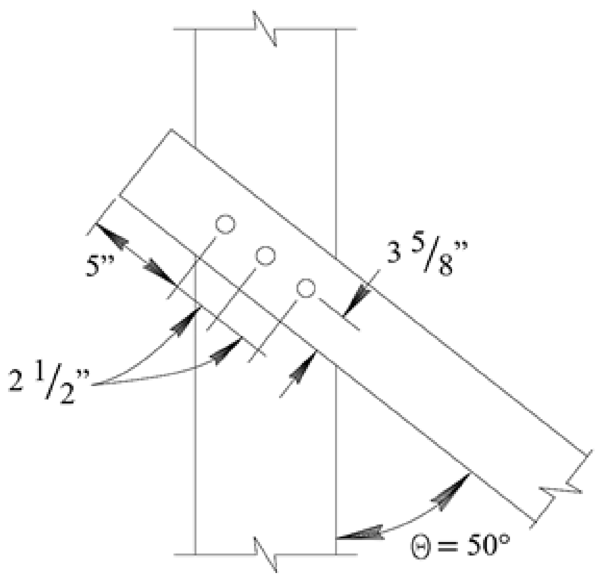
Appendix D Example 12 – Multiple Fastener Connection - Single Shear

Refer to *Falsework Manual,* Section 5-3, *Timber Fasteners*. This example demonstrates how to calculate the capacity of a multiple fastener connection between a single diagonal brace and post. For this example, 2 % dead load is the governing load.

Given Information



Posts:
12 x 12 Rough Douglas Fir-Larch #1 (G=0.50)

Diagonal Braces:
2x8 S4S Douglas Fir-Larch #2 (G=0.50)

Connectors:
Three 5/8" Ø Bolts in a single row
Center of gravity of the bolt group coincides with the center of gravity of the members.

Figure D-12-1. Post and Brace with Multiple Fasteners

Determine the connection capacity between brace and post for 2% Dead Load

Main Member Properties

l_m = 12 in	*thickness (12x12)*
$t_m = l_m$ = 12 in	
θ_m= 50°	*angle between direction of loading & direction of grain*
E_m = 1600000 psi	*modulus of elasticity NDS Table 4D*
G = 0.50	*Specific Gravity NDS Table 12.3.3*

Side Member Properties

l_s = 1.5 in	*thickness (2x8)*
$t_s = l_s$ = 1.5 in	
θ_s= 0°	*angle between direction of loading & direction of grain*
E_s = 1600000 psi	*Modulus of elasticity NDS Table 4A*

Connector Properties

D = 0.625 in	*connector diameter*
n = 3	*number of fasteners*
F_{yb} = 45000 psi	*Yield Strength (NDS table 12A, footnote 2)*
$F_{e.pll}$ = 11200G psi = 5600 psi	*Dowel Bearing Strength Parallel to Grain (NDS table 12.3.3 footnote 2)*

$$F_{e.perp} = \frac{6100G^{1.45}}{\sqrt{\frac{D}{in}}} = 2824 \text{ psi}$$

Dowel Bearing Strength Perpendicular to Grain (NDS table 12.3.3 footnote 2)

Compare values to NDS Table 12.3.3:

$F_{e.pll\ (NDS\ Table\ 12.3.3)}$ = 5600 psi

$F_{e.perp\ (NDS\ Table\ 12.3.3)}$ = 2800 psi

Use calculated value for F_{perp} = 2824 psi

Find Dowel Bearing Strength at an Angle to Grain (NDS Section 12.3.4):

$$F_{em} = \frac{F_{e.pll}F_{perp}}{F_{e.pll}(\sin(\theta_m))^2 + F_{perp}(\cos(\theta_m))^2} = 3551 \text{ psi}$$

$$F_{es} = \frac{F_{e.pll}F_{perp}}{F_{e.pll}(\sin(\theta_s))^2 + F_{perp}(\cos(\theta_s))^2} = 5600 \text{ psi}$$

Find Reduction Term, R_d (NDS Table 12.3.1B):

