

20.32 SEISMIC DESIGN GUIDANCE FOR TUNNELS

20.32.1 GENERAL

This BDM provides seismic design guidance to analyze, design, and detail new tunnels.

20.32.2 DEFINITIONS

Tunnel — For seismic design purposes, an enclosed roadway with vehicle or pedestrian access that is restricted to portals regardless of the type of structure or method of construction.

Ovaling/Racking Deformation — Deformation/strain in the cross-section plane of the tunnel caused primarily by seismic shear waves propagating perpendicular to the tunnel's longitudinal direction, causing deformation in the plane of the tunnel cross section. Vertically propagating shear waves are generally considered the most critical type of wave for this mode of deformation.

Axial Deformation — Compressive and tension deformation/strain in the longitudinal direction of the tunnel caused by components of seismic waves that propagate along the tunnel's longitudinal direction.

Curvature Deformation— Flexural and shear deformation/strain in the longitudinal direction of the tunnel caused by components of seismic waves that propagate along the tunnel's longitudinal direction.

For definitions not included, see AASHTO, 2017, and SDC.

20.32.3 NOTATIONS AND ABBREVIATIONS

- C = Compressibility ratio
- *D_{avg}* = average outside diameter of circular tunnel or average of height & width of non-circular tunnel
- *F* = Flexibility ratio
- H_{soil} = soil cover thickness measured from ground surface to top of the tunnel
- *L* = longitudinal length of tunnel measured along the centerline of the roadway
- s_{CL} = cross tie spacing in the longitudinal direction
- s_{CT} = cross tie spacing on a vertical plane normal to the longitudinal direction
- s_L = longitudinal reinforcement spacing on a vertical plane normal to longitudinal direction
- s_{T} = transverse reinforcement spacing in the longitudinal direction



V_{EQ}	=	Seismic Shear Demand			
Vn	=	Nominal Shear Capacity			
γmax	=	maximum free-field ground shear strains			
Esteel	=	peak strain demand for reinforcing steel from analysis			
Econcrete=		peak strain demand for confined concrete from analysis			
ρι	=	longitudinal reinforcement ratio			
ρτ	=	transverse reinforcement ratio			
FEE	=	Functional Evaluation Earthquake			
GD	=	Geotechnical Designer			
PSDC =		Project-Specific Design Criteria			
PSHA =		Probabilistic Seismic Hazard Analysis			
SEM	=	Sequential Excavation Method (also known as New Austrian Tunneling			
		Method, NATM)			
SCM	=	Seismic Critical Member			
SEE	=	Safety Evaluation Earthquake			
SSI	=	Soil Structure Interaction			
ТВМ	=	Tunnel Boring Machine			

For notations not included, see AASHTO, 2017 and SDC.

20.32.4 SEISMIC ANALYSIS

20.32.4.1 Ground Motion Parameters

The displacement time histories should be prepared by GD and should incorporate threedimensional effects of ground motions, local site effects, including spatially varying effects along the tunnel alignment, and wave traveling/phase shift effects. Seismic analysis of tunnels requires seven sets of free-field ground displacement time history profiles in the longitudinal, transverse, and vertical directions within the limits of the entire soil-tunnel structure system.

To develop the displacement time histories, a site response analysis may be needed to account for subsurface conditions, including groundwater, complex site geometry, and dynamic soil/rock properties at the project site. Seed time histories at the bedrock level that spectrally match the design ARS are required to perform site response analysis.

The seed time histories may be obtained from records under conditions comparable to the seismic characteristics at the site. The seismic characteristics include tectonic environment – subduction zone, shallow crustal faults in the Western U.S.; seismic magnitude, type of faulting; seismic source-to-site distance; and local site conditions.



Rayleigh damping may be used to model overall damping in the site response analysis. The mass and stiffness proportional damping coefficients are determined using a damping ratio of 5% at two frequencies: the first mode of the site's natural frequency and a frequency equal to five times the first mode frequency (Kwok, 2007).

