considered in the calculations. The total elongation calculated above must be reduced by 20 percent to account for take-up and reorienting of prestressing strand at the beginning of the stressing operation. The measurable elongation, \( \Delta_{80\%} \), for this example problem is shown below:

\[
\Delta_{80\%} = 0.80 \Delta_{100\%} = 0.80 \times 24.39 = 19.51
\]

\[\Delta_{80\%} = 19.51 \text{ in.}\]
Example 2 – Anchor Set Calculation:

The contract plans usually identify an anchor set length of $3/8$ inch (10 mm). This length represents the distance the strand slips back into the anchor head during the seating process. Using the results from Example 1, what is the change in stress at the jacking end of the frame, and how far into the frame does anchor set loss affect the stress in the tendon?

Given:

- $E = 28,000$ ksi
- $\Delta L = \frac{3}{8}$ in.
- Friction loss in length $L = 202.5$ ksi – $192.0$ ksi = $10.5$ ksi = $d$

\[
x = \sqrt{\frac{E(\Delta L)L}{d}} = \sqrt{\frac{(28,000 ksi)(0.375 in)(144 ft)}{(10.5 ksi)(12 in / ft)}} = 109.5 ft
\]

\[
\Delta f = \frac{2dx}{L} = \frac{(2)(10.5 ksi)(109.5 ft)}{144 ft} = 15.97 ksi
\]

The stress at the anchorage after seating must be less than $0.70f'_s$:

\[
\{202.5 \text{ ksi} – 15.97 \text{ ksi} = 186.53 \text{ ksi}\} < \{0.70f'_s = 0.70 (270 \text{ ksi}) = 189 \text{ ksi}\} \therefore \text{OK}
\]
Example 3 – Simple Span Box Girder Stressed from One End:

Given:
- 140 ft long simply supported CIP P/S Box Girder = $L$.
- 270 ksi low relaxation strand.
- $P_{jack} = 12,600$ kips.
- Area of 0.5 inch diameter strand = $0.153$ in$^2$.
- Anchor set length = $0.375$ in = $\Delta L$
- One end stressing.
- $\mu = 0.15$, $K = 0.0002$

Find:
1. How many 0.5 inch diameter strands are required?
2. Find the initial and final stress distribution in the prestressing steel.
3. Find the final working force at the centerline of the span.
4. Find the theoretical and measurable elongation.

Part 1: Number of strands required:

\[ 0.75 f_s = \text{The jacking stress on the contract plans} = 0.75 \times (270 \text{ ksi}) = 202.5 \text{ ksi} \]
\[ A_{p/s} = \frac{P_{jack}}{f_{jack}} = \frac{12,600 \text{ kips}}{202.5 \text{ ksi}} = 62.22 \text{ in}^2 \]
\[ n_{p/s} = \text{number of strands} = \frac{A_{p/s}}{A_{strangd}} = \frac{62.22 \text{ in}^2}{0.153 \text{ in}^2} = 407 \text{ strands} \]
Part 2: Initial and Final Stresses in Prestressing Steel:

Stress at dead end:

\[ \alpha_{ab} = \alpha_{bc} = 2\frac{y}{L} = 2\frac{2.5}{70} = 0.0714 \]

At dead end, \( \alpha_{ac} = 2 \cdot 0.0714 = 0.1428 \)

\[ T_x = T_{oe}e^{-\left(\mu\alpha+KL\right)} \]

At dead end, \( T_c = 202.5 \cdot e^{-\left[(0.15)(0.1428)+(0.0002)(140)\right]} = 202.5 \cdot e^{-0.0494} = 192.73 \text{ ksi} \)

Effect of anchor set:

\[ x = \sqrt{\frac{E(\Delta L) L}{d}} = \sqrt{\frac{(28,000 \text{ ksi})(0.375 \text{ in})(140 \text{ ft})}{(9.77 \text{ ksi})(12 \text{ in} / \text{ ft})}} = 112 \text{ ft} \]

\[ \Delta f = \frac{2dx}{L} = \frac{(2)(9.77 \text{ ksi})(112 \text{ ft})}{140 \text{ ft}} = 15.63 \text{ ksi} \]
Stress at jacking end:

\[ T_a = f_{jack} - \Delta f = 202.5 \text{ ksi} - 15.63 \text{ ksi} = 186.87 \text{ ksi} \]

\[ \{ T_a = 186.87 \text{ ksi} \} \leq \{ 0.70 f_s' = 0.70 (270 \text{ ksi}) = 189 \text{ ksi} \} \therefore \text{OK} \]

