

11-34 HINGE CURL

Introduction

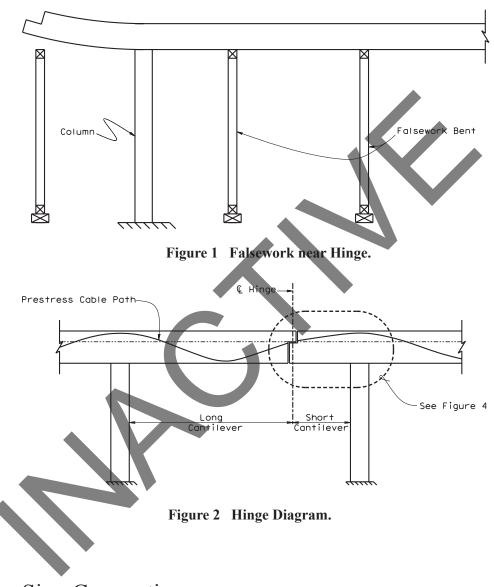
This memo addresses the deformation behavior of in-span hinges for cast-in-place prestressed concrete box girder bridges. This behavior consists of the upward deflection of the unloaded short cantilever of the box girder bridge, as well as the downward deflection of the short cantilever when it is loaded by the long cantilever. This deformation behavior is commonly referred to as "hinge curl".

The designer is reminded that there is a variable period of time, usually between 30 and 180 days, in which the short cantilever remains unloaded after it has been stressed. Experience indicates that the duration over which the prestressed short cantilever is left unloaded influences the final location of the hinge. In general, shorter durations would produce a final deflection that is downward from its initial formed location, while longer durations may result in a final deflection that is upward from its initial formed location. This period of time and the extent of the deformation is not known until the contractor's schedule is finalized. Therefore, a table of time dependent camber values is typically provided as part of the contract plans.

The procedure and calculations for estimating hinge curl are presented in this memo, and it simplifies a complicated analysis process by using deflection factors instead of considering the time dependent changes in concrete modulus of elasticity, creep, shrinkage, and steel relaxation. The procedure described in this memo provides the design engineer with a method for predicting the "hinge curl" deflections in order to provide the associated camber values on the contract plans. The procedure assumes that falsework will remain in the adjacent spans until load transfer to the hinge takes place (Figure 1). This assumption is based on the requirements of the standard specifications. It is worth noting that after the falsework is removed, the top of columns may rotate due to unbalanced spans. This effect is accounted for using the procedures presented in this memo.

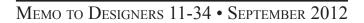
Should it be desirable to remove falsework prior to load transfer, the procedure presented in this memo should not be used. Instead, consideration should be given to either tying down the short cantilever or producing the camber values using a time dependent analysis program. Both alternatives are beyond the scope of this memo.





Sign Convention

It is important to note that deflection and camber carry opposite sign conventions. Specifically, a downward deflection is considered positive, and corresponds to a positive camber in the upward direction. Positive camber requires setting screed line elevations higher than profile grade.





Method of Calculation

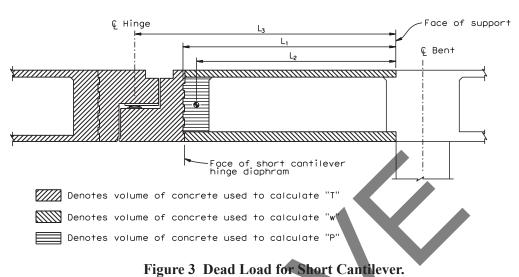
1. Approximate the deflection of the short cantilever, in inches, at the centerline of hinge due to dead load (See Figure 3).

$$\Delta_{DL} = \underbrace{\frac{wL_1^3}{24EI}(4L_3 - L_1)}_{(1)} + \underbrace{\frac{PL_2^2}{6EI}(3L_3 - L_2)}_{(2)}$$