The detailed site response analysis procedure is as follows:

- 1. Evaluate subsurface conditions and select a reference bedrock elevation or the elevation of competent soil that is at least 50 feet below the tunnel invert.
- 2. Determine the V_{s30} of the ground conditions at the reference rock elevation.
- 3. Develop design ARS for the FEE and SEE events by performing PSHA for the respective return periods, using V_{s30} of the reference bedrock and the site location as inputs.
- 4. Select seven sets of seed time histories based on the controlling seismic event characteristics (i.e., earthquake magnitude and distance). The seed time histories should be rotated to principal axes. Near fault effects should be included as needed.
- 5. Perform spectrum matching of the rotated seed time histories to the reference bedrock ARS developed in Step 3 above. Preserve seismic characteristics (such as the velocity pulse) of the seed time histories during spectrum matching.
- 6. Perform site response analysis that reflects the effects of wave propagation through the site-specific soil/rock conditions from the reference bedrock elevation, past the tunnel depth, and up to the ground surface. Outputs from the site response analysis are either depth- and spatially-varying time histories that can be used for time history SSI analysis or free-field shear strains of the ground surrounding the tunnel and associated strain-compatible shear modulus values that can be used for 2*D* Pseudo-Dynamic (push-over) analysis as specified Section 20.32.4.2.2.1.
- 7. As an alternative to step 6, an analysis using a coupled model shown in Section 20.32.4.2.1 can be performed. The base of the model should be the reference bedrock elevation. The spectrum-compatible motions at the bedrock elevation developed in Step 5 should be applied as inputs at the base.

20.32.4.2 Evaluation of Ground Shaking Effects

Seismic analysis and design of underground structures are mostly controlled by the deformation of the surrounding ground as opposed to loads typically applied for aboveground structures. During a seismic event, underground structures move with the surrounding ground. The structures must be designed to accommodate the deformations imposed by the ground.

Seismic design and analysis of tunnels are based on three primary modes of tunnel deformation induced by the FEE and SEE events. The three primary deformation modes are:



- Ovaling and Racking
- Axial
- Curvature



(c) Axial (Longitudinal) Deformation (d) Curvature (Bending) Deformation

Figure 20.32.4.2-1 Tunnel Deformation Modes (AASHTO, 2017)

For tunnels constructed in uniform competent ground conditions, seismically induced deformations/strains are generally small. However, the deformations and the associated force demands can be substantial for tunnels under the following conditions:

- Loose, soft, or non-uniform ground
- Abrupt changes in stiffness of structures (i.e., tunnel transition or junction)
- Abrupt changes in stiffness of the surrounding ground profile (i.e., soil-to-rock transition)
- Ground failure effects due to liquefaction, fault crossing, or landslide

The above geologic and seismic conditions should be considered when selecting tunnel alignment and location.

20.32.4.2.1 Analysis

Road tunnels are classified as either long or short tunnels for seismic analysis purposes. Long tunnels have a ratio of the longitudinal length (L) to the average diameter or average



of width & height (D_{avg}) equal to or greater than 8 ($L/D_{avg}>8$). Otherwise, tunnels are considered short.

For long tunnels, a three-dimensional site-specific time history analysis is required. The entire soil-structure system is subjected to dynamic excitations using free-field ground displacement time histories as input at the boundary of the soil-structure system in longitudinal, transverse, and vertical directions. The three orthogonal ground motions need not be applied to the model simultaneously for analysis. Instead, one motion at a time is applied in a direction where demands need to be assessed.

For short tunnels, simplified methods are allowed if uniform and competent ground conditions are present within the distance of three times the D_{avg} of the tunnel measured from the outside face of tunnel lining and no ground failures described in Section 20.32.4.3 are expected. A 2-D pseudo-dynamic analysis, as described in Section 10.8.3.1.3 of AASHTO, 2017, may be used to evaluate ovaling/racking in the transverse plane if the effect of the structure inertia is insignificant.