Assume long term losses = 20 ksi

---

**Part 3:** Final working force at the centerline of span.

\[ f_b - 166.87 = \frac{174.68 - 166.87}{112} \]

\[ f_b - 166.87 = \frac{70(7.81)}{112} \]

\[ f_b = 4.88 + 166.87 = 171.75 \text{ ksi} \]

**Part 4:** Theoretical and measurable elongation.

\[ \Delta_{100\%} = \left[ \frac{(202.5 + 192.73) \text{ ksi}}{2} \right] \left( \frac{140 \text{ ft}}{12 \text{ in}} \right) \left( \frac{12 \text{ in}}{ft} \right) = 11.86 \text{ in.} \]

\[ \Delta_{80\%} = 0.80(11.86 \text{ in}) = 9.49 \text{ in.} \]
Example 4 – Continuous Four Span CIP Box-Girder Stressed from Both Ends

Given:
- 818 ft long continuous 4 span CIP P/S box girder frame.
- Two end stressing, with first stage jacked from left end.
- 270 ksi Low Relaxation strand.
- Jacking stress = 202.5 ksi.
- Area of 0.5 inch diameter strand = 0.153 in².
- The initial force coefficient \( FC_i \) at the point of no movement = 0.802.
- \( \mu = 0.20, \ K = 0.0002 \) (informational only).
Find: What is the total theoretical (expected) 1\textsuperscript{st} stage elongation?

1. What is the measurable 1\textsuperscript{st} stage elongation?
2. What is the theoretical (expected) 2\textsuperscript{nd} stage elongation?

**Part 1:** Total Theoretical (expected) 1\textsuperscript{st} Stage Elongation:

When calculating the first stage elongation, it is common practice to break the force coefficient diagram into two parts, identified as areas A and B in the above diagram. The equation for calculating tendon elongations is shown as follows:

$$\Delta = \frac{PL}{AE}$$

When jacking to 202.5 ksi, and using a strand nominal area of 0.153 in\textsuperscript{2} the jacking force per strand is calculated below:

$$P_{strand} = (202.5 \text{ ksi})(0.153) = 30.98 \text{ kips/strand}$$

When calculating $\Delta_A$, it is important to include the length of tendon within the jack. The strand movement will be measured relative to the end of the ram, which generally results in 2½ to 3 feet of extra strand within the length of the jack.

$$\Delta_A = \frac{(30.98 \text{ kips})(1 + 0.802)}{2} \times \frac{(416 \text{ ft} + 3 \text{ ft})(12)}{(0.153 \text{ in}^2)(28,500 \text{ ksi})} = (27.91)(1.153) = 32.2 \text{ inches}$$

In order to find the first stage elongation for area B, it is necessary to extrapolate the $FC_i$ out to the dead end of the first stage post tensioning:

$$FC_{i_{dead}} = 1 - [(2)(1 - 0.802)] = 0.604$$

$$\Delta_B = \frac{(30.98 \text{ kips})(0.802 + 0.604)}{2} \times \frac{(402 \text{ ft})(12)}{(0.153 \text{ in}^2)(28,500 \text{ ksi})} = (21.78)(1.106) = 24.1 \text{ inches}$$

$$\Delta_{A+B} = \Delta_{1st \text{ stage \_theo}} = 32.2 + 24.1 = 56.3 \text{ inches}$$

**Part 2:** Measurable 1\textsuperscript{st} Stage Elongation:

The total theoretical elongation does not have direct practical application because it does not take into account slack or strand reorientation in the tendon. The measurable elongation is determined to be 80\% of the theoretical, as strands are marked with paint after being stressed to 20\% of $P_{jack}$. In this case, after stressing the tendon to 20\% of $P_{jack}$, the remaining 80\% stressing should yield an elongation of:
\[ \Delta_{\text{1st stage meas}} = (0.80)\Delta_{\text{1st stage theo}} = (0.80)(56.3 \text{ inches}) = 45.0 \text{ inches} \]

**Part 3:** Theoretical (expected) 2\(^{nd}\) Stage Elongation:

Once the first stage stressing operation is complete, and the Engineer is satisfied with the physical measurements obtained, the second stage stressing operation can begin. Theoretical 2\(^{nd}\) stage elongations must be calculated before stressing, to serve as a tool to guarantee that the proper amount of P/S force is being delivered to the structure. The second stage elongation equates to Area C in the force coefficient diagram. Again, the length of the tendon within the jack must be included in the calculation.

\[
\Delta_c = \Delta_{\text{2nd stage theo}} = \frac{(30.98 \text{ kips})(1 - 0.604)}{2} \times \frac{(402 \text{ ft} + 3 \text{ ft})(12)}{(0.153 \text{ in}^2)(28,500 \text{ ksi})} = (6.13)(1.115) = 6.8 \text{ inches}
\]