Where,

- (1) Deflection of short cantilever due to self weight (in.)
- (2) Deflection of short cantilever due to weight of short cantilever portion of hinge diaphragm (in.)
 - w = Uniform self weight of the prismatic section of the short cantilever (kips/in.)
 - P = Weight of the portion of the hinge diaphragm that fills the voids of the prismatic section; short cantilever side only (kips)
 - L_1 = Length of short cantilever measured from the end of the hinge diaphragm to the face of support (in.)
 - L_2 = Length of short cantilever measured from the face of support to the centroid of the short cantilever hinge diaphragm (in.)
 - L_3 = Length of short cantilever measured from the face of support to the centerline of the hinge (in.)
 - = Concrete modulus of elasticity (ksi)
 - Average moment of inertia of short cantilever span (in⁴.)





2. Approximate the deflection of the short cantilever, in inches, at the centerline of hinge due to prestress force.

$$\Delta_{PS} = \frac{-P_j L_1 FC}{12EI} \Big[e_1 (8L_3 - 3L_1) + e_2 (4L_3 - 3L_1) \Big]$$

Where,

- P_i = Design jacking force (kips)
- FC = Average initial force coefficient at time of stressing that adjusts for anchor set, tendon friction a wobble in the short cantilever (unitless)
- e_1 Eccentricity at centerline of bent, positive up (in.). See Figure 4
- e_2 = Eccentricity at anchorage in hinge diaphragm, positive up (in.). See Figure 4

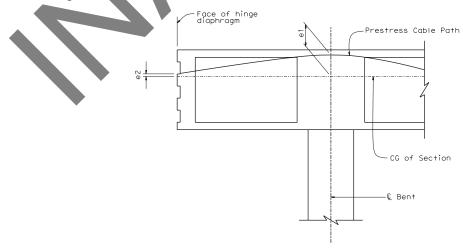


Figure 4 Prestress Cable Path.



3. Approximate the deflection of the short cantilever due to load transfer from the long cantilever (See Figure 3).

$$\Delta_{reaction} = \frac{TL_3^3}{3EI}$$

Where,

- T = Transfer load from long cantilever; dead load and prestress only (kips). "T" includes the weight of the cast-in-place hinge. The transfer load may be estimated from the longitudinal model as the shear demand at the face of the short cantilever hinge diaphragm.
- 4. Calculate Δ_{curl} .

$$\Delta_{curl} = \Delta_{DL} + \Delta_{PS}$$

5. Calculate Adjustment "SC" and Adjustment "LC" using the following formulas:

Adjustment "SC" is the profile adjustment required for the Short Cantilever (may be positive or negative value).

0 day value = $3.00\Delta_{reaction} + 3.00\Delta_{curl}$ (theoretical; included here for illustrative purposes only.) 30 day value = $2.60\Delta_{reaction} + 3.00\Delta_{curl}$ 60 day value = $2.20\Delta_{reaction} + 3.00\Delta_{curl}$ 90 day value = $1.80\Delta_{reaction} + 3.00\Delta_{curl}$

- 120 day value = $1.60\Delta_{reaction} + 3.00\Delta_{curl}$
 - 180 day value = $1.55\Delta_{reaction} + 3.00\Delta_{curl}$
 - 240 day value = $1.50\Delta_{reaction} + 3.00\Delta_{curl}$
 - 360 day value = $1.40\Delta_{reaction} + 3.00\Delta_{curl}$
 - 720 day value = $1.25\Delta_{reaction} + 3.00\Delta_{curl}$

1440 day value = $1.00\Delta_{reaction} + 3.00\Delta_{curl}$ (theoretical; included here for illustrative purposes only.)

Calculate the difference between Adjustment "SC" values of the 0-day and 720-day. If this difference is less than or equal to 1/2", it is reasonable to assume that hinge curl effects are negligible, and that a time dependent camber table is not necessary. However, it is still advisable to utilize the recommendation described in the section on "Construction Details".



Adjustment "LC" is the profile adjustment required for the Long Cantilever (may be positive or negative value).