In the simplified methods, the shear demand associated with curvature deformation in the longitudinal direction is $1.2V_{EQ}$. The V_{EQ} is a peak ground spectral acceleration at the tunnel's mid-height times the tunnel's mass. The nominal shear capacity in the longitudinal direction, V_n , is determined as defined in SDC.

For both long and short tunnels, geotechnical and structural modeling must include the SSI effects as specified in AASHTO, 2017, and FHWA, 2009.

20.32.4.2.1.1 Geotechnical modeling

Use the same set of geological material engineering properties throughout the analysis and design. For de-coupled modeling, develop discrete ground springs in the longitudinal, transverse, and vertical directions. In deriving ground springs, account for ultimate frictional (drag) resistance between tunnel structures and ground to allow for a slippage mechanism. The nonlinear and hysteretic behavior of the ground should be reflected in the ground springs. The discrete ground springs should be spaced less than two times the tunnel lining thickness.

20.32.4.2.1.2 Structural modeling

The structural components may be modeled as a beam or solid elements. For both long and short tunnels, include geometric and material nonlinearities of the tunnel structure in the model. Incorporate plastic hinges where frame action (i.e., slab-to-wall connection for cut and cover tunnel) of structural components is expected in a transverse direction. For site-specific time history analysis, Rayleigh damping is recommended to model general damping per SDC.

If large axial and/or curvature responses are expected (i.e., ground failures specified in Section 20.32.4.3), nonlinearity in the longitudinal direction needs to be captured by including transverse (circumferential) seismic joints that are modeled as nonlinear springs.



Two different numerical modeling techniques can be used for seismic analysis: 1) decoupled modeling and 2) coupled modeling.

In the de-coupled modeling, ground deformations around the tunnel are calculated using site response analysis in a geotechnical model. The calculated ground deformations are then used to generate SSI models. The SSI is then applied to the structure model that includes tunnel lining and other relevant structural elements. The following steps summarize the de-coupled approach:

- 1. Perform two-dimensional geotechnical site response analysis to obtain time histories of seismic displacements around the perimeter of the tunnel. This step is also referred to as "scattering" analysis, where a cavity is included in the site response analysis model to represent the tunnel opening. The input displacement time histories required for such analyses are described in Section 20.32.4.1.
- 2. Develop SSI models, both normal and tangential components, at structural nodes around the perimeter of the tunnel. Where appropriate, tangential SSI models should allow for slippage, and normal SSI models should allow for gapping.
- 3. Develop a structural model representing the tunnel lining and other relevant structural elements as discussed above. The model should be able to adequately account for the nonlinearity of the structure.
- 4. Apply the normal and tangential components of displacement time histories obtained from Step 1 to the structural model to analyze structure response.

The coupled modeling incorporates both geotechnical and structural components in one finite element or a finite difference numerical model. Ground seismic response, deformations, and structural response are analyzed in the single model. The ground surrounding the tunnel is modeled using continuum elements with appropriate nonlinear constitutive models and properties, and structural components, such as tunnel liner and relevant internal components, are modeled using nonlinear structural behavior models. The soil-structure interface should be defined to simulate the interaction between the tunnel and the surrounding ground. Overall model dimensions should be determined to reduce boundary effects.

20.32.4.2.2 Methods for Independent Check Analysis

Section 20.32.4.2.2.1 describes a closed-form solution that may be used as the independent and preliminary check. The results of the closed-form solution may serve as a preliminary guidepost for the work to be performed as described in Section 20.32.4.2.1

20.32.4.2.2.1 Ovaling and Racking Deformation

Select a suitable independent check analysis from the following.



Closed-Form Solution

A closed-form solution analysis may be performed in accordance with Section 13.5.1.1 thru 13.5.1.3 of FHWA, 2009, if the design meets the following conditions:

- Structural cross-sections are uniform along the longitudinal direction, and geologic materials surrounding the tunnel structure are uniform.
- Cross-sections are rectangular, circular, horseshoe, or oval-shaped.
- There are no adjoining structures or interactions with other tunnels, and
- A short tunnel.