$\theta = \max(\theta_m, \theta_s) = 50°$ — *Maximum angle between direction of load and direction of grain for any member in connection (See Table 12.3.1B)*

$$K_\theta = 1 + 0.25\frac{\theta}{90\ \text{deg}} = 1.139$$

$R_{d\_I} = 4\ K_\theta = 4.56$ — *Reduction Term for Yield Mode I_m and I_s*

$R_{d\_II} = 3.6\ K_\theta = 4.10$ — *Reduction Term for Yield Mode II*

$R_{d\_III.IV} = 3.2\ K_\theta = 3.64$ — *Reduction Term for Yield Mode III_m, III_s, and IV*

Find Yield Limit Equations for Single Shear (NDS Table 12.3.1A):

$$R_e = \frac{F_{em}}{F_{es}} = 0.6341$$

$$R_t = \frac{l_m}{l_s} = 8$$

$$k_1 = \frac{\sqrt{R_e + 2R_e^2\left(1 + R_t + R_t^2\right) + R_t^2 R_e^3} - R_e(1 + R_t)}{(1 + R_e)} = 1.8305$$

$$k_2 = -1 + \sqrt{2(1 + R_e) + \frac{2F_{yb}(1 + 2R_e)D^2}{3F_{em}l_m^2}} = 0.8221$$

$$k_3 = -1 + \sqrt{\frac{2(1 + R_e)}{R_e} + \frac{2F_{yb}(2 + R_e)D^2}{3F_{em}l_s^2}} = 2.0029$$

$$Z_{Im} = \frac{Dl_mF_{em}}{R_{d\_I}} = 5846\ \text{lb}$$ *NDS Eqn 12.3-1*

$$Z_{Is} = \frac{Dl_sF_{es}}{R_{d\_I}} = 1152\ \text{lb}$$ *NDS Eqn 12.3-2*

$$Z_{II} = \frac{k_1Dl_sF_{es}}{R_{d\_II}} = 2344\ \text{lb}$$ *NDS Eqn 12.3-3*

$$Z_{IIIm} = \frac{k_2Dl_mF_{em}}{(1 + 2R_e)R_{d\_III.IV}} = 2649\ \text{lb}$$ *NDS Eqn 12.3-4*

$$Z_{IIIs} = \frac{k_3Dl_sF_{em}}{(2 + R_e)R_{d\_III.IV}} = 695\ \text{lb}$$ *NDS Eqn 12.3-5*

$$Z_{IV} = \frac{D^2}{R_{d\_III.IV}}\sqrt{\frac{2F_{em}F_{yb}}{3(1 + R_e)}} = 865 \text{ lb}$$ *NDS Eqn 12.3-6*

The controlling value is the minimum single shear capacity from the above equations.

$Z_{control} = \min(Z_{Im}, Z_{Is}, Z_{II}, Z_{IIIm}, Z_{IIIs}, Z_{IV}) = 695$ lb *(Yield Mode IIIs controls)*

<u>Find Adjusted Lateral Design Value, Z':</u>

Adjustment factors from NDS Table 11.3.1:

$C_D = 1.25$ *Duration Factor for 2% lateral loading*

$C_M = 1.0$ *Wet Service Factor NDS 11.3.3 (Assume < 19% moisture content)*

$C_t = 1.0$ *Temperature Factor NDS 11.3.4 (Temp up to 100ºF)*

$C_{eg} = 1.0$ *End Grain Factor NDS 12.5.2 (Does not apply)*

$C_{di} = 1.0$ *Diaphragm Factor NDS 12.5.3 (Does not apply)*

$C_{tn} = 1.0$ *Toe Nail Factor NDS 12.5.4 (Does not apply)*

<u>Find the Group Action Factor C_g (NDS Section 11.3.6)</u>:

The Group Action Factor, Cg, accounts for load distribution within a fastener group.

$$C_g = \left[\frac{m(1 - m^{2n})}{n\left[(1 + R_{EA}m^n)(1 + m) - 1 + m^{2n}\right]}\right]\left(\frac{1 + R_{EA}}{1 - m}\right) = 0.98$$ *Group Action Factor NDS Eqn. 11.3-1*

where:

n	$= 3$	*Number of fasteners in a row*
A_m	$= t_m^2 = 144$ in^2	*Area of post*
A_s	$= t_s$ x brace width $= 10.88$ in^2	*Area of brace*
E_m	$= 1600000$ psi	*Modulus of elasticity NDS Table 4D*
E_s	$= 1600000$ psi	*Modulus of elasticity NDS Table 4A*
R_{EA}	$= \min\left(\frac{E_s A_s}{E_m A_m}, \frac{E_m A_m}{E_s A_s}\right) = 0.08$	
D	$= 0.625$ in	*connector diameter*