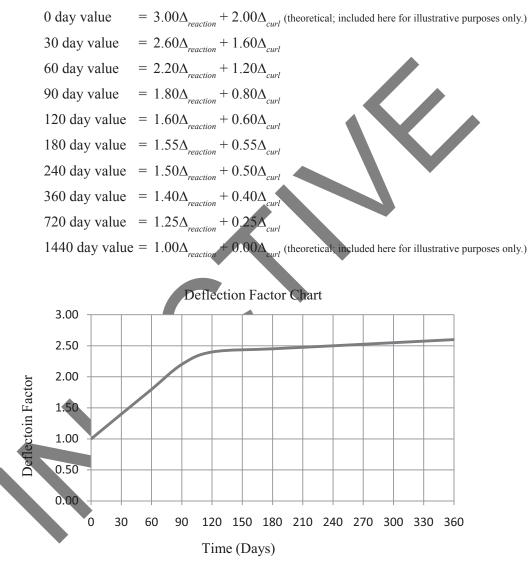


Figure 5 Deflection Factor Chart.

The long term effects that are incorporated into the Adjustment "SC" and Adjustment "LC" calculations are derived from the Deflection Factor Chart (Figure 5). The Deflection Factor curve represents the total amount of deflection a cast-in-place prestressed concrete element undergoes with respect to time. The long term effect of creep and shrinkage is assumed to



result in a total deflection that is three times that of immediate elastic deflection, and this will occur over a four year period. Although Figure 5 shows up to 360 days, the Deflection Factor curve approaches 3.0 by day 1440. The curve starts at a value of 1.00 since it represents immediate elastic deflection for a given load at day "0". Day "0" is considered the day that the short hinge is prestressed.

The Deflection Factor chart may be used thus:

On day "0", $\Delta_{total, day 0} = 1.00 \Delta_{elastic}$.

If that given load is left for 60 days after day "0", the deflection factor grows to 1.80, thus:

On day "60", $\Delta_{total, day 60} = 1.80 \Delta_{elastic}$.

Adjustment "SC" and Adjustment "LC" utilize the Deflection Factor graph in the same manner as described. The difference is that the adjustment calculations capture a change in the loading condition sometime after the initial loading condition. The initial loading condition is the prestressing and self weight of the short cantilever, and the change in loading condition pertains to the addition of the transfer load from the long cantilever. The time value is the elapsed time between the initial loading condition and the time that the long cantilever load is transferred to the short cantilever. Each loading component results in an elastic deflection (Δ_{can} and $\Delta_{reaction}$), and each deflection component is multiplied by a deflection factor associated with the time that the load is applied.

For example, the 30-day value for Adjustment "SC" is $2.60\Delta_{reaction} + 3.00\Delta_{curl}$. The factor 3.00, applied to Δ_{curl} , represents the notion that the short cantilever will be loaded by its self weight and prestressed immediately after it has cured sufficiently, therefore the maximum deflection factor of 3.00 is applied. The $2.60\Delta_{reaction}$ represents the notion that 30 days has elapsed since prestressing the short cantilever, and the component of the deflection factor representing creep and shrinkage, in the amount of 0.40 (1.40-1.00), has already occurred in the short cantilever. Thus, the transfer load component, $\Delta_{reaction}$, will only be subjected to the remaining deflection factor of 2.60 (3.00-0.40).

Accordingly, Adjustment "LC" for the 30-day value is $2.60\Delta_{reaction} + 1.60\Delta_{curl}$. Adjustment "LC" signifies the amount of camber correction that the long cantilever needs in order to match the location of the short cantilever when the load is transferred. The factor 1.60, applied to Δ_{curl} , represents the notion that at 30 days, the short hinge has already undergone $1.40\Delta_{curl}$ of deflection, and what remains is $(3.00-1.40)\Delta_{curl}$. The $2.60\Delta_{reaction}$ signifies that the transfer load component, $\Delta_{reaction}$, will only be subjected to the remaining deflection factor of

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2.60 (3.00-0.40). In short, Adjustment "LC" results in the contact of Points 2 and 3 (Figure 6) when the transfer load from the long cantilever occurs on the anticipated schedule.

The factors used for Adjustment "SC" and Adjustment "LC", and the calculation methods presented herein, may be adjusted if more accurate site-specific and material-specific deflection curves can be generated.

Development of the Plan Camber Diagram

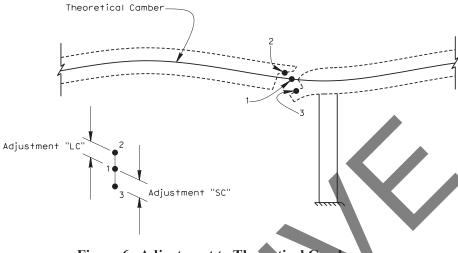
This step involves incorporating the Adjustment "SC" and Adjustment "LC" values with the theoretical camber of the hinge span. Once the Adjustment values are added to the theoretical camber of the span, we refer to the values as Camber "SC" and Camber "LC". Load transfer from the long cantilever will usually occur sometime in the period of 30 to 180 days after prestressing the short cantilever span. Thus, tabulated camber values shall be shown on the plans. The designer may optionally provide camber values for up to 720 days.

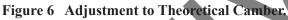
Figure 6 shows the relationship between Adjustment "SC", Adjustment "LC", and the theoretical camber of the span.

- **Point 1** Represents the theoretical adjustment to theoretical camber if load transfer is immediate.
- **Point 2** Represents the adjustment to theoretical camber, up or down, at the end of the long cantilever, which is dependent on the time of load transfer (Adjustment "LC").

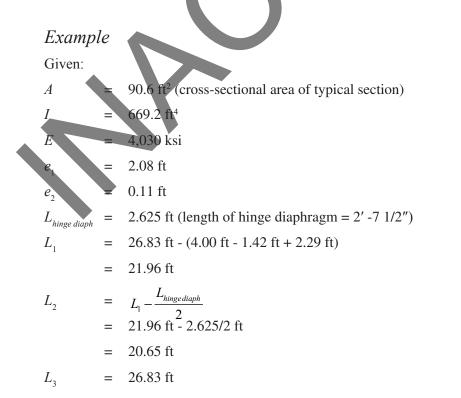
Point 3 - Represents the adjustment to theoretical camber up or down at the end of the short cantilever (Adjustment "SC").



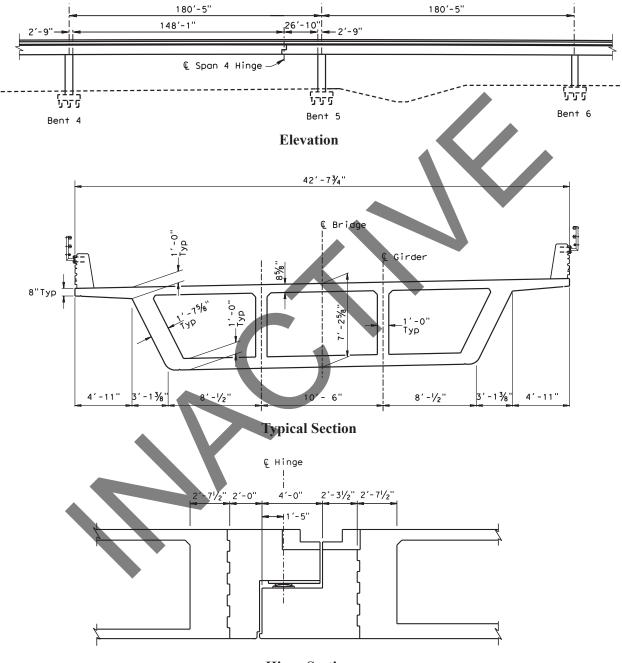




The generation of tabulated camber values and the Camber Diagram is best illustrated in an example. The following example shows the steps involved in calculating the Adjustment "SC" and Adjustment "LC" values, as well as incorporating them with the theoretical camber to generate Camber "SC" and Camber "LC" values for various durations of time before load transfer.







Hinge Section

Figure 7 Example bridge.



Step 1 – Calculate Δ_{DI} $= 90.6 \text{ ft}^2 \times 0.15 \text{ kip/ft}^3 = 13.6 \text{ kip/ft}$ w $A_{voids} = 133.4$ ft² (area of voids of the typical section) Р $= A_{voids} \times L_{hinge \, diaph} \times 0.150 \, \mathrm{kip}/\mathrm{ft}^3$ = 133.4 ft² × 2.625 ft × 0.150 kip/ft³ = 52.5 kips (hinge diaphragm weight on short cantilever side only) $\Delta_{DL} = \frac{wL_1^3}{24EI}(4L_3 - L_1) + \frac{PL_2^2}{6EI}(3L_3 - L_2)$ $\Delta_{DL} = \frac{(13.6)(21.96^3)}{(24)(4030)(669.12)(144)} [4(26.83) - 21.96](12)$ $+\frac{52.5(20.65^2)}{6(4030)(669.2)(144)}[3(26.83)-20.65](12)$ $\Delta_{DL} = 0.016'' + 0.007'' = 0.023''$ Step 2 – Calculate Δ_{ps} $P_i = 8,992 \text{ kips}$ FC = 0.876 $\Delta_{PS} = \frac{-P_{I}L_{1}FC}{12EI} \left[e_{1}(8L_{3} - 3L_{1}) + e_{2}(4L_{3} - 3L_{1}) \right]$ $\frac{-8992(0.876)(21.96)}{12(4030)(669.2)(144)} \Big\{ 2.08 \big[8(26.83) - 3(21.96) \big] + 0.11 \big[4(26.83) - 3(21.86) \big] \Big\} (12)$ $= -0.000037 \times 3767.75 = -0.140''$ Step 3 – Calculate $\Delta_{reaction}$ T = 809 kips**TT** 3

$$\Delta_{reaction} = \frac{IL_3}{3EI}$$



 $\Delta_{reaction} = \frac{809(26.83^3)}{3(4030)(669.2)(144)} (12)$ $\Delta_{reaction} = 0.161''$

Step 4 – Calculate Δ_{curl} $\Delta_{curl} = \Delta_{DL} + \Delta_{PS}$ $\Delta_{curl} = 0.023 + (-0.140) = -0.117''$

Step 5 – Calculate Adjustment "SC"

	reaction curl	
Adjustment "SC", 0-day	$= 3.00 \times (0.161) + 3.00 \times (-0.117)$	= 0.13"
Adjustment "SC", 30-day	$= 2.60 \times (0.161) + 3.00 \times (-0.117)$	= 0.07"
Adjustment "SC", 60-day	= 2.20 × (0.161) + 3,00 × (-0.117)	= 0.00"
Adjustment "SC", 90-day	$= 1.80 \times (0.161) + 3.00 \times (-0.117)$	= -0.06"
Adjustment "SC", 120-day	$= 1.60 \times (0.161) + 3.00 \times (-0.117)$	= -0.09"
Adjustment "SC", 180-day	$= 1.55 \times (0.161) + 3.00 \times (-0.117)$	= -0.10"
Adjustment "SC", 240-day	$= 1.50 \times (0.161) + 3.00 \times (-0.117)$	= -0.11"
Adjustment "SC", 360-day	$= 1.40 \times (0.161) + 3.00 \times (-0.117)$	= -0.13"
Adjustment "SC", 720-day	$= 1.25 \times (0.161) + 3.00 \times (-0.117)$	= -0.15"

Calculate the difference between Adjustment "SC" values of the 0-day and 720-day: Difference = 0.13 - (-0.15) = 0.28''

This difference is less than 1/2'', so hinge curl effects need not be considered. However, the time dependent camber table and diagram will be generated for demonstrative purposes.



Step 6 - Calculate Adjustment "LC"

 $\Delta_{reaction}$ Δ_{curl} $= 3.00 \times (0.161) + 2.00 \times (-0.117)$ Adjustment "LC", 0-day = 0.25''Adjustment "LC", $30\text{-day} = 2.60 \times (0.161) + 1.60 \times (-0.117)$ = 0.23''Adjustment "LC", 60-day = $2.20 \times (0.161) + 1.20 \times (-0.117)$ - 0.21" = 0.20" Adjustment "LC", 90-day = $1.80 \times (0.161) + 0.80 \times (-0.117)$ Adjustment "LC", 120-day = $1.60 \times (0.161) + 0.60 \times (-0.117)$ = 0.19" Adjustment "LC", 180-day = $1.55 \times (0.161) + 0.55 \times (-0.117)$ - 0.19" Adjustment "LC", 240-day = $1.50 \times (0.161) + 0.50 \times (-0.117)$ = 0.18''Adjustment "LC", 360-day = $1.40 \times (0.161) + 0.40 \times (-0.117)$ = 0.18''Adjustment "LC", 720-day = $1.25 \times (0.161) + 0.25 \times (-0.117)$ = 0.17''

Step 7 – Obtain Long Cantilever Camber from CTBridge at 1/4 points The camber at 1/4 points includes a deflection factor of 3.0.

LC $_{0.25}$ (at 37.7') = 0.87" LC $_{0.50}$ (at 75.4') = 1.19" LC $_{0.75}$ (at 113.1') = 1.02" LC $_{1.00}$ (at 150.8') = 0.63" SC (at 150.8') = 0.63"

Where, $LC_{0.25}$ represents unadjusted camber at the quarter point along the length of the long cantilever calculated from the CTBridge program. SC is the unadjusted camber at the tip of the short cantilever. SC, by definition, is equal to $LC_{1.00}$. However, for a reverse configuration, in which the short cantilever is on the left side of the span, SC is equal to $LC_{0.00}$.



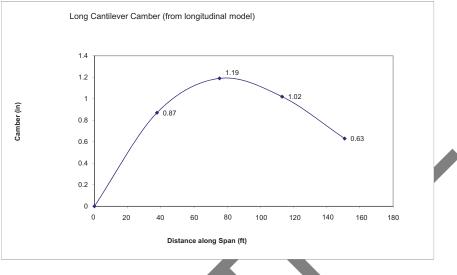


Figure 8 Long Cantilever Camber.

Step 8 – Adjust Short Cantilever Camber for Time-Dependent Correction

This step calculates Camber "SC" for the short cantilever. Because the calculations for Δ_{curl} assume the adjacent spans are supported on falsework, it negates the effects of joint rotation on span deflection. Generally, unbalanced spans will generate deflections that differ from the Adjustment "SC" values. Therefore, one can estimate the deflection due to joint rotation, δ_{sC} , by calculating the difference between the camber determined by the longitudinal analysis program, CTBridge, and Adjustment "SC" at 0-day (see Figure 9).

Adjustment "SC" at 0-day = 0.13" (from Step 5)

= SC - Adjustment "SC" = 0.63" - 0.13" = 0.50"

Camber "SC" values are calculated as such:

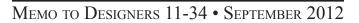
SC, 30-day	=	Adjustment "So	$C'' + \delta_{SC}$
	=	0.07'' + 0.50''	= 0.57"
SC, 60-day	=	0.00'' + 0.50''	= 0.50"
SC, 90-day	=	-0.06" + 0.50"	= 0.44"
SC, 120-day	=	-0.09" + 0.50"	= 0.41"
SC, 180-day	=	-0.10" + 0.50"	= 0.40"

 Q_{SC}



SC, 240-day = $-0.11'' + 0.50'' = 0.39''$				
SC, $360\text{-day} = -0.13'' + 0.50'' = 0.37''$				
SC, 720-day = $-0.15'' + 0.50'' = 0.35''$				
Camber Line based Profile Line				
t + sc osc osc osc osc osc osc osc osc osc				
Figure 9 Short Cantilever Camber. Step 9 – Adjust Long Cantilever Camber for Time-Dependent Correction				
Camber "LC" is calculated similarly to Camber "SC"				
At $LC_{1,0}$ (at hinge):				
Camber Line based on longitudinal model Profile Line				
Bent 5 Bent 5 Bent 6 Bent 6 Bent 4 Bent 4 C ¹ /2 LC ¹ /2 C				
Long Cantilever, L Cantilever				

Figure 10 Long Cantilever Camber.





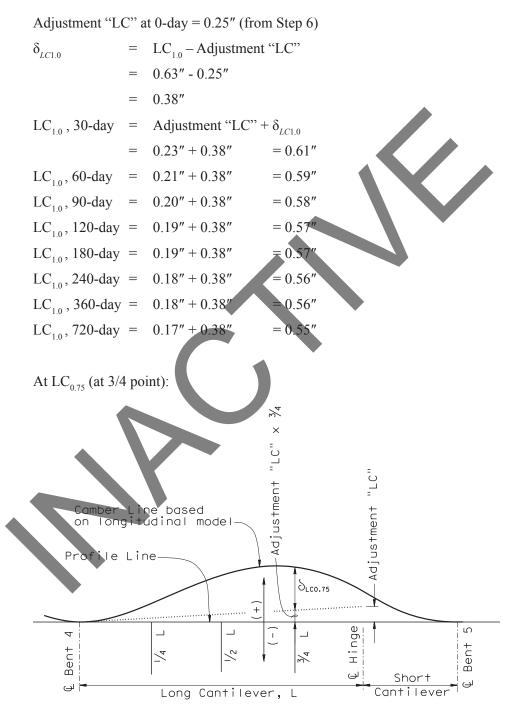


Figure 11 Long Cantilever Camber at 3/4 Point.



At locations along the long cantilever, other than at the hinge, time dependent camber values are adjusted by linearly interpolating Adjustment "LC". At the 3/4 L point, Adjustment "LC" is factored by 3/4. Note that if the hinge span configuration is reversed, in which the short hinge is on the left, the factor applied to Adjustment "LC" at the 3/4 L point would be 1/4 instead of 3/4.

Adjustment "LC" at 0-day = $0.25'' \times 3/4 = 0.19''$ = $LC_{0.75} - (Adjustment "LC" \times 3/4)$ $\delta_{_{LC0.75}}$ = 1.02'' - 0.19''= 0.83" $LC_{0.75}$, 30-day = Adjustment "LC" × 3/4 + $\delta_{LC0.75}$ $= 0.23'' \times 3/4 + 0.83'' = 1.00''$ $LC_{0.75}, 60\text{-day} = 0.21'' \times 3/4 + 0.83'' = 0.99''$ $LC_{0.75}, 90\text{-day} = 0.20'' \times 3/4 + 0.83'' = 0.98''$ $LC_{0.75}, 120$ -day = $0.19'' \times 3/4 + 0.83'' = 0.97''$ $LC_{0.75}$, 180-day = $0.19'' \times 3/4 + 0.83'' = 0.97''$ $LC_{0.75}$, 240-day = $0.18'' \times 3/4 + 0.83'' = 0.97''$ $0.18'' \times 3/4 + 0.83'' = 0.97''$ $LC_{0.75}$, 360-day $LC_{0.75}$, 720-day = 0.17" × 3/4 + 0.83" = 0.96" At LC_{0.50} (at 1/2 point) Adjustment "LC" at 0-day = $.25'' \times 1/2 = 0.13''$ (A diustment "I C" \times 1/2) IC

$$EC_{0.50} = EC_{0.50} - (Adjustment EC \times 1/2)$$

$$= 1.19'' - 0.13''$$

$$= 1.06''$$

$$EC_{0.50}, 30 \text{-day} = \text{Adjustment "}\text{LC"} \times 1/2 + \delta_{LC0.50}$$

$$= 0.23'' \times 1/2 + 1.06'' = 1.18''$$

$$EC_{0.50}, 60 \text{-day} = 0.21'' \times 1/2 + 1.06'' = 1.17''$$

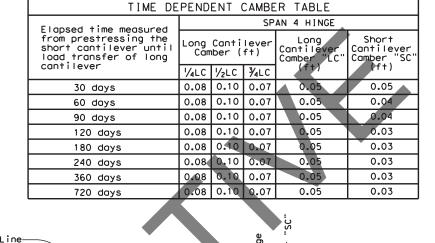
$$EC_{0.50}, 90 \text{-day} = 0.20'' \times 1/2 + 1.06'' = 1.16''$$

$$EC_{0.50}, 120 \text{-day} = 0.19'' \times 1/2 + 1.06'' = 1.15''$$



LC _{0.50} , 180-day	=	$0.19'' \times 1/2 + 1.06'' = 1.15''$
LC _{0.50} , 240-day	-	$0.18'' \times 1/2 + 1.06'' = 1.15''$
LC _{0.50} , 360-day	-	$0.18'' \times 1/2 + 1.06'' = 1.15''$
LC _{0.50} , 720-day	=	$0.17'' \times 1/2 + 1.06'' = 1.15''$
At LC _{0.25} (at 1/4	1 poi	int):
Adjustment "Le	C" a	$t \ 0 - day = .25'' \times 1/4 = 0.06''$
$\delta_{_{LC0.50}}$	=	$LC_{0.25}$ – (Adjustment "LC" × 1/4)
	=	0.87" - 0.06"
	=	0.81"
LC _{0.25} , 30-day	=	Adjustment "LC" $\times 1/4 + \delta_{LC0.50}$
	=	$0.23'' \times 1/4 \neq 0.81'' = 0.87''$
LC _{0.25} , 60-day	=	$0.21'' \times 1/4 + 0.81'' = 0.86''$
LC _{0.25} , 90-day	=	$0.20'' \times 1/4 + 0.81'' = 0.86''$
LC _{0.25} , 120-day	/ =	$0.19'' \times 1/4 + 0.81'' = 0.86''$
LC _{0.25} , 180-day	r=	$0.19'' \times 1/4 + 0.81'' = 0.86''$
LC _{0.25} , 240-day	/=	$0.18'' \times 1/4 + 0.81'' = 0.86''$
LC _{0.25} , 360-day	/ =	$0.18'' \times 1/4 + 0.81'' = 0.85''$
LC _{0.25} , 720-day	, =	$0.17'' \times 1/4 + 0.81'' = 0.85''$





The Camber Diagram and Time Dependent Camber Table for the hinge span are shown below (Figure 12).

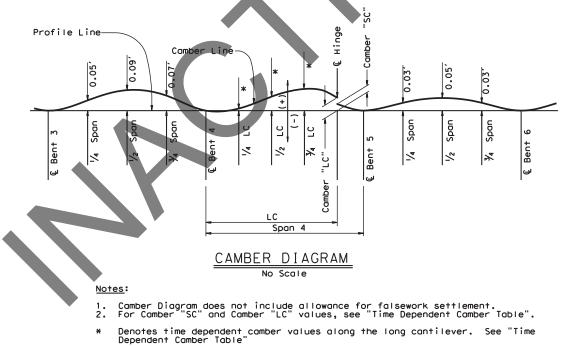


Figure 12 Camber Diagram and Time Dependent Camber Table.



Construction Details

Although efforts are made to provide accurate time dependent camber adjustments for hinge spans, it is prudent to provide construction details that accommodate variations to the final product. Therefore, it is recommended that designers provide an additional 1" of concrete cover for the top deck reinforcement so that grinding can be performed if necessary. The additional cover should extend over a distance no less than the full length of the hinge diaphragm.

Original signed by Barton J. Newton

Barton J. Newton State Bridge Engineer Deputy Division Chief, Structure Policy & Innovation Division of Engineering Services