This analysis provides seismic displacement demands under FEE and SEE, accounting for SSI effects. In this analysis, seismic displacement demands on tunnel lining are defined as an elongation in the tunnel diameter for bored circular sections and a relative lateral displacement at the top of the tunnel to the tunnel bottom for rectangular sections. The calculation steps are as follows:

- 1. Calculate the expected free-field ground shear strains caused by the vertically propagating shear waves of the design earthquakes for both FEE and SEE. The maximum free-field ground shear strains, γ_{max} , should be derived at the elevation of the tunnel.
- 2. Estimate a displacement demand considering *F* and *C* (for circular sections).

2-D Pseudo-Dynamic or Dynamic Time History Analysis (Numerical Modeling)

Perform nonlinear inelastic pseudo-dynamic (push-over) analysis or dynamic time history analysis for the tunnel structure. A push-over analysis may be performed by applying the expected peak seismic displacement profile to either a de-coupled or coupled model if the effect of the structure inertia is insignificant. Otherwise, dynamic time history analyses are required using the ground motion time history sets.

20.32.4.2.2.2 Axial and Curvature Deformation

Select a suitable independent check analysis method from the following list.

Closed-Form Solution

Perform closed-form solution analysis following Section 13.5.2.1 and 13.5.2.2 of FHWA, 2009, if the design meets the following conditions:

- Structural cross-sections are uniform along the longitudinal direction, and geologic materials surrounding the tunnel structure are uniform.
- Cross-sections are rectangular, circular, or horseshoe shaped.
- There are no jointed structures or interactions with other tunnels, and
- A short tunnel.



3-D Pseudo-Dynamic or Dynamic Time History Analysis (Numerical Modeling)

Perform 3-D pseudo-dynamic or dynamic time history analysis if a tunnel has crosssections, structural stiffness, or geotechnical conditions that vary abruptly along the tunnel alignment.

Highly variable subsurface conditions or variable structural stiffness include:

- Transitions at portals
- Transitions to differing tunnel cross sections or connections to another tunnel
- Intersections of cross passages
- Changes in ground motion due to both wave propagation and rock attenuation effects
- Change of ground conditions
- When a tunnel traverses two distinct geologic materials with sharp contrast in stiffness, such as passing through a soil/rock interface

20.32.4.3 Evaluation of Ground Failure Effects

The major risk to tunnel structures is the potential for large permanent ground movements caused by unstable ground conditions, such as liquefaction, lateral spreading, landslides, and fault displacements. GD should evaluate the probable magnitude of permanent deformation and wedge block loading caused by potential ground failure. The tunnel structure and its joints must be designed for the deformation and the wedge block loading.

20.32.4.3.1 Fault Rupture

If a tunnel can be designed to accommodate the magnitude of the fault displacement, or the sheared fault zone is substantially wide such that the displacement is dissipated gradually over a distance, perform analysis and design with the steps below instead of relocating the tunnel alignment to avoid the fault.

- Evaluate the angle of the fault plane intersecting the tunnel, the width of the fault, and the magnitude and orientation of the fault offset.
- Evaluate the free-field fault displacement where the fault zone crosses the tunnel.
- Use the finite element method to evaluate the effects of the displacement on the tunnel structures. Incorporate the nonlinear behavior of the structural elements and ground in the finite element model.

20.32.4.3.2 Landslide and Liquefaction

For potential effects of a landslide or lateral spreading movements intersecting the tunnel, follow the procedure described in Section 20.32.4.3.1. Evaluate the impact of liquefaction on the tunnel, including resulting pressure on the tunnel and potential uplift or settlement.



20.32.5 SEISMIC DESIGN OF TUNNEL STRUCTURES

The static loads, such as structure dead load plus earth and water pressure, should be combined with the seismic load effect in the seismic design.

Seismic design should be performed for the temporary structures, portals and portal walls, and permanent tunnel structures.

20.32.5.1 Temporary Structures

The temporary structures include temporary (initial) support elements and temporary support of excavation defined in AASHTO, 2017, such as bracing and shoring for cut and cover tunnel, vertical shafts for TBM mobilization for a bored tunnel, and initial lining for SEM. Seismic design for these structures should be performed in accordance with the seismicity level and the target performance specified in Section 10.9 of AASHTO, 2017.

20.32.5.2 Portals and Portal Walls

See Section 10.10 of AASHTO, 2017, for the seismic design requirement.

20.32.5.3 Permanent Tunnel Structures

Components of tunnel structures that support the earth, roof slabs, or crown/shoulder, such as tunnel side walls and columns, are SCMs defined in SDC. The SCMs should be designed and detailed to form plastic hinges in the transverse direction such that ductile behavior can be expected under SEE.

For the performance levels established in STP 20.32 Section 20.32.4.2, the SCMs should be designed not to exceed the following strain limits in plastic hinge regions of an SCM:

For FEE event: $\epsilon_{steel} < \epsilon_{ve}$ and $\epsilon_{concrete} < 0.003$

For SEE event: $\varepsilon_{steel} < \frac{1}{2} \varepsilon_{su}^{R}$ and $\varepsilon_{concrete} < \frac{2}{3} \varepsilon_{cu}$

For the tunnel lining segments outside the plastic hinge regions, the $\varepsilon_{concrete}$ induced by compressive deformation should be less than 0.003 under FEE and SEE levels.

The target damage state for a SEE event is between DS-2 (Minor Spalling and Possible Shear Cracks) and DS-3 (Extensive Cracks and Spalling) (Refer to SDC for definitions of DS-2 and DS-3). The performance evaluation criteria in this BDM are that a strain demand computed under the SEE is to be below a strain expected at a damage state between DS-2 to DS-3. The strain limits are set based on a combination of experimental data and past PSDC with a primary focus on shallow cut and cover tunnels.

Doyle Drive and Devil's Slide (Arup, 2009 & HNTB, 2005) used a similar performance level under the SEE event. Both projects used 2/3 of ε_{cu} as a strain limit for the SEE event. Note ε_{cu} is based on reduced ultimate hoop strain. The damage level at ε_{cu} is estimated to be between DS-3 and DS-4.



A 1/3 scale reinforced concrete tunnel specimen which was based on the as-built plans of the Doyle Drive Batter Tunnel, was tested at UCSD (Kim et al., 2016). The test results showed that the maximum strain of the reinforcements in the side walls was about 2/3 of ε_{su}^{R} when the major cracks occurred, and the concrete began to bulge.

The SCMs should carry the shear demand in the transverse and longitudinal directions by meeting a shear force ratio, D/C, less than 1.0. The roof slab, floor slab, and crown/shoulder should be designed as capacity protected members to withstand overstrength shear and flexural demand as specified in SDC.

The potential plastic hinges forming in the sidewalls due to the transverse and vertical deformation demands should be capable of carrying the longitudinal shear force demands.

20.32.6 SEISMIC DETAILING FOR TUNNEL STRUCTURES

20.32.6.1 Cut-and-Cover and Jacked Tunnels

Cut-and-cover and jacked tunnels are typically box type. Two layers of reinforcements in the SCMs and roof slabs should be provided. The main reinforcement for ovaling/racking is generally transverse reinforcing steel. Transverse reinforcement from the SCMs should be fully developed into the top & bottom slabs and should be #11 or smaller.

Confinement in plastic hinges consists of cross ties and longitudinal reinforcements. Reinforcement should be provided in compliance with confinement requirements in SDC. The confinements should be at least #4 bars. The confinements required for plastic hinge areas of SCMs should extend to the roof slab within the same plastic hinge length defined for the SCMs to prevent possible large-size debris from falling.

Cross ties and longitudinal reinforcement should have a minimum hook angle of 135 degrees on one end and a 90-degree hook on the opposite end. The hooks should alternate in both directions. The 90-degree hook of the cross-tie bars may be replaced with a "T-head" from the Caltrans Authorized Material List. Cross ties should hook around both transverse and longitudinal reinforcement.

The spacing and reinforcement ratio of longitudinal reinforcement, transverse reinforcement & cross ties should meet the requirements shown in Table 20.32.6.1-1.

No splices of longitudinal and transverse reinforcements are allowed within the plastic hinge region.

If splicing of longitudinal reinforcements is unavoidable, use staggered ultimate butt splices. Longitudinal reinforcement should have staggered service splices outside plastic hinge regions.

The transverse expansion joint should be detailed to sustain seismic shear and tension demands and remain watertight after a SEE event.



Reinforcement	Reinforcement Parameters	Outside plastic hinge region	Plastic hinge region
Transverse	ρτ	$0.004 \le \rho_T < 0.04$	
Reinforcement	ST	≤ 12"	≤ 8"
Longitudinal	ρι	0.0025 ≤ ρ _L	
Poinforcomont	SL	≤ 12"	≤ 6"
Reiniorcement	ρς	0.0025 ≤ ρ _C	
Cross Tios	SCL	≤ 12"	≤ 8"
CIUSS HES	SCT	≤ 12"	≤ 6"

Table 20.32.6.1-1: Reinforcement Ratio and Spacing Recommendations ^a

^a the above reinforcement recommendations are based on the tests at UC Irvine (Haroun et al., 1994).





20.32.6.2 Bored or Mined Tunnels

The final lining should have two layers of reinforcement. The transverse and longitudinal reinforcements for the final lining should be at least 0.004 times the gross sectional area. Reinforcement should be continuous and distributed uniformly across the lining section (See Fig. 20.32.6.2-1). The reinforcements may be service spliced outside the expected plastic hinge areas. The splices should be staggered. The spacing of the reinforcing steel should follow the recommendation in Table 20.32.6.1-1. Reinforcing steel size should be #11 bar or smaller.



For a two-pass lining tunnel which includes initial lining, the minimum thickness of the final concrete lining should be 16" for ease of concrete placement.

Single-pass lining tunnels are bolted and gasketed steel plate or precast concrete segmented liners for the bored tunnels. The segments along the length of the tunnel should be staggered so the segments' longitudinal (radial) joints are not continuous. Continuity of the lining should be provided by bolting the segments of the liners together to reduce the potential for segment fall out in case of a large tunnel deformation.

All tunnel joints should be watertight throughout the design life and should accommodate the shear and tension demand caused by the SEE event. Joint shear capacity should account for the influence of normal and bending moments on the shear capacity of the section.



Figure 20.32.6.2-1 Example of continuous hoop reinforcements



20.32.6.3 Immersed Tube

Immersed tunnels can be constructed using structural steel or precast concrete segmental lining. Tunnels constructed using structural steel, usually in the form of stiffened plates, perform compositely with the interior concrete as the structural system.

On the other hand, tunnels with concrete elements rely on concrete reinforced by reinforcing steel or prestressing strands. The connection joints are weak points of such tunnel during a SEE event. Design the joints to accommodate seismic displacements and remain watertight during and after a SEE event. The seismic joint system should be prooftested before accepting it for design.

20.32.6.4 Tunnel Seismic Joints for Special Conditions

At tunnel sections where the following conditions are expected, a seismic joint allowing differential movement should be used to avoid unacceptable shear and flexural demands:

- An abrupt change of tunnel or ground stiffness, such as the transition from the tunnel to portal or tunnel junctions and soil/rock transition.
- Presence of soft soils.
- Presence of the ground failures described in Section 20.32.4.3.

The seismic joints should be designed to support the expected static and dynamic earth and water pressure during and after the SEE event and should remain watertight.

In soil/rock transition, at least 2-foot-thick isolation from rock or rock ridge within the soil is required by overcutting and backfilling with competent materials. A sufficient length of the isolation in the longitudinal direction should be determined to prevent a sudden change in ground stiffness. If isolation is not possible, such as in a bored tunnel, design and install seismic joints in soil/rock transition.

Examples of such joint details for the above cases can be found in FHWA, 2004. The seismic joint should be proven by testing before accepting it for design.

20.32.7 REFERENCES

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