γ	$= 180000 \frac{lb}{in}\left(\frac{D}{in}\right)^{1.5} = 88939 \frac{lb}{in}$	*Load/Slip modulus for connection Dowel-type fasteners in wood-to-wood connections*
s_{bolt}	$= spacing_{in.a.row\_actual} = 2.5$ in	*Center to center spacing between adjacent fasteners in a row*
u	$= 1 + \gamma \frac{s_{bolt}}{2}\left(\frac{1}{E_m A_m} + \frac{1}{E_s A_s}\right) = 1.007$	
m	$= u - \sqrt{u^2 - 1} = 0.888$	

Find the Geometry Factor C_Δ (NDS Section 12.5.1):

The Geometry Factor, C_Δ, is based on the end distance, edge distance and spacing of the dowel-type fasteners. For $C_\Delta = 1.0$, the following requirements must be met:

1. End Distance Requirements (NDS Table 12.5.1A):

   For softwood (DF-L) with the force acting Parallel to Grain in Tension, for $C_{\Delta\ end} = 1.0$, the minimum end distance must be 7D.

   $dist_{end} = 7D = 7(0.625\ in) = 4.38$ in
   $dist_{end\_actual} = 5$ in

   $$dist_{end} < dist_{end\_actual} \therefore C_{\Delta\ end} = 1.0$$

   Note: If $dist_{end\_actual}$ was between the minimum end distances for $C_{\Delta\ end} = 0.5$ and 1.0, $C_{\Delta\ end}$ would be determined as follows:

   $$C_{\Delta\ end} = \frac{dist_{end\_actual}}{\text{minimum end distance for } C_{\Delta\ end}} = 1.0$$

2. Shear area requirements (NDS Section 12.5.1(b)):

   In this case, the dowel-type fastener is not being loaded at an angle as shown in NDS Figure 12E. Therefore, the shear area geometry factor is $C_{\Delta\ shear\_area} = 1.0$.

3. Spacing Requirements for Fasteners in a Row (NDS Table 12.5.1B):

   Similar to the end distance requirements, the brace member is loaded parallel to grain. According to NDS Table 12.5.1B, the minimum spacing between fasteners in a row for $C_{\Delta\ in.a.row} = 1.0$ is 4D.

$spacing_{in.a.row}$ = 4D = 4(0.625 in) = 2.5 in

$spacing_{in.a.row\_actual}$ = 2.5 in

$$spacing_{in.a.row} = spacing_{in.a.row\_actual} \therefore C_{\Delta\ in.a.row} = 1.0$$

4. Edge Distance Requirements (NDS Table 12.5.1(c)):

   The edge distance requirement is determined by $\frac{l_s}{D}$ or $\frac{l_m}{D}$, whichever is smaller. For this case, $\frac{l_s}{D}$ is the smaller ratio. For the parallel to grain loading on the brace:

   $\frac{l_s}{D} = 2.4 \le 6$ ➔ the minimum edge distance is 1.5D

   $dist_{edge}$ = 1.5D = 1.5(.625 in) = 0.94 in

   $dist_{edge.\ \_actual}$ = 3.625 in

$$dist_{edge} < dist_{edge\_actual} \therefore C_{\Delta\ edge} = 1.0$$

5. Spacing Requirements Between Rows (NDS Table 12.5.1D):

   Since there is only one row of bolts, this condition does not apply.

$$C_{\Delta\ row} = 1.0$$

The Geometry Factor is the minimum factor of all the conditions.

$C_{\Delta} = \min(C_{\Delta\ end}, C_{\Delta\ shear\_area}, C_{\Delta\ in.a.row}, C_{\Delta\ edge}, C_{\Delta\ row}) = 1.0$

Adjusted lateral design value Z’

$Z' = nZ_{control}(C_D)(C_M)(C_t)(C_g)(C_{\Delta})$ = 3(695 lb)(1.25)(1.0)(1.0)(0.98)(1.0) = 2554